DOI: 10.24996/ijs.2021.62.12.23

ISSN: 0067-2904

On Estimating Reliability of a Stress – Strength Model in Case of Rayleigh Pareto Distribution

Emad Sh. M. Haddad *, Feras Sh. M. Batah
Department of Mathematics, University Of Anbar, Anbar, Iraq

Received: 23/11/2020 Accepted: 28/2/2021

Abstract
The stress – strength model is one of the models that are used to compute reliability. In this paper, we derived mathematical formulas for the reliability of the stress – strength model that follows Rayleigh Pareto (Rayl. – Par) distribution. Here, the model has a single component, where strength Y is subjected to a stress X, represented by moment, reliability function, restricted behavior, and ordering statistics. Some estimation methods were used, which are the maximum likelihood, ordinary least squares, and two shrinkage methods, in addition to a newly suggested method for weighting the contraction. The performance of these estimates was studied empirically by using simulation experimentation that could give more varieties for different-sized samples for stress and strength. The most interesting finding indicates the superiority of the proposed shrinkage estimation method.

Keywords: Rayleigh – Pareto distribution, Stress – Strength model, Order statistic, Maximum likelihood, Estimation methods, Shrinkage Coefficient.

حول تقدير موثوقية نموذج القوة والإجهاد في حالة توزيع رايلي باريتو

عماد شاكر محمود حداد*، فراس شاكر محمود بطاح
قسم الرياضيات ، كلية التربية للعلوم الصرفة، جامعة الانبار، الانبار، العراق

الخلاصة
نموذج القوة والاجهاد احد النماذج التي استخدمت في حساب المعولية . يتضمن هذا البحث اشتقاق صيغ تقدير المعولية في نموذج القوة والاجهاد تتبع توزيع رايلي باريتو . يحتوي نموذجنا على القوة Y مقابلة الإجهاد X والتي تمثله العزوم ودوال الموثوقية والغاية والاحصاءات المرتبة. تم استخدام بعض طرائق التقدير متمثلة بطريقة الامكان الاعظم وطريقة المربعات الصغرى وطريقتان للانكماش اضافة لاقتراح طريقة جديدة موزونة . تمت دراسة هذه المقدرات تجريبيا باستخدام تجارب المحاكاة بافتراض عدد من الحالات وعدد من احجام العينات المختلفة . لتأكيد افضلية طريقة تقدير الانكماش المقترحة.

Introduction

Researchers have focused on the expansion of the most common distributions that are not sufficiently flexible in real life. We need to formulate new distributions and introduce new, more suitable, and simpler established methods. The generalized families of distribution were established by Marshall and Olkin [1] and Gupta et al, [2]. Recently, several attempts have been made to propose methods for increase the flexibility of the exponential distribution, which gave rise to the beta exponential distribution [3]. This enhanced the importance of

*Email: ema19u2009@uoanbar.edu.iq

using the distribution in many areas of life, such as biometric, reliability, and survival analyses. Therefore, the more recent Rayleigh Pareto distribution was proposed, which is a simple alternative in some situations and may be well suited to failure data and to provide more appropriate information about reliability, risk, and rates[4].

In the concept of reliability, the stress-strength model describes the lifetime of a component that has a random variable strength (y) that is put in danger of a random variable stress (x). The failure of the component at the moment is applied to it exceeds the strength and the component will operate at (Y>X). Thus, (Y>X) is the scale of the component reliability. This distribution model has many applications in engineering sciences, rocket engine degradation, and monitoring in hospitals ... etc,. For instance, in a clinical study, X and Y can be assumed as the outcomes of treatment and a control groups, respectively. Then, the quantity R= P(Y>X) can be considered as the effectiveness and the treatment [5]. Several authors assumed different lifetime distributions of random stress variables [6-10]. The objective of this paper is to derive a formula of system reliability in the stress – strength model, when the strength Y and stress X independently follow Rayleigh – Pareto distribution with two parameters. The model involves a system that contains one component with strength Y subjected to stress X. The structure of this paper is as follows. In Section 2, we calculate the distribution function and density functions, along with their properties, of Rayleigh – Pareto distribution. In Section 3, we derive the stress-strength (R) model by using estimation methods that included the maximum likelihood estimation, ordinary least squares, and two types of shrinkage methods. We propose a new type of shrinkage estimator depending on different shrinkage weight coefficients. In Section 4, we conduct a Monte Carlo simulation study to investigate the performance of the proposed estimator using the mean squares error criteria. The results of the analysis showed that the new type of shrinkage estimator, which utilizes shrinkage weight functions, is the most efficient among the other estimators. The conclusions are presented in Section 5.

