On Bayesian Estimation in Multi - Component Inverted Exponential Stress-Strength Model

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Abstract. In this paper we find the mathematical formula of the reliability system for Multi component in stress-strength model R_p , when the stress and strength are identical random variables distributed as inverted exponential distribution, in addition estimated the reliability R_p using : the maximum likelihood estimator (MLE) and the Bayes (B) method, and made a comparisons among the estimation methods using Monte-Carlo simulation depend on the mean square Error (MSE) criteria.
keyword; stress-strength, Inverted exponential distribution, Bayes Method, Monte Carlo Simulation

1. Introduction

The term stress – strength refers to a component that has a random strength X subject to a random stress Y to evaluate the reliability. The component fails if the stress applied to it exceeds the strength, while the component works whenever Y less than X (Y<X) [1], [9].
Several researchers assuming various life time distributions for the stress – strength random. However, because modern engineering systems may have more than two components. For instant bridges, car engines, air conditioning systems, the elements are often arranged in mechanical or logical series or parallel system [3] , [4].
In 2016 Karam exposed the reliability function for a component that has strength independent and three stresses, using two parallel components $R_1 = p(\max(X_1, X_2, X_3) < Y)$, $R_2 = p(x < \max(Y_1, Y_2))$.
In 1999 Hanagal considered the estimating $p[X_{k+1} < \min(X_1, \dots, X_k)]$ and $p[X_{k+1} < \max(X_1, \dots, X_k)]$ when $X_1, \dots, X_k$ with respect to common stress X_{k+1} [8].
In 2003 Hanagal obtained MLE for $p[X_{k+1} < \min(X_1, \dots, X_k)]$ and $p[X_{k+1} < \max(X_1, \dots, X_k)]$ such that $X_1,\dots, X_k$ are strength with respect to common stress X_{k+1} [7].
In this paper, a stress – strength model is a multiple component (k), we consider $(Z_1, Z_2, \dots, Z_k)$ he a random strength with respect to one of random stresses $(Z_{k+1}, Z_{k+2}, \dots, Z_{k+r})$. So, the estimation of $R_p = p[[\max(Z_{k+1}, Z_{k+2}, \dots, Z_{k+r})] < \max(Z_1, Z_2, \dots, Z_k)]$ is studied, we let $Z_1, Z_2, \dots, Z_{k+r}$ follow Inverted Exponential distribution. The estimation of stress – strength model based on MLE and Bayes estimator by using two functions non-Informal Jeffery and Gamma prior information when the system is parallel with assume that strengths and stresses follow Inverted Exponential distribution. On the other hand, we consider the Inverted Exponential distribution as life distribution with probability density function and distribution function on respectively [2], [6] ,[11], [12];

$$f(z, \alpha_1) = \frac{\alpha_1}{z^2} e^{-\alpha_1/z} \quad z \geq 0 \quad \alpha_1 > 0$$
$$F(z, \alpha_1) = e^{-\alpha_1/z} \quad z \geq 0 \quad \alpha_1 \geq 0$$

2. The Reliability System in Stress - Strength Parallel Model

Consider the Parallel stress -strength model is a combination of K^{th} independent components which are subject to one of r^{th} independent stresses. If (Z_1, Z_2, …, Z_k) be strengths having Inverted Exponential distribution with parameter α_1, with respect to one of the stresses (Z_{k+1}, Z_{k+2} ,…, Z_{k+r}) that follows Inverted Exponential distribution with parameter α_2 [9].

The probability density function and the distribution function of Z_i having the form:

$$f_i(z) = \frac{\alpha_1}{z^2}\ e^{-\alpha_1/z} \qquad z \geq 0\ \ \alpha_1 > 0$$

$$F_i(z) = e^{-\alpha_1/z} \qquad , i = 1,2,\dots,k$$

and

$$f_i(z) = \frac{\alpha_2}{z^2}\ e^{-\alpha_2/z} \qquad z \geq 0\ \ \alpha_2 > 0$$

$$F_i(z) = e^{-\alpha_2/z} \qquad , i = k+1, k+2, \dots, k+r$$

So, the distribution function of $V = \max\,(Z_1, Z_2, \dots, Z_k)$ will be:

$$G_1(v) = p(V < v) = p(Z_i < v)$$
$$= p(Z_1 < v).p(Z_2 < v)\dots\dots p(Z_k < v)$$
$$= \prod_{i=1}^{k} p(Z_i < v)$$
$$= \prod_{i=1}^{k} e^{-\alpha_2/v}$$
$$= e^{-k\,\alpha_1/v}$$

