Estimating Maximum Magnitude Earthquake in the Southern Red Sea Region using Extreme Value Statistics

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Abstract

Extreme value statistics was used to estimate maximum magnitude earthquakes in Southern Red sea region (12-22° N and 37-46°E). The seismicity file for the period (1900-2000) was used to determine the recurrence relationship as well as the parameter of Gumbel type I asymptotic distribution .The statistical parameters are estimated by least squares technique. The seismicity file of the area is complete in earthquake magnitude beyond 4.8 Ms over 100 years sample. The relation between earthquake magnitude Ms and cumulative frequency Nc is Log Nc = 5.83 – 0.80 Ms. The expected largest magnitude earthquake in the area since 1900 is 7.3 Ms. This value is in a good agreement with the values calculated by other authors. There is a general agreement of the observed return periods with those estimated, except for the higher magnitudes. This agreement indicates that the extreme value statistics is an apt technique to estimate the return periods of the moderate magnitude. The probability of non- exceedence of 7.0 Ms equals to 0.84 for the design period of 50 years.

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المستخلص

استخدم احصاء القيمة القصوى لتخمين أقصى مقدار زلزالي في منطقة البحر الأحمر الجنوبي (١٢°- ٢٢° شمالا و ٣٧°-٤٦°شرقا). استخدمت الملفة الزلزالية للفترة من سنة ١٩٠٠ – ٢٠٠٠ لتعيين علاقة اعادة الحدوث بالاضافة الى معاملات التوزيع الخطي المقارب لجومبل من نوع –١ . خمنت المعاملات الاحصائية بواسطة تكنيك المربعات الدنيا .تكون الملفة الزلزالية لمنطقة الدراسة كاملة للمقدار الزلزالي الذي يزيد عن ٤,٨ عبر فترة نمذجة تبلغ ١٠٠ سنة .كانت العلاقة بين المقدار الزلزالي والتكرار التراكمي للهزات الأرضية في منطقة الدراسة بالشكل التالي :

Log Nc = 5.83 – 0.80 Ms

أكبر مقدار زلزالي متوقع في منطقة الدراسة منذ عام ١٩٠٠ يساوي ٧,٣ . تكون هذه القيمة في توافق جيد مع مثيلاتها المستنتجة من قبل باحثين آخرين. يوجد توافق عام بين فترات اعادة الحدوث الملحوظة مع تلك المستنتجة لجميع المقادير الزلزالية باستثناء المقادير المرتفعة . يدل هذا الاتفاق الى أن احصاء القيمة القصوى يمثل تكنيك ملائم لتخمين فترات اعادة الحدوث للمقادير الزلزالية المتوسطة.تكون احتمالية عدم التجاوز لمقدار زلزالي (٧,٠) لفترة تصميم من ٥٠ سنة مساوية لـ ٨٤% .

Introduction

The principal engineering use of earthquake, both macroseismic and instrumental, is to establish in a given geographic region the recurrence laws as well as to define and assess acceptable seismic risk levels for the design of different type of engineered structures. According to Ambraseys (1978) the estimation of the maximum magnitude earthquake and its occurrence in space and time is the most difficult of all the seismic risk parameters to assess. Amongst the different methods employed in estimating the largest possible earthquake magnitude, that which is based on the theory of extremes is in frequent use (Karnik and Algermissen 1978). Extreme value statistics developed by Gumbel (1958) provide a convenient method to obtain estimates of the frequencies of occurrence of natural events (e.g. meteorological hydrological, geological, etc.) on the extreme of a statistical distribution and to estimate recurrence times for these events.

The application of Gumbel's theory to earthquake magnitude data dates back to the early work of Nordquist (1945). The significance of this study induced later investigations along similar lines which covered various regions of the world over different time samples (see, e.g.Lomnitz 1966; Karnik 1971; Shakal and Willis 1972; Lilwall 1976; Burton 1979; Burton, et al., 1983; Al- Abbasi and Fahmi 1985; Fahmi and Al- Abbasi 1990).

The purpose of this paper is use of Gumbel type asymptotic distribution in estimating maximum magnitudes and return periods for earthquakes occurring in the southern Red sea regions.

Seismic activity and the earthquake data file

Previous research on the seismicity of Africa and Arabia indicates the presence of intermediate to shallow focus earthquake which are restricted to plate boundaries. Thus, the Red sea was recognized early in the development of the global plate tectonic theory as part of the world wide rift system (Barazangi 1981; Sykes 1967).