2- Statistical Properties of the Rayleigh – Pareto Distribution

The distribution function (cdf) of the Rayleigh Pareto distribution can be written as:

$$D(x;\ \tau,\zeta) = 1 - e^{-(\frac{x}{\tau})^{\zeta}} \tag{1}$$

where $0 < x < \infty\,, \zeta > 0\,, \tau > 0\,, \tau$ is the scale parameter and ζ is the shape parameter. Here, the probability density function (p.d.f) of this distribution can be written as:

$$d(x,\tau,\zeta) = \frac{\zeta}{\tau^{\zeta}} x^{\zeta-1} e^{-(\frac{x}{\tau})^{\zeta}} \tag{2}$$

where $0 < x < \infty\,, \zeta > 0\,, \tau > 0$. The limitation behavior of p.d.f. and c.d.f. can be written as:

$$\lim_{x\to 0} d(x;\ \tau,\zeta) = \lim_{x\to 0} \frac{\zeta}{\tau^{\zeta}} x^{\zeta-1} e^{-(\frac{x}{\tau})^{\zeta}} = 0 \tag{3}$$

We notice that $lim_{x\to\infty} \frac{\zeta}{\tau^{\zeta}} x^{\zeta-1} e^{-(\frac{x}{\tau})^{\zeta}}$ is not an integer, then $\lim_{x\to\infty} d(x;\ \tau,\zeta) =$ NAN . Also, the limit of c.d.f . can be written as:

$$\lim_{x\to 0} D(x;\ \tau,\zeta) = 0 \ and \ \lim_{x\to\infty} D(x;\ \tau,\zeta) = 1 \tag{4}$$

Here, the r-th central moment is given by

$$E(x-\mu)^k = \sum_{i=0}^{k} \binom{k}{i} (\tau)^i (-\mu)^{k-i}\ \Gamma(\frac{i+\zeta}{\zeta})\ , k = 1,2,3, \dots.. \tag{5}$$

Also, $\mu = E(x)$. Therefore, the raw moment around the origin can be obtain as,

$$E(x^k) = \tau^k \Gamma \frac{\zeta + k}{\zeta}\,. \tag{6}$$

In particular, the mean and variance are presented by

$$Mean = E(x) = \tau\Gamma\frac{\zeta+1}{\zeta}\ ,\ Variance\ (x) = \tau^2\Gamma\left(\frac{\zeta+2}{\zeta}\right) - \left[\tau\Gamma\left(\frac{\zeta+1}{\zeta}\right)\right]^2 \quad (7)$$

The coefficients of Skewness (sk) and kurtosis (kr) of Rayl. – Par. random variable can be obtained as:

$$sk = \frac{\Gamma\frac{\zeta+3}{\zeta} - 3\Gamma\frac{\zeta+2}{\zeta}\Gamma\frac{\zeta+1}{\zeta} + 2\left(\Gamma\frac{\zeta+1}{\zeta}\right)^3}{\left(\Gamma\frac{\zeta+2}{\zeta} - \left(\Gamma\frac{\zeta+1}{\zeta}\right)^2\right)^{\frac{3}{2}}} \quad (8)$$

$$kr = \frac{\left(-3\Gamma\frac{\zeta+1}{\zeta}\right)^4 + 6\left(\left(\frac{\zeta+1}{\zeta}\right)^2\right)\Gamma\frac{\zeta+2}{\zeta} - 4\Gamma\frac{\zeta+4}{\zeta}\Gamma\frac{\zeta+1}{\zeta}\Gamma\frac{\zeta+2}{\zeta}}{\left(\Gamma\frac{\zeta+2}{\zeta} - \left(\Gamma\frac{\zeta+1}{\zeta}\right)^2\right)^2} - 3 \quad (9)$$

And the coefficient of variation (cv) is presented by

$$cv = \frac{\sqrt{\Gamma\frac{\zeta+2}{\tau} - \left(\Gamma\left(\frac{\zeta+1}{\zeta}\right)^2\right)^2}}{\Gamma\frac{\zeta+1}{\zeta}} \quad (10)$$

So that, the median x_{med} and mode x_{mod} of the Rayl. – Par. random variable can be written as

$$x_{med} = \tau\,(\ln 2)^{\frac{1}{\zeta}}\ ,\ x_{mod} = \tau\left(\frac{\zeta-1}{\zeta}\right)^{\frac{1}{\zeta}} \quad (11)$$

The formula of the geometric mean is presented by

$$\mu_G = E\left(\sqrt{x}\,\right) = \tau^{\frac{1}{2}}\,\Gamma\frac{2\zeta+1}{2\zeta} \quad (12)$$

Therefore, the formula of Shannon Entropy of Raly. – Par. random variable can be written as

$$E_\tau(x) = E(-\ln d(x)) = -\ln(\frac{\zeta}{\tau}) - (\zeta-1)\left(\ln(\tau) + \Gamma\frac{\zeta+1}{\zeta}\right) - \ln(\tau) + 1 \quad (13)$$

Consider $x_1, x_2, \dots, x_n$ as a random sample of size n of Raly. – Par. distribution. Let $x_{1:n}, x_{2:n}, \dots, x_{n:n}$ be the corresponding order statistics. Then, the formula of p. d. f. of the i^{-th} order statistics can be found via the following standard formula statistics:

$$d_{i,n}(x) = \frac{n!}{(i-1)!\,(n-1)!}\left(\frac{\zeta}{\tau^\zeta}\ x^{\zeta-1}\ e^{-\left(\frac{x}{\tau}\right)^\zeta}\right)\left(1 - e^{-\left(\frac{x}{\tau}\right)^\zeta}\right)^{i-1}\left(e^{-\left(\frac{x}{\tau}\right)^\zeta}\right)^{n-i} \quad (14)$$

If i=1, then the p. d. f. of minimum is formulated as

$$d_{1,n}(x) = \frac{n\zeta}{\tau^\zeta}\ x^{\zeta-1}\ e^{-\left(\frac{x}{\tau}\right)^\zeta}\left(e^{-\left(\frac{x}{\tau}\right)^\zeta}\right)^{n-1} \quad (15)$$

If i=n, then the p. d. f. of maximum is formulated as

$$d_{n,n}(x) = \frac{n\zeta}{\tau^\zeta}\ x^{\zeta-1}\ e^{-\left(\frac{x}{\tau}\right)^\zeta}\left(1 - e^{-\left(\frac{x}{\tau}\right)^\zeta}\right)^{n-1} \quad (16)$$

Therefore, if i = m+1, then the p.d.f. of the median is formulated as

$$d_{m+1,n}(x) = \frac{n!}{(i-1)!\,(n-1)!}\left(\frac{\zeta}{\tau^\zeta}\ x^{\zeta-1}\ e^{-\left(\frac{x}{\tau}\right)^\zeta}\right)\left(1 - e^{-\left(\frac{x}{\tau}\right)^\zeta}\right)^{m}\left(e^{-\left(\frac{x}{\tau}\right)^\zeta}\right)^{n-m-1} \quad (17)$$

where n , m $\in$ R. The reliability $R(x)$ and hazard $h(x)$ functions can be written as

$$R(x) = e^{-\left(\frac{x}{\tau}\right)^{\zeta}} \quad , \quad h(x) = \frac{\zeta}{\tau^{\zeta}} x^{\zeta-1} \tag{18}$$

3- The single system of reliability estimated in Stress – strength model

Generally, the stress – strength model is used in many applications in engineering, such as strength failure and system collapse. Let Y be the strength and X be the stress random variables that are independent of each other and follow the Rayl. – Par., respectively, with two different parameters. The p. d. f. of strength Y and the P. d. f. of stress X are given as follows [11]:

$d(y) = \frac{\zeta_1}{\tau^{\zeta_1}} y^{\zeta_1-1} e^{-\left(\frac{y}{\tau}\right)^{\zeta_1}}$ and $d(x) = \frac{\zeta_2}{\tau^{\zeta_2}} x^{\zeta_2-1} e^{-\left(\frac{x}{\tau}\right)^{\zeta_2}}$, for $0 < Y < \infty, \zeta_1 > 0, \zeta_2 > 0$, $\tau > 0$

where τ is the scale parameter and ζ is the shape parameter. The system reliability (R) in the first stress – strength model is defined as

$$R = p(Y > X) = \int_0^{\infty}\int_0^{y} d(x)d(y) \quad dxdy$$

$$R = \int_0^{\infty} \frac{\zeta_1}{\tau^{\zeta_1}} y^{\zeta_1-1} e^{-\left(\frac{y}{\tau}\right)^{\zeta_1}} \left(1 - e^{-\left(\frac{y}{\tau}\right)^{\zeta_2}}\right) \quad dy \tag{19}$$

$$= 1 - \int_0^{\infty} \frac{\zeta_1}{\tau^{\zeta_1}} y^{\zeta_1-1} e^{-\left(\frac{y}{\tau}\right)^{\zeta_1}} \; e^{-\left(\frac{y}{\tau}\right)^{\zeta_2}} \, \mathrm{dy}$$

$$= 1 - \int_0^{\infty} e^{-z} e^{-z\left(\frac{\zeta_2}{\zeta_1}\right)} dz, \text{ where } z = \left(\frac{y}{\tau}\right)^{\zeta_1}$$

$$R = \frac{\zeta_2}{\zeta_1 + \zeta_2} \tag{20}$$

3-1- Maximum Likelihood Estimator (MLE) [12]

The maximum likelihood estimation method is carefully used in complete samples to estimate the parameters of Raly. – Par. distribution. Let $y_1, y_2, \ldots, y_{n1}$ be a Raly. – Par. random complete sample with parