

A NOVEL APPROACH OF MOBILE SECURITY ROBOTICS MOVED BASED ON GRAPH THEORY

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ABSTRACT

Mobile computing plays vital role in our daily life. It has a lot of important applications in modern technology such as, scientific discoveries, rescue operations, and scientific research. The movement direction of mobile robots is considered very important issue because of their direct relationship to the amount of energy consumed. Therefore, computerized direction movement of mobile computer must be measured before transfer from one point to others. Because of any random movement of robots will have negative and direct impact on nodes life. In this paper, mobile computing is waiting in stand-by mode to obtain a new control data at critical time for moving from one point to other in a specific zoon. In more details, graph theory is utilised in positioned of mobile nodes. In addition, it has the ability in determine mobile computer movement without any energy losses. Our simulations result of the new movement system show that the proposed approaches possess outstanding result with a reduction in energy consumption.

Keywords: *Image Compression, Video Compression, Frame Compression, Frame Extraction 2D-DWTAd hoc network; mobile computing; graph theory; search and rescue.*

1. INTRODUCTION

Robots are considered optimal choice in a search and rescue tasks in modern applications. It has the ability to complete some difficult tasks that cannot does by humans. In addition, these mobile computing suffer from certain limitations, such as energy, direction, movement, capacity and adaption with environment [1]. Recently, determining process of robots' movement from one location to other is considered tricky issue in harsh environment [2].

As we know, Leonhard Euler formed the basis of graph theory by 1707 [3]. Basically, it is settled for fixing the problem of Konigsberg Bridge. In more details, it is formulated to find the best path between two large islands that connected by seven bridges. Graph theory has established as a subsection of mathematics, but it has been widely utilised in many scientific filed such as, engineering, biological, social systems, chemistry and computer science [4] [5]. However, it is become a significant filed of robotic application in our daily life.

Graph theory has the ability to find the relation between networks objects as well as it allows the representation of network properties formally [6]. In graph theory, we are trying to find relation between network basis and graph features that represented by vertices/ robots (V)and edges/ connection link E. Generally, graph is demonstrated with two basic notation which are $G=(V,E)$. Moreover, the location of robots is represented by V whereas link connection or flux of energy are characterised by E.

In this paper, a new algorithm is extracted from graph theory that provided better comprehension between network objects and less time complexity. Therefore, it can to calculate the shortest path between inquire node and available robot. These mobility nodes will not allocate randomly as like traditional wireless network scenario but will be distribute based on graph theory. Thus, this distribution and selection play a vital role in providing response time at suitable time with any delay for inquire nodes. The relation

between graph theory and network objects is represented in figure 1.

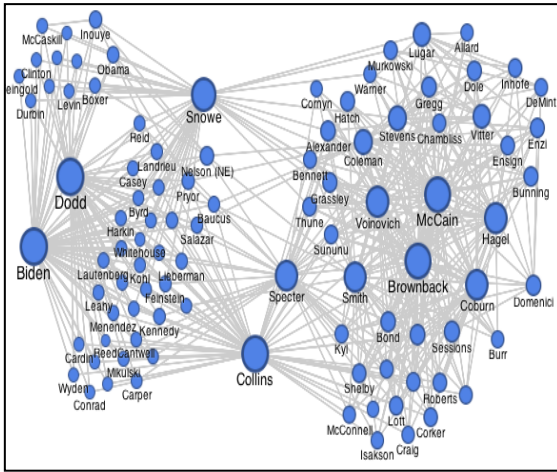


Figure 1. Relation between Graph Theory and Wireless Network [7].

According to the figure 1, we can easily imagine size of the selection problem of the suitable robot between inquire node and available robots that have the same coverage radio area. For this, graph theory is employed in this paper to help us in locate and select the best robot can response on inquire via measure the shortest path between them. As a result, this integrate between graph theory and wireless network will have direct and positive impact on computerize performance.

Our contribution in this paper will distinct us from other by reducing of response time, increase efficiency, optimal exploitation of bandwidth, cost and provide safety of human life at natural disasters.

This research organises as follows; Section two related works. Section three presents more details about graph theory and its applications. Mobile computing applications is presented in section four. Section five explains implemented process of the process system in locate, select and find the best path between inquire node and closes robot. Section six presents the simulation results while section seven will provide discuss our results and compare it with others. Whereas, section eight is explained aspects and advantages of the proposed system. Conclusion and future works is mentioned in section nine.