Evidence of seismic activity in the southern Red sea and western Arabia is found in diverse sources indicating its occurrence from historical time to the present (e.g. Ambraseys 1988; Ambraseys and Melville 1983; El-Isa and Al-Shanti 1989). More studies started on local seismicity and seismic hazard assessment (e.g. Barazangi 1981, 1983; Merghelani 1979; Al-Haddad et al .,1994; Al-Amri 1994; Al-Amri et al., 1998).

Earthquake Database of the Arabian Peninsula issued by Seismic Studies Center – King Saud University, Saudi Arabia Kingdom, was the source of seismic data for the southern Red Sea region (12° - 22° N and 37° - 46° E). This database depended upon a number of seismological catalogues and bulletins including those of the International Seismological Center (ISC), Preliminary Determination of Epicenter (PDE) and related publications. A time period of observation of approximately 100 years (1900 – 2000) for the seismic data was used in the required calculations. Figure 1 shows a seismicity map of the southern Red Sea region.

It is important to note that historical earthquakes prior to twentieth century have not been included in the compiled file due to the scarcity of reporting and because magnitude determinations for these events remain rather speculative.

Magnitude- frequency relationship

It is well understood that earthquakes are not independent events, but tend to cluster in space and time. The knowledge of these tends in time or in space helps in defining the source regions of future shocks. Gutenberg and Richter (1954) found that within a certain magnitude range a straight line on semilogue paper fits well most magnitude- frequency distrubutions.

Typically log N (N= number of events > M) falls off linearly with an increase in M (earthquake magnitade) and the relationship is usually written as:

$$\text{Log } N(M) = a - bM$$

Where the constants a and b characterize the studied area.

The recurrence relationship for the investigated geographic region is depicted (Figure 2). The cumulative frequency of earthquake occurrence is plotted for the period 1900-2000 at intervals of 0.1 unit of magnitude beginning at Ms=4.0.

The regression fit is given by:

$$\text{Log}_{10}\, Nc\,(M) = 5.83 - 0.80\, Mb \quad (r^2 = 0.95) \ldots\ldots\ldots\ldots\ldots \quad (1a)$$

Where r^2 is the correlation coefficient.

Now in order to establish the limit of complete detectibility of earthquake magnitudes in the region (Båth 1981) single frequencies N(M) rather than cumulative frequencies Nc(M) are considered, as shown by the lower straight line fit of (Figure 2). The equation above Ms=4 is:

$\mathrm{Log}_{10}\,N\,(Ms) = 3.50 - 0.51\,Ms \quad (r^2 = 0.92)$………………… (1b)

The N (M) plot clearly illustrates that for $Ms > 4.8$ complete detection is a chieved. It thus follows that the file of the Red sea regions is complete in earthquake magnitudes beyond 4.8 Ms over the 100 years sample.

From equation (1a) we may determine the expected largest magnitude earthquake in the area since 1900 to be:

$a/b = 7.3$

Extreme value statistics

According to Yegulalp and Kuo (1974), for a relatively long sampling period Gumbel's theory of extremes assumes that the occurrence of a maximum magnitude earthquake in a given region within such a time interval is a random independent event, and that the behaviour of the maximum magnitude earthquake in the future will be similar to that of the past number of observational years.

Many workers (see, e.g. Yegulalp and Kuo, 1974; Burton 1979) give an excellent review of the mathematics behind Gumbel's theory as applied to earthquake magnitudes. For our purpose we make use of the simplified treatment of Lilwall (1976) and Burton (1978) to explain the same concept.

After marking the annual extremes of earthquake magnitudes as given in the seismicity file of the investigated area for a sample of n consecutive years, the readings are arranged in:

$M_1 < M_2 <$……………….Mn

Then the ith largest value of magnitude has a probability Pi of being an exterme in any one year and is given by:

$Pi\,(Ms) = i/n+1$ ……………….. (2)

Where Pi is the effective plotting position of the ith magnitude observation on the special Gumbel probability paper.

It follows that the return period Ti (Ms), which is the mean number of intervals required for a largest value greater than or equal to Ms to be observed, is monotonically increasing function Ms defined by:

$Ti\,(Ms) = 1/\,1 - Pi$ ………… (3)

According to Gumbel (1958) there are three types of extremal asymptotic distributions each corresponding to a specific type of behavior of maximum magnitude. In the first type (type I) the magnitude series is unlimited and the distribution of the largest value is defind by:

$P^{I}\,(Ms) = \exp\,[-\exp\,\{-\alpha\,(Ms-U)\,\}]$ ………..(4)

Where α and U are constants.