And the distribution function of $U = \max\,(Z_{k+1}, Z_{k+2}, \dots, Z_{k+r})$ will be:

$$G_2(u) = p(U < u)$$
$$= \prod_{j=k+1}^{k+r} p(Z_j < u)$$
$$= p(Z_{k+1} < u).p(Z_{k+2} < u)\dots p(Z_{k+r} < u)$$
$$= e^{-\alpha_2/u}.e^{-\alpha_2/u}\dots\dots e^{-\alpha_2/u} = e^{-r\,\alpha_2/u}$$

Then, the parallel stress – strength model is given by:

$$R_p = p[[\max(Z_{k+1}, Z_{k+2}, \dots Z_{k+r})] < \max(Z_1, Z_2, \dots Z_k)] = p(u < v)$$
$$= \int_0^{\infty} G_2(v) d\, G_1(v)$$
$$= \int_0^{\infty} e^{-r\,\alpha_2/v}\ d\,(e^{-k\,\alpha_1/v})$$
$$= k\alpha_1 \int_0^{\infty} e^{-r\,\alpha_2/v}.e^{-k\,\alpha_1/v}\frac{1}{v^2}dv$$
$$= k\alpha_1 \int_0^{\infty} e^{-(r\,\alpha_2 + k\,\alpha_1)/v}.\frac{1}{v^2}\,dv$$
$$= \frac{k\,\alpha_1}{r\,\alpha_2 + k\,\alpha_1}\ e^{-\frac{r\,\alpha_2 + k\,\alpha_1}{v}}$$

Therefore:

$$R_p = \frac{k\,\alpha_1}{r\,\alpha_2 + k\,\alpha_1} \qquad (1)$$

3.Maximum Likelihood Estimators Under α_1, α_2

Let $(Z_{i1}, Z_{i2}, \dots, Z_{ik})$ $(i = 1,2,\dots,n)$ be a strength random sample from Inverted Exponential distribution with parameter α_1 and $Z_{ik+1}, Z_{ik+2}, \dots, Z_{ik+r}$ $(i = 1,2,\dots,n)$ be a stresses random sample from Inverted Exponential distribution with parameter α_2.

Therefore, the likelihood function of $X_{ij} (i = 1,\dots,n, j = 1,\dots,k, k+1, k+2,\dots,k+r)$ is given as;

$$L(\alpha_1, \alpha_2 / X_{ij}) = \alpha_1^{nk} \prod_{i=1}^{n} \prod_{j=1}^{k} \frac{1}{X_{ij}^2} \quad e^{-\sum_{i=1}^{n}\sum_{j=1}^{k} \alpha_1 / X_{ij}} \, . \, \alpha_2^{nr} \prod_{i=1}^{n} \prod_{j=1}^{r} \frac{1}{X_{ij}^2} \quad e^{-\sum_{i=1}^{n}\sum_{j=1}^{r} \alpha_2 / X_{ij}}$$

Take (ln) to both sides will be;

$$\ln L = n\,k \ln \alpha_1 + n\,r \ln \alpha_2 - 2\sum_{i=1}^{n}\sum_{j=1}^{k} \ln X_{ij} - 2\sum_{i=1}^{n}\sum_{j=1}^{r} \ln X_{ij} - \alpha_1 \sum_{i=1}^{n}\sum_{j=1}^{k} \frac{1}{X_{ij}} - \alpha_2 \sum_{i=1}^{n}\sum_{j=1}^{r} \frac{1}{X_{ij}}$$

(lnL) partial derivative with respect to α_1 and α_2 on respectively, is given by:

$$\frac{\partial \ln L}{\partial \alpha_1} = \frac{nk}{\alpha_1} - \sum_{i=1}^{n}\sum_{j=1}^{k} \frac{1}{X_{ij}}$$

$$\frac{\partial \ln L}{\partial \alpha_2} = \frac{nr}{\alpha_2} - \sum_{i=1}^{n}\sum_{j=1}^{r} \frac{1}{X_{ij}}$$

Now, $\frac{\partial \ln L}{\partial \alpha_1}$ and $\frac{\partial \ln L}{\partial \alpha_2}$ equate to zero on respectively, we have:

$$\hat{\alpha}_{1\,mle} = \frac{nk}{\sum_{i=1}^{n}\sum_{j=1}^{k} \frac{1}{X_{ij}}} \qquad (2)$$

$$\hat{\alpha}_{2\,mle} = \frac{nr}{\sum_{i=1}^{n}\sum_{j=1}^{r} \frac{1}{X_{ij}}} \qquad (3)$$

Substitute equations (2) and (3) into equation (1) to obtain an estimate of parallel stress – strength model will be:

$$R_p = \frac{k\hat{\alpha}_{1\,mle}}{r\hat{\alpha}_{2\,mle} + k\hat{\alpha}_{1\,mle}} \qquad (4)$$

4.Bayesian Estimator [13], [14], [15]

Let $(Z_{i1}, Z_{i2}, \dots, Z_{in1})$ $(i = 1,2,\dots,k)$ be a random sample on strength of the i[th] component and $(Y_{i1}, Y_{i2}, \dots, Y_{in2})$ $(i = 1,2,\dots,r)$ be a random sample on stresses of i[th] component.

Now, we have to find the Bayes estimator for α_1 and α_2 using non – informative prior distribution $g(\alpha_1)$ and $g(\alpha_2)$ based on modified extension of Jeffery prior as follow:

The modified extension of Jeffery prior can be found by:

$g(\alpha_1) \propto [I(\alpha_1)]^{c_1}$

Where, $I(\alpha_1)$ is fisher information.

$$I(\alpha_1) = -n_1 E\left[\frac{\partial^2 \ln f(x_i;\, \alpha_1)}{\partial \alpha_1^2}\right] = \frac{n_1}{\alpha_1^2}$$

In the same way, we have

$$I(\alpha_2) = \frac{n_2}{\alpha_2^2}$$

Therefore;

$$g(\alpha_1) \propto \left[\frac{n_1}{\alpha_1^2}\right]^{c_1} \; and \; g(\alpha_1) = k_1\, n_1^{c_1}\, \alpha_1^{-2c_1}$$

$$g(\alpha_2) \propto \left[\frac{n_2}{\alpha_2^2}\right]^{c_2} \; and \; g(\alpha_2) = k_2\, n_2^{c_2}\, \alpha_2^{-2c_2}$$

The joint probability density function $H_1\,(Z_{ij}, Y_{ij}\,, \alpha_1\,, \alpha_2)$ is given by :

$$H_1\,(Z_{ij}, Y_{ij}; \alpha_1\,, \alpha_2) = \left[\prod_{i=1}^{k}\prod_{j=1}^{n_1} \alpha_1/Z_{ij}^2 \;\; e^{-\alpha_1/Z_{ij}}\right] . \left[\prod_{i=1}^{r}\prod_{j=1}^{n_2} \alpha_2/Y_{ij}^2 \;\; e^{-\alpha_2/Y_{ij}}\right] . k_1 . k_2 . n_1^{c_1} . n_2^{c_2} \alpha_1^{-2c_1} . \alpha_2^{-2c_2}$$

Then we have;

$$H_1\left(Z_{ij}, Y_{ij}\,; \alpha_1\,, \alpha_2\right) = \alpha_1^{kn_1-2c_1} . \alpha_2^{rn_2-2c_2} k_1 . k_2 . n_1^{c_1} . n_2^{c_2} . \prod_{i=1}^{k}\prod_{j=1}^{n_1} 1/Z_{ij}^2 \; . \prod_{i=1}^{r}\prod_{j=1}^{n_2} 1/Y_{ij}^2 \; e^{-\alpha_1 \sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}} - \alpha_2 \sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}}$$

The marginal probability density function of Z_{ij} *and* Y_{ij} will be:

$$p_1\left(Z_{ij}, Y_{ij}\right) = \int_0^{\infty}\int_0^{\infty} H_1\left(Z_{ij}, Y_{ij}\,; \alpha_1\,, \alpha_2\right) d\alpha_1\, d\alpha_2$$

$$= k_1 . k_2 . n_1^{c_1} . n_2^{c_2} \prod_{i=1}^{k}\prod_{j=1}^{n_1} \frac{1}{Z_{ij}} . \prod_{i=1}^{r}\prod_{j=1}^{n_2}\frac{1}{Y_{ij}}$$

$$\int_0^{\infty} \alpha_1^{kn_1-2c_1-1+1} \;\; e^{-\alpha_1 \sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}}\, d\alpha_1 \; . \int_0^{\infty} \alpha_2^{rn_2-2c_2-1+1} \;\; e^{-\alpha_2 \sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}}\, d\alpha_2$$

$$= k_1 . k_2 . n_1^{c_1} . n_2^{c_2} \prod_{i=1}^{k}\prod_{j=1}^{n_1}\frac{1}{Z^2{}_{ij}} \prod_{i=1}^{kr}\prod_{j=1}^{n_2}\frac{1}{Y^2{}_{ij}}$$

$$. \left[\Gamma(kn_1 - 2c_1 + 1)\left(\sum\sum\frac{1}{Z_{ij}}\right)^{-(kn_1-2c_1+1)}\right]$$

$$. \left[\Gamma(rn_2 - 2c_2 + 1)\left(\sum\sum\frac{1}{Y_{ij}}\right)^{-(rn_2-2c_2+1)}\right]$$

Then, the posterior distribution of $\alpha_1\,, \alpha_2$ is given as:

$$f_1(\alpha_1, \alpha_2\,, Z_{ij}\,; Y_{ij}) = \frac{H_1\left(Z_{ij}\,, Y_{ij}; \alpha_1, \alpha_2\right)}{p_1\,\left(Z_{ij}\,, Y_{ij}\right)}$$

$$= \alpha_1^{kn_1-2c_1} \; . \alpha_2^{rn_2-2c_2} . \left(\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)^{kn_1-2c_1+1} . \left(\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)^{rn_2-2c_2+1}$$

$$. e^{-\alpha_1\left(\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right) - \alpha_2\left(\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)} . \frac{1}{\Gamma(kn_1 - 2c_1 + 1)\Gamma(rn_2 - 2c_2 + 1)}$$

Now, we let;

$$B_1 = \left(\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)^{(kn_1-2c_1+1)}$$

$$B_2 = \left(\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)^{(rn_2-2c_2+1)}$$

$$B_3 = e^{-\alpha_1\left(\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)-\alpha_2\left(\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)}$$

Therefore, the Bayes of $\hat{R_{pBJ}}$ *will be*:

$$\hat{R_{pBJ}} = \int_0^\infty\int_0^\infty R_p\ f_1(\alpha_1,\alpha_2;Z_{ij}\,,Y_{ij})d\alpha_1,d\alpha_2$$

$$= \frac{B_1\ B_2\ k}{\Gamma(kn_1-2c_1+1)\Gamma(rn_2-2c_2+1)}$$

$$\int_0^\infty\int_0^\infty \frac{B_3\ \alpha_1^{kn_1-2c_1+1}\ .\alpha_2^{rn_2-2c_2}}{r\alpha_2+k\alpha_1}\ d\alpha_1\ d\alpha_2 \qquad (5)$$

Also, we have to find the Bayes estimator for $\alpha_1\,,\alpha_2$ using Gamma prior information as follow:
We take:

$$g(\alpha_1) = \frac{1}{F(\alpha)}\alpha_1^{\alpha-1}\ e^{-\alpha_1} \qquad \alpha_1>0\,,\alpha>0$$

$$g(\alpha_2) = \frac{1}{F(\beta)}\alpha_2^{\beta-1}\ e^{-\alpha_2} \qquad \alpha_2>0\,,\beta>0$$

The joint probability density function is given by :

$$H_2\left(Z_{ij},Y_{ij};\ \alpha_1\,,\alpha_2\right) = \frac{\alpha_1^{kn_1+\alpha-1}\ \alpha_2^{rn_2+\beta-1}}{F(\alpha)\ F(\beta)}\prod_{i=1}^{k}\prod_{j=1}^{n_1}\frac{1}{Z_{ij}^2}\cdot\prod_{i=1}^{r}\prod_{j=1}^{n_2}\frac{1}{Y_{ij}^2}\ .\ e^{-\alpha_1\left(1+\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)-\alpha_2\left(1+\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)}$$

The marginal probability density function of Z_{ij},Y_{ij} will be:

$$p_2(Z_{ij},Y_{ij}) = \int_0^\infty\int_0^\infty H_2(Z_{ij},Y_{ij};\ \alpha_1\,,\alpha_2)\ d\alpha_1\ d\alpha_2$$

$$= \frac{\prod_{i=1}^{k}\prod_{j=1}^{n_1}\frac{1}{Z_{ij}^2}\cdot\prod_{i=1}^{r}\prod_{j=1}^{n_2}\frac{1}{Y_{ij}^2}}{F(\alpha)\ F(\beta)}$$

$$\int_0^\infty \alpha_1^{kn_1+\alpha-1}\ \ e^{-\alpha_1\left(1+\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)}\ d\alpha_1$$

$$\int_0^\infty \alpha_2^{rn_2+\beta-1}\ \ e^{-\alpha_2\left(1+\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)}\ d\alpha_2$$

$$= \frac{\prod_{i=1}^{k}\prod_{j=1}^{n_1}\frac{1}{Z_{ij}^2}\cdot\prod_{i=1}^{r}\prod_{j=1}^{n_2}\frac{1}{Y_{ij}^2}}{F(\alpha)\ F(\beta)}$$

$$\left[\Gamma(kn_1+\alpha)\left(1+\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)^{-(kn_1+\alpha)}\right]$$

$$.\left[\Gamma(rn_2+\beta)\left(1+\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)^{-(rn_2+\beta)}\right]$$

Then, the posterior distribution of α_1 , α_2 given as:

$$f_2(\alpha_1,\alpha_2\,;\,Z_{ij}\,,Y_{ij}) = \frac{H_2(Z_{ij}\,,Y_{ij};\alpha_1,\alpha_2)}{p_2\,(Z_{ij}\,,Y_{ij})}$$

$$= \alpha_1^{kn_1+\alpha-1}\;.\alpha_2^{rn_2+\beta-1}\;\frac{\left(1+\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)^{(kn_1+\alpha)}.\left(1+\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)^{(rn_2+\beta)}}{\Gamma(kn_1+\alpha).\Gamma(rn_2+\beta)}$$

$$.\,e^{-\alpha_1\left(1+\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)-\alpha_2\left(1+\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)}$$

We let:

$$m_1=\left(1+\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)^{(kn_1+\alpha)}$$

$$m_2=\left(1+\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)^{(rn_2+\beta)}$$

$$m_3=e^{-\alpha_1\left(1+\sum_{i=1}^{k}\sum_{j=1}^{n_1}\frac{1}{Z_{ij}}\right)-\alpha_2\left(1+\sum_{i=1}^{r}\sum_{j=1}^{n_2}\frac{1}{Y_{ij}}\right)}$$

Therefore the Bayes of $\hat{R}_{PBG}$ will be:

$$\hat{R}_{pBG}=\int_0^\infty\int_0^\infty R_p\;f_2(\alpha_1,\alpha_2\,;\,Z_{ij}\,,Y_{ij})d\alpha_1.d\alpha_2$$

$$=\frac{m_1\,m_2\,k}{\Gamma(kn_1+\alpha)\Gamma(rn_2+\beta)}\int_0^\infty\int_0^\infty\frac{m_3\;\alpha_1^{kn_1+\alpha}\;.\alpha_2^{rn_2+\beta-1}}{r\alpha_2+k\alpha_1} \tag{6}$$

5. Simulation study

A simulation study was carried out to calculate MSES reliabilities of parallel with generating 1000 sample of size n_l=4, 5, 7, 9 various values of k, r and parameters (α_1 , α_2) found in tables (1-5). To achieve the numerical results, MATLAB Version 2016b is used. We take n_l=n=4,5,7,9 Various values of k, r. After that, the Monte Carlo simulation to comparison between these methods. It can be observed that maximum likelihood estimated are an efficient estimator than Jeffery's prior formulation and gamma priors.

Table 1.MLE, Bayes estimators and MSE for $\hat{R}_{pg}$ and $\hat{R}_{pj}$

$\alpha_1 = 1.5\,,\alpha_2 = 3\,,k = 4\,,\ r = 3\ \ \alpha = 1.7\,,\beta = 2\,,c_{1=1}\,,c_2 = 1\,,R_p = 0.4000$						
$n_1 = n_2$	$\hat{R}_p$	$\hat{R}_{pg}$	$\hat{R}_{pj}$	$MSE(\hat{R}_p)$	$MSE(\hat{R}_{pg})$	$MSE(\hat{R}_{pj})$
4	0.4004	0.4247	0.4116	0.0076	0.0060	0.0074
5	0.4010	0.4211	0.4099	0.0062	0.0050	0.0060
7	0.3969	0.4123	0.4033	0.0047	0.0039	0.0045
9	0.3990	0.4110	0.4039	0.0033	0.0029	0.0032

Table 2. MLE, Bayes estimators and MSE for $\hat{R}_{pg}$ and $\hat{R}_{pj}$

$\alpha_1 = 1.5, \alpha_2 = 3, k = 4, r = 4 \ \alpha = 1.7, \beta = 2, c_1 = 1, c_2 = 1, R_p = 0.3333$						
$n_1 = n_2$	$\hat{R}_p$	$\hat{R}_{pg}$	$\hat{R}_{pj}$	$MSE(\hat{R}_p)$	$MSE(\hat{R}_{pg})$	$MSE(\hat{R}_{pj})$
4	0.3378	0.3555	0.3422	0.0057	0.0048	0.0055
5	0.3380	0.3525	0.3415	0.0047	0.0041	0.0045
7	0.3355	0.3464	0.3380	0.0034	0.0031	0.0034
9	0.3344	0.3431	0.3364	0.0028	0.0026	0.0028

Table 3. MLE, Bayes estimators and MSE for $\hat{R}_{pg}$ and $\hat{R}_{pj}$

$\alpha_1 = 1.5, \alpha_2 = 3, k = 5, r = 4 \ \alpha = 1.7, \beta = 2, c_1 = 1, c_2 = 1, R_p = 0.3846$						
$n_1 = n_2$	$\hat{R}_p$	$\hat{R}_{pg}$	$\hat{R}_{pj}$	$MSE(\hat{R}_p)$	$MSE(\hat{R}_{pg})$	$MSE(\hat{R}_{pj})$
4	0.3827	0.4020	0.3903	0.0057	0.0047	0.0055
5	0.3882	0.4034	0.3942	0.0046	0.0040	0.0045
7	0.3840	0.3955	0.3883	0.0034	0.0030	0.0034
9	0.3859	0.3949	0.3892	0.0027	0.0025	0.0027

Table 4. MLE, Bayes estimators and MSE for $\hat{R}_{pg}$ and $\hat{R}_{pj}$

$\alpha_1 = 1.5, \alpha_2 = 3, k = 5, r = 3 \ \alpha = 1.7, \beta = 2, c_1 = 1, c_2 = 1, R_p = 0.4545$						
$n_1 = n_2$	$\hat{R}_p$	$\hat{R}_{pg}$	$\hat{R}_{pj}$	$MSE(\hat{R}_p)$	$MSE(\hat{R}_{pg})$	$MSE(\hat{R}_{pj})$
4	0.4467	0.4729	0.4611	0.0080	0.0058	0.0076
5	0.4517	0.4726	0.4631	0.0064	0.0050	0.0062
7	0.4506	0.4661	0.4586	0.0044	0.0036	0.0043
9	0.4533	0.4653	0.4596	0.0038	0.0033	0.0038

Table 5. MLE, Bayes estimators and MSE for $\hat{R}_{pg}$ and $\hat{R}_{pj}$

$\alpha_1 = 1.5, \alpha_2 = 3, k = 5, r = 5 \ \alpha = 1.7, \beta = 2, c_1 = 1, c_2 = 1 \ R_p = 0.3333$						
$n_1 = n_2$	$\hat{R}_p$	$\hat{R}_{pg}$	$\hat{R}_{pj}$	$MSE(\hat{R}_p)$	$MSE(\hat{R}_{pg})$	$MSE(\hat{R}_{pj})$
4	0.3385	0.3530	0.3420	0.0046	0.0041	0.0045
5	0.3330	0.3453	0.3359	0.0038	0.0033	0.0037
7	0.3337	0.3427	0.3358	0.0027	0.0024	0.0026
9	0.3361	0.3431	0.3377	0.0022	0.0021	0.0022

6.Conclusion

From the numerical results, we concluded that the reliability estimation parallel system (Rp) for Inverted Exponential distribution using different two functions as Jeffery's prior information and gamma priors with maximum likelihood estimated. The model $R_p = p[[\max(Z_{k+1}, Z_{k+2}, \dots Z_{k+r})] < \max(Z_1, Z_2, \dots Z_k)$ using different samples size. In addition It can be observed that maximum likelihood estimated are an efficient estimator than Jeffery's prior formulation and gamma priors.

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