2. RELATED WORKS

Today, this scientific research field is considered a significant issue in modern

technology. The importance of this field increased after the great growth in the revolution of information technology. All these factors encouraged us to work in this field. The integrate process in this paper are made between graph theory and wireless network to get optimal solution in selection and distribution of mobile robots. Some related survey is mentioned in this subsection that worked on this field recently.

Wang and et al. have designed of an energy routing protocol that based on graph theory in an energy local area network [8]. It plays important role in reducing amount of consumer energy for nodes. The section processor of a lowest- cost is based on some features that selected from network. These features are power transmission and load connection condition on node channels. Graph theory had an important role in enhance energy consumer of wireless nodes.

In [9], the authors can employ one of the most important fields of the mathematics with computer sciences which is graph theory. Shirinivas and et al can present the vital role of graph theory ideas in different areas of information technology applications for researchers. They have been able to demonstrate this effective role of graph theory in computer sciences applications. The researchers related between some important information of the graph theory and its applications in computer science.

Wei and et al have presented a new scheme that explained the role of graph theory in future of network [10]. The authors can explain significant role in networks future that play important role in our delay life. In more details, current comparison studies show allocated bandwidth has direct impact on throughput of whole networks. moreover, this mathematical model has direct role in develop and spread network communication.

In [11], role of graph theory is shown in enhancing security matters of computer. The authors try to apply graph theory to explain the effect of graph theory on network security. However, they provide pointers to important key in this scientific area as well as it shows the recent survey that has related to the same research area. This mathematical model was clarified an important influence in the most important security failures of the current modern systems.

Webb and et al have reviewed the significant key application of graph theory applications in wire and wireless networks security [12]. Algorithm aspects of graph theory are presented to find the

relation between network coding and its reasonable relation to routing protocols.

According to the previous studies, we can easily notice that the most mobility mode of robotics are based on direct techniques. In this case, we are trying to full this gap via employing graph theory in distribute security robots mathematically. In this case, we will avoid a lot of common problems that faced mobility system of robots.

Intelligent integration between graph theory and network is distinguished our work from others. In addition, the proposed algorithm in this paper will identify us from other by reducing of response time, increase efficiency, optimal exploitation of bandwidth, cost and provide safety of human life at natural disasters.

3. GRAPH THEORY

In mathematics, Graph theory is analysis of graphs that are mathematical structures utilised to paradigm relationship between available objects [13]. Generally, a graph is composed of vertices, robots, or nodes are connected by virtual line, connection link, or edges [14]. The relation between vertices and edge in graph is divided into two categories which are direct and undirect. Moreover, undirect graph is no associated between nodes and connection link whereas direct graph is distinction between the two nodes associated with links [15]. The direct and undirect graph are shown in figure 2.

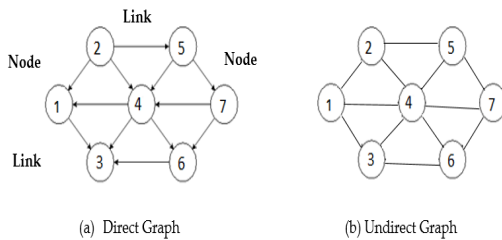


Figure 2. Types of Graph.

Graph theory has the ability to find the relation between networks objects as well as it allows the representation of network properties formally [6]. It is often expressed with two matrix which are incidence matrix T and adjacency matrix A [16]. This matrix has size $n \times n$, where n refers to the number of robots in the network. However, type of this matrix heavily depends on class of graph which are symmetric or non-symmetric. In addition, symmetric matrix is created when graph is undirect whereas direct graph generates non-symmetric [17].