The second type (type II) introduces a lower limit and the third type (type III) imposses an upper limit W. Where an upper limit exists, P follows the type III distribution which is given by:

$P^{III}\,(Ms) = \exp\,[(W-Ms)\,/\,(W-U)^{K}]$…………………..(5)

Where W represents the upper limit to the range of extreme value, U is the characteristic exterme value associated with unit time, and K is the curvature parameter allowing for curvature asymptotic to W at low annual probabilities or large return periods. P is a probability of non- exceedence of a magnitude Ms. It thus follows that P (W) =1 and P (U) =1/e (Burton et al, 1983).

As expected P approaches unity for the larger values of Ms and tends to zero for the smaller values. This method gives n estimates of P for various values of Ms, these estimates are plotted on special extremal probability paper with Ms as ordinate and –ln (-lnp) as abscissa.

If the distribution is of type I, the points lie on a straight line and the constants α and U are estimated from a least equares fit to (4):

$$-\ln [-\ln p^{I} (Ms)] = \alpha (Ms-U) \ldots\ldots\ldots\ldots (6)$$

If, on the other hand, the distribution is of type III, the plot follows a curve concave downward and is given by:

$$-\ln [-\ln p^{III} (Ms)] = K \ln (W-U) - K \ln (W-Ms) \ldots\ldots\ldots (7)$$

Methods of fitting this curve to the data and hence estimating the parameters W, U and K are described by Yegulalp and Kuo (1974). Onc these parameters are determined, it is possible to predict the mode, median or mean earthquake for a given return period as will be shown later.

There are a number of techniques for estimating the extremal distribution parameters (see, e.g. Gumbel, 1958; Yegulalp and Kuo, 1974). Amongst these the least squares and maximum like- lihood are considered outstanding in accuracy and the least squares method is empolyed in this study.

The numerial linearization techniques of parameter estimation for Gumbel type I mentioned in this study was written into the appropriate computer software (Al-Abbasi, 1984). This program was run for the 100 years earthquake magnitude sample.

Statistical prediction and Risk calculation

For engineering seismological purposes, it is useful to use extreme magnitude predictions in order to determine earthquake risk at specific return and design periods.

According to Lomnitz (1974) earthquake risk may be defined as the probability (percent) of the occurrence of a critical earthquake characterized by a design magnitude during a specific period of time.

Invoking the return period relationship (4) and using lomintz definition of earthquake risk, we have

$$R_D(Ms) = 1- [P(Ms)]^{D} \ldots\ldots\ldots\ldots (8)$$

Where R_D is the earthquake risk in the design period D (yr) available expression of the occurrence of the largest magnitudes in a given return period, we may write (9) as:

$$R_D (Ms) = 1-\exp \{-D \exp- [\alpha (Ms-U)]\} \ldots\ldots\ldots (9)$$

and provided that the return period T>D we can express earthquake risk as :

$$RT (Ms) = 1- \exp \{- T \exp-[\ldots(Ms-U)]\} \ldots\ldots\ldots (10)$$

Solving equation (9) and (10) for R_D we get:

$$R_D = 1-\exp [D/T \,\mathrm{lin}\, (1-Rt)] \ldots\ldots\ldots\ldots (11)$$

Thus for a given design period D it is possible to estimate the risk of occurrence of maximum magnitude earthquake using the analysis extremes.These and other estimates are reported in the next section.

Results and discussion

The upper straight line of Figure 2 gives the cumulative frequency of earthquakes occurrence in the southern Red Sea for the period 1900 – 2000. From Figure 2 we get long Nc = 5.83 – 0.80Ms, where a and b equal to 5.83 and -0.80, respectively. Thenhaus et al.(1986) estimated b-value for the southern Red Sea to be between -0.89 to -0.1.11. El-Isa and Al-Shanti (1989) and Al-Haddad et al. (1994) showed that the b-values for the whole southern Red Sea are in the range – 0.51 to – 0.80. Al-Amri (1994) found that a and b values for the whole southern Red Sea are 6.22 and – 0.91, respectively. Al-Amri et al. (1998) found that the range of the obtained b values in the southern Red Sea was from – 0.59 to – 2.07. Al-Heety (2005) found that the range of the b- value in the southern Red Sea was between -0.5 to – 0.7.

Different b- values for the same area and the same data set may vary according to different assumptions, such as maximum magnitude and the different techniques of treating (least squares and maximum likelihood). For different global catalogues, Frohlich and Davis (1993) found values of b ranging from 0.79 to 1.25. Since the frequency – magnitude relation is not linear, the b-value varies by 15% or more, even for individual catalogues, depending on the exact range of magnitude used for its determination (Al-Amri et al. 1998). However, results of this study show good agreement with previous works done in the studied area.

The N (Ms) plot (the lower straight line fit of Figure 2) clearly illustrates that for Ms > 4.8 complete detection is achieved. This means the file of the southern red Sea region is complete in earthquake magnitudes beyond 4.8 Ms over the 100 years sample.

The expected largest magnitude earthquake in the area since 1900 is 7.3 Ms. The observed largest earthquake occurred in the region has magnitude of 6.7 Ms. If we take into account the accuracy in magnitude estimation of (0.5 unit) we will find that the expected largest magnitude earthquake (7.3 Ms) in the study area agrees with the observed largest magnitude earthquake (6.7 Ms).

Using the plotting position given by equation (3), Figure 3 illustrates Gumbel type I distribution for the least squares techniques. This figure gives n estimates of probability (P) for various values of Ms. As expected P approaches unity for the larger values of Ms and tends to zero for the smaller values.

Table 1. lists the observed return period (yr) versus the estimated return period (yr) of earthquakes with magnitudes > Ms for type I distribution through the use of least squares technique .There is a general agreement of the observed return periods with those estimated , except for the higher magnitude . This agreement indicates that the extreme value statistics is an apt technique to estimate the return periods of the moderate magnitude.

As explained earlier it is possible to get an idea of the risk of occurrence of a certain magnitude earthquake by application of equation (9). Table 2. shows just that for type I distribution using least squares technique. This table lists the probability (P) of non-exceedence of a magnitude Ms for different design periods (D yr), for example, the (P) of non-exceedence of 7.0 Ms equals to 0.32 , 0.61 and 0.84 for design periods of 20 , 25 and 50 years , respectively .

Preliminary seismic hazard assessment of the southern Red Sea region was carried out by Al-Amri (1995). The results of this study were presented in the form of iso-acceleration maps for the return period of 475 years. These results indicate that relative level of ground motion in southern Red Sea is found to be moderate and subjected to more severe seismic hazard compared with the Arabian shield. The maximum magnitude (Mmax) was determined to be 7.0 Ms in the southern Red Sea which are about of half a magnitude unit higher than the highest instrumentally determined magnitudes (Al-Amri, 1995). The spatial distribution of estimated maximum magnitude and expected magnitude at 90% non-exceedence in 50 years for the southern Red Sea was prepared by Al-Amri et al. (1998) They found that the maximum magnitude at 90% non – exceedence in 50 years for the southern Red Sea is 7.0 . In our study, the maximum magnitude at 84% non-exceedence in 50 years for the southern Red Sea is 7.0.There is a good consistence between our results and results of the previous works.

Conclusions

In this investigation, we have used the extreme value statistics to estimate the maximum magnitude earthquake in the southern Red Sea region in order to determine earthquake risk at specific return and design periods. From this study, we conclude the following:

1. Using the single frequencies N(Ms) in the frequency –magnitude relation, we found that the file of the southern Red Sea region is complete in earthquake magnitudes beyond 4.8Ms over 100 years sample .
2. The relation between earthquake magnitude Ms and cumulative frequency Nc is Log Nc = 5.83 – 0.80 Ms . The b-value for the southern Red Sea region is – 0.80.
3. The expected largest magnitude earthquake in the southern Red Sea region since 1900 is 7.3. This value is about half a magnitude unit higher than the highest instrumentally determined magnitude (6.7Ms).
4. There is a general agreement of the observed periods with those estimated except for the higher magnitudes. This agreement indicates that the extreme value statistics is an apt technique to estimate the return periods of the moderate magnitude.
5. The probability (P) of non-exceedence of 7.0Ms equal to 0.84 for the design return period of 50 years.

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Table 1. Observed versus estimated return periods for Gumbel type I distribution using the least squares technique

Magnitude (> Ms)	Observed T(Ms) (yr)	Estimated T(Ms) (yr)
4.0	2.0	2.3
4.5	2.5	2.8
5.0	3.3	3.6
5.5	5.5	5.7
6.0	14.3	9.5
6.5	33.3	15.9

Table 2. Risk estimation during certain design periods for Gumbel type I distribution.

Magnitude (>Ms)	Probability of non- exceedence (P)		
	1	2	3
5.0	0.96	1.00	1.00
5.5	0.85	0.99	1.00
6.0	0.67	0.94	1.00
6.5	0.48	0.80	0.96
7.0	0.32	0.61	0.84

Design period (D): (1) 20years ; (2) 25years; (3) 50years

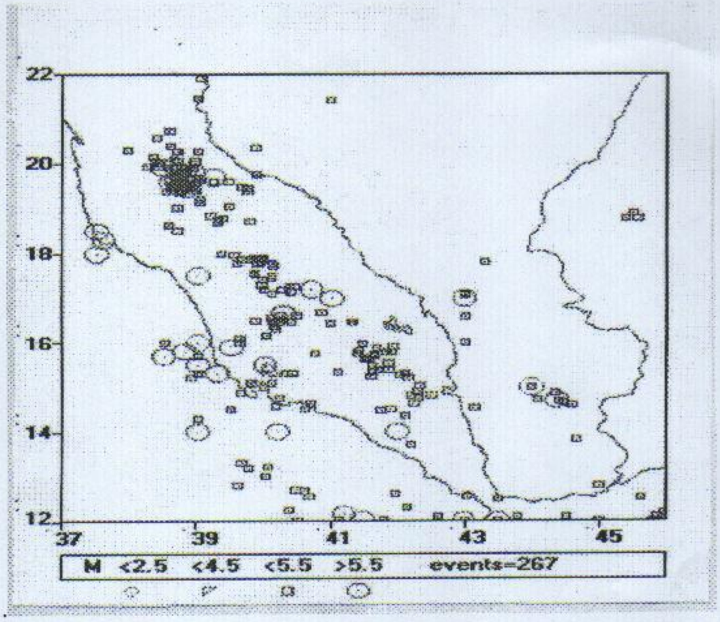

**Figure 1. Seismicity map of the southern Red Sea region (12° - 22° and 37°- 46°) over the period (1900-2000)
The symbol (M) means earthquake magnitude.**

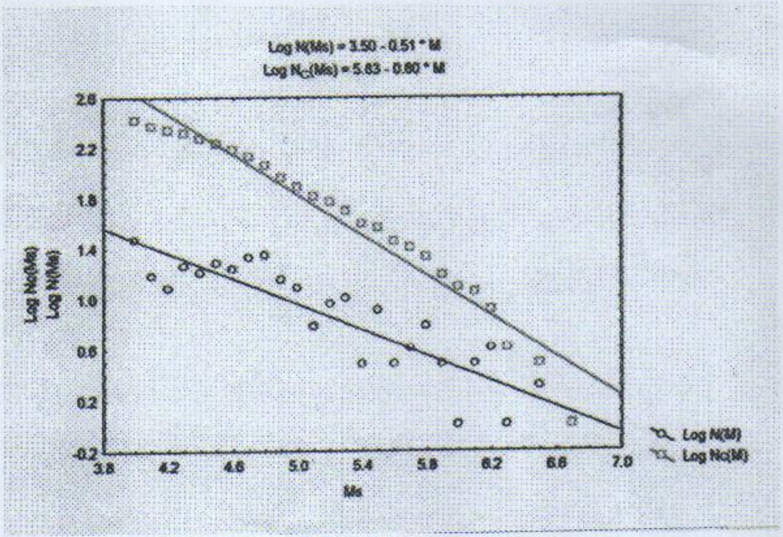

Figure 2. Recurrence relationship for the southern Red Sea region.

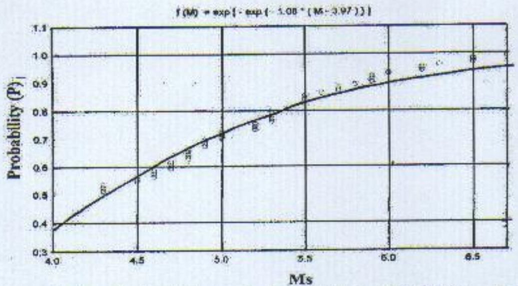

Figure 3.Gumbel type I extremal distribution for earthquake magnitudes in the southern Red Sea region over the period (1900-2000).