Undirect graph is utilised in our proposal to applied graph theory in select optimal robot to do specific task at situatable time without any delay. This selection of robots will be introduced other nodes in sleep mode that played role in maintain on the amount of available energy. The mathematical expression of undirect graph is represented below:

An undirected graph $G = (V, E)$ is made of set of vertices (a finite set of elements);

$$\text{Robot Number} = \{R_1, R_2, R_3, R_4, \dots, R_n\} \cong V = \{V_1, V_2, V_3, V_4, \dots, V_n\} \dots (1)$$

$$\text{A connection link is subset of unordered pairs of } E^2 = \{(R_i, R_j)\}, \quad i=1,2,\dots,N; \quad j=1,2,3,\dots,N; \quad i \neq j \dots (2)$$

If $(R_i, R_j) \in L$, then $(R_j, R_i) \in L$

To generate A and T matrixes of graph theory, we need create simple graph that reflect network topology. It is shown in figure 3.

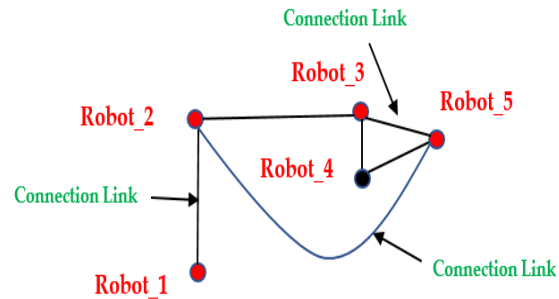


Figure 3. Simple Graph.

Robots/ vertices set, and connection line/ edges set can be defined in figure 3 as following:

$$\text{Robot Number} = \{R_1, R_2, R_3, R_4, \dots, R_7\}$$

$$\text{Connection_Link} = \{(R_1, R_2), (R_2, R_3), (R_2, R_5), (R_3, R_5), (R_3, R_4), (R_4, R_5)\}$$

The adjacency matrix that is defined $A = [(a)_{ij}]$ of order n from robots. However, these letters are associated with link connection of network topology. The mathematical expression below shows us how can generate this matrix.

$$a_{(i,j)} = 1 \text{ if } (V_i, V_j) \in E. \\ = 0 \text{ otherwise}$$

This type of matrix is symmetric as well as all elements are one or zero. Here, adjacency matrix is generated from figure 2.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Graph theory needs another matrix to show relation between mobility robots and their connection link which is incidence matrix [18]. In this matrix, robots/ vertices are represented by rows whereas connection links/ edges are represented by column. In more details, these matrixes will have two 1's if the connection links are connected to a robot in both endings. According to equation 4, we can generate T matrix for undirect graph.

$$t(i,w) = t(j,w) = 1 \quad \text{if and only if } ew = (R_i, R_j)$$

=0 otherwise

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Graph theory has another important parameter is degree of graph [8]. In this graph, minimum and maximum degrees are calculated $d(R)$ which are $\delta(G)$ and $\Delta(G)$ respectively.

Graph theory will support wireless network in distributing and select mobile nodes for any emergency request at a suitable time without any delay.

4. MOBILE COMPUTING APPLICATIONS

Networking and mobile computing play important role in our daily life. Mobile robotics has a lot of vital applications in modern technology. However, security mobile robotics are considered one of the best application in mobile computing [19] [20]. It has the ability to move around in a physical area. In this paper, a novel movement of mobile security robots are proposed to enhance its response time for any urgent request. Moreover, it is heavily based on graph theory in distribute robots on e-map of University of Anbar.

In general, two types of mobile robots which are non - autonomous and autonomous [21] [22]. Each of them has different behaviour in mobility on

physical environment. On one side, non-autonomous robots utilise some kind of guidance approach to explore their movement. On the other side, autonomous robots can determine their physical movement. In this paper, graph theory is utilised as guidance system in explore physical environment of security non-autonomous robots.

The integrated process of graph theory with mobile computing that proposed in this research will play significant role in reduce fail/ delay case for emergency request. The graph theory will apply mathematical model on position of robotics on e-maps. Thus, it will guaranty in set optimal dimensions of robots on map of University.

Mobility of each robots will determine by number of parameters, such as location, speed, radio coverage area and request area. All these parameters have direct impact on select and move robot on map. However, our work is different from other by utilising graph theory in distribute robots on map.

In this paper, two scenarios are generated to imagine role of the new mobility system that proposed in this paper. In other words, normal and attacks behaviour are created to evaluate the role of the proposed system. Therefore, it is evaluated under behaviour and with same certain condition. In more details, the security robots will expose for attacks behaviour as well as it will evaluate under two approaches: first, the security robots will move based on radio coverage area whereas second one it will move based on new graph theory.

5. IMPLEMENTED SYSTEM

In this paper, it is considered that there are rescue robots that are employing as service mobile robots in a defined coverage area. These nodes have the ability to respond to any emergency/ urgent request from any a victim asks for help. In this paper, graph theory is employed in finding the suitable position of these robots depends on features that were collected from the network topology. However, network topology plays important role in distributing these rescue robots. In addition, this mathematical model will gain any wireless communication system ability to choose the appropriate robots for the inquire point. As a result, this will reduce request time, save human life, increase response time, save memory, increase efficiency for network, and optimal use of carrier medium.

A novelty in this paper is employed graph theory for wireless network for finding a suitable position of mobile nodes from request areas at a natural disaster, volcanoes, floods, earthquakes, and wars. The lifecycle of propose system is shown in figure 4.

user chooses the positions of mobility robots on network topology for once time only.

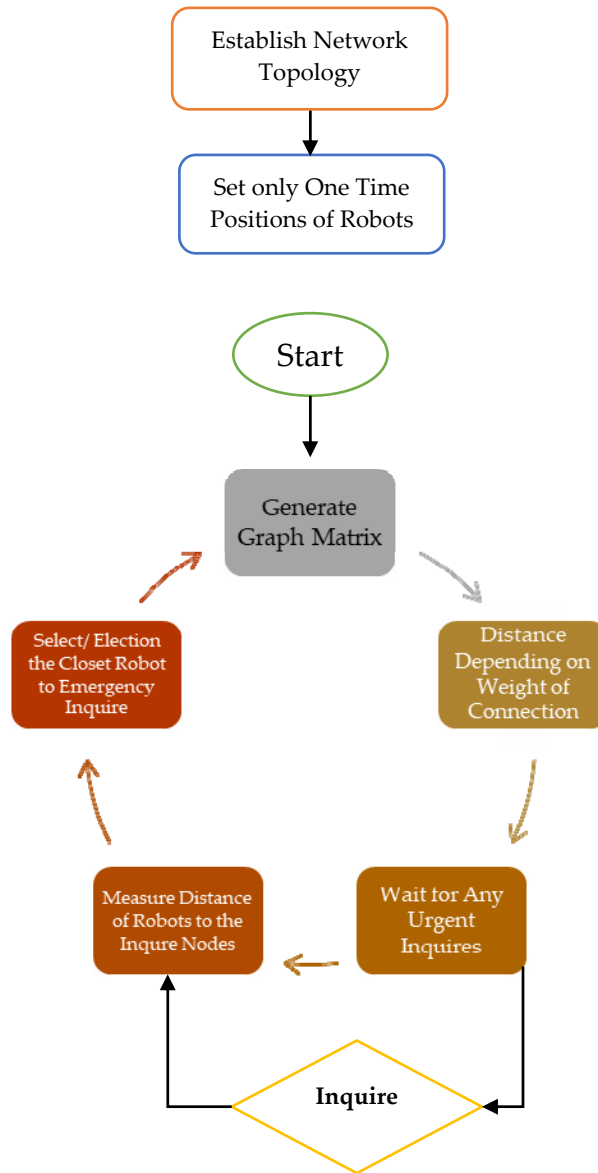


Figure 4. Flow Chart of the Proposed System.

According to figure 4, we can easily notice the role of the proposed system in enhancing efficiency and effectiveness of the network through shortest time for making the decision to select closet robot for request area. On one side, establishing network topology is required for the proposed system with

On the other side, it has the ability to settle services robots at a suitable place on e-maps for recusing team as well as it has the ability to select the best robot for doing specific task. The proposed system is employed on e-map of Anbar University as shown in figure 5.

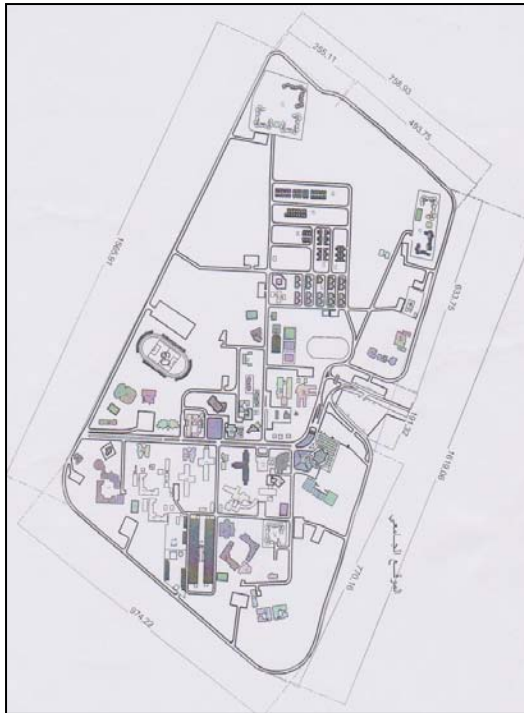


Figure 5. E-Map Of Anbar University.

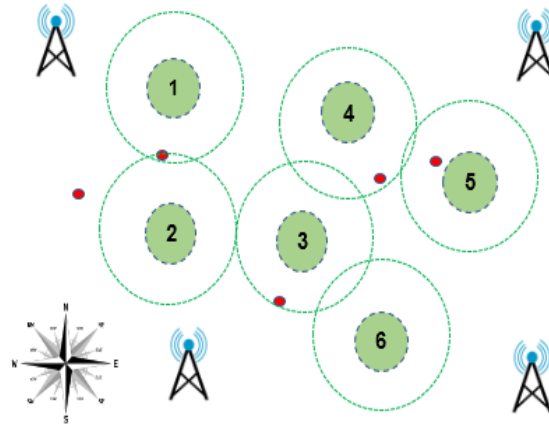


Figure 6. Security Robots Topology.

This Bus-topology is applied on e-map of University of Anbar. However, each security robots have specific location on this map of University. In this paper, new mobility system is proposed in enhance security role of these robots as well as it has positive and direct impact on overall performance of security system. The graph theory in this paper will utilised in distribute security robots on e-map of University as shown in figure 7.

6. Experimental Results

The proposed algorithm will sign each mobile robot with priority number that depends on the calculating distance between the robot and inquires area in that zone. Therefore, these values will update regularly at each request. The proposed system, in this paper, plays important role in canalizing the mobile robots to the attacks area at suitable time. It has primary idea about positions of mobile nodes on e-maps of University of Anbar. In this simulation, University of Anbar map is utilised in determine robots' locations that required to determine at only first time by technician. It utilises bus topology in distribute mobile nodes on e-map as shown in figure 6. Thus, the possible paths are drawn between source and destination after that it will be selected the shortest route to the attacks area.

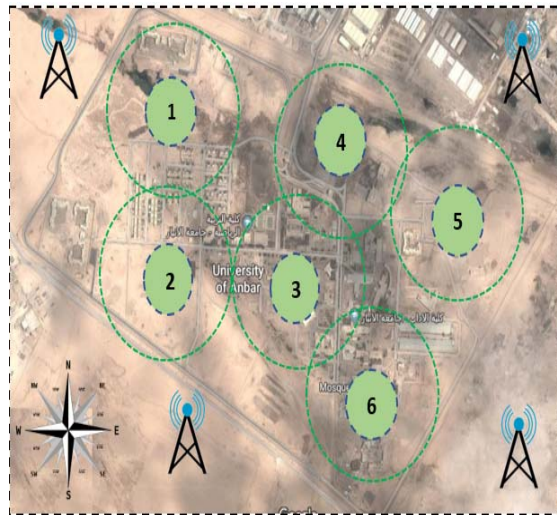


Figure 7 Robots' Location On University Of Anbar Map.

In a request for help or assistance zone, the proposed system which based on the mathematical model will calculate the distances between robots “edges” terms in graph theory. It must select the shortest path between source robots that send to the specific zone and attack area. In other words, the proposed approach will draw the moving path for

each robot on e-map according to the priority value that attached to each node. All robots will be ready if the first robot with the highest priority cannot do the mission at the suitable time to avoid any failure case. Therefore, it needs to determine the distance between mobile robots after each positioning loop algorithm. However, the proposed algorithm makes a measuring for planning which mobile robot to canalizing to defined zone. Matrices and features of graph theory have been utilised for this mentioned process. In this paper, abnormal behaviour is required to explain the significant role of the proposed system in reducing response time that has direct and positive impact on human life.

Mobility robots play a security role that employed in this simulation to the defence any intruder. In this scenario, we need to establish abnormal behaviour/ attacks from any direction of the university zone to evaluate the performance of the proposed system and compare it with the traditional system that without graph theory. Figure 8 shows robots' positions and attack zone that notified by generate alarm to all security points at university.

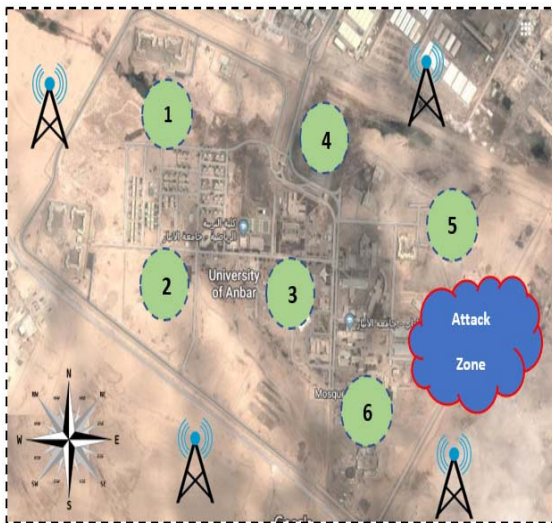


Figure 8 Robots' location on University of Anbar Map and Attack zone.

In this scenario, the proposed system is composed from two phases which are re-distribute and select the best solution. In this case, it utilises graph theory to find the best location of mobile robot as shown in figure 8. Thus, it needs to calculate the distance between all robots and attack zone. The proposed system calculates all distance between mobile security robots and breakthrough zone as shown in table 1.

Table 1 Distance value of security robots.

Security Robot ID	Distance/ m
Robot_1	1820
Robot_2	1647
Robot_3	1325
Robot_4	1410
Robot_5	230
Robot_6	315

The proposed system will have allocated one or more form security robot and re-sign priority number for each robot as shown in table 2.

Table 2 Priority Number of security robots.

Security Robot ID	Priority Number
Robot_1	0
Robot_2	0
Robot_3	1
Robot_4	2
Robot_5	4
Robot_6	3

According to table 2, the proposed system will directly notify security robots that have the highest priority to moving to the attack zone which are robot_5 and 6. In addition, it will notify another security robot to become ready for any urgent request as shown in figure 9.

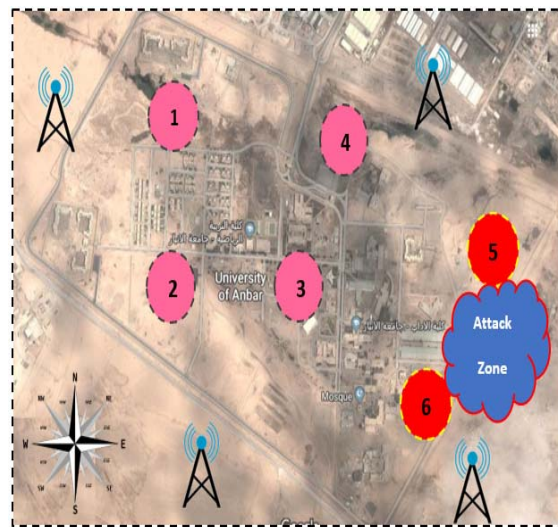


Figure 9. Canalizing Security Robot to the Attack zone.

The performance metrics is required to evaluate the efficiency of the proposed system and

compare it with another system. These metrics such as, response time, end-to-end delay and waiting time that shown in table 3.

Table 3 Performance Metrics.

Metrics	System with Graph Theory	System without Graph Theory
Response Time	0.89s	7.0s
Delay	1s	12.0s
Waiting Time	0.6s	8.33s

Table 3 shows the efficiency and effectivity of the proposed system when compared it with the traditional system without mathematical model.

7. DISCUSSION

The lightweight mathematical model that proposed in this paper will be integrated with the current algorithm to obtain more reliable impact on the real-time applications. Graph theory is utilised to calculate the distance in term of edges between security robots. These mobility nodes share in important data, such as control data, warning messages, notification messages, and cooperative awareness messages. In addition, security robots try to exchange positions information with their neighbours in that radio coverage area.

The adjacency matrix and eigenvalues of this matrix are utilised in obtain distance information and communication data of these security robots. This matrix shows the distance between security robots that create the communication link with each other. According to the graph theory, security robots in this paper occupy the place of vertices and the best path of neighbour nodes take the place of edges. These vertices and edges are formed in adjacency matrix. It has the ability in calculating the eigenvalues that play direct role in support graph theory [26].

Connection information and neighbour numbers of security robots are obtained from index and principle eigenvector values [27] [28] [29] [30]. In addition, Topal algorithm is employed in extracted these values. The index values are extracted from the edges of graph theory. Therefore, the principle eigenvector values are generated from the number of neighbour robots that connected to a security robot. These values of graph theory distinguish security robots that have the

same distance value. However, the proposed algorithm cannot depend on robots that outside communication area. All these per-knowledge making connection of a security robot could be calculated according to another node [23] [24] [25].

In this research, the principal eigenvector value is utilised in calculated important values that employed in making a decision of which security robot to be canalizing to intrude zone. The calculating process of principal eigenvector components is heavily based on equation 5.

$$m(i) = \left(\sum_{n=1}^k |{}^n P_i(t) - {}^i P_i(t)| \right) + \frac{{}^i P_i(t)}{2}$$

In this simulation, the proposed system will detect attack zone at the east direction of university map. In this case, it will identify and send closest security robots to the nearest area. However, the algorithm gives the support mission to the robots to fixed this breakthrough problem as shown in figure 7. After this phase, it waits for any other alarm that generated from any place of the university area. The proposed system is compared with tradition approaches to measure throughput rate as shown in figure 10. It is easily observed that our propose system more efficient from others [28] [29] [30].

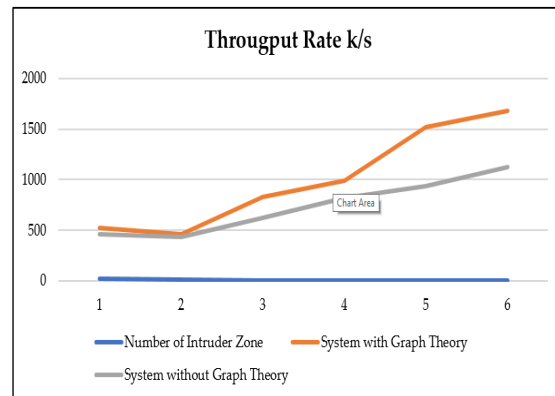


Figure 10. Throughput Rate.

According to figure 10, we can easily notice a significant role of the proposed system that based on graph theory in increasing throughput rate of the response time for any security system. Security department of university can determine any mobile robots at any place and time on the e-map. Moreover, the proposed system has the ability to calculate the closest suitable security robot and sent a new one to specific zone without any delay.

8. ASPECTS AND ADVANTAGES

Most of the current related works are studies security robots and its mobility mode depended on traditional techniques. This scheme deals with graph theory via minimum delay waiting and increase efficiency of repose time that adapted for mobile robotics. The various of this proposal is its integrated between the security robots and the graph theory.

9. CONCLUSIONS AND FUTURE WORKS

In this research, a new mobility algorithm is proposed of robots that is distributed based on graph theory. It is utilised to connection of mobile nodes and identifying of the closest robot to intervene any emergency case. According to the simulation result, we can easily notice that our proposed system more efficient from others. Graph theory plays important role in enhance mobility of robots with reducing the amount of power consumed. The hybrid process between the mathematical model and mobility algorithm of distributed nodes has had a large reflection on the efficient and effective of robots. As a result, it has direct and positive impact on increasing battery life, reducing disconnection and lost in the connection range rate between robots, as well as it can decline breaking down of robots. In addition, this light mathematical model does not need more computing time for calculating and finding closest robot to request zoon.

As a future direction, we are trying to integrate graph theory with routing protocol of robots' wireless communication system. Thus, it can determine and re-distribute robots even in the outside of coverage radio area. On the other side, this mathematical mobility algorithm is intended to enhance its performance to avoid the cases of losses of connection of distributed nodes. The limitation of this proposal system adapts with various types of routing protocols. In future work.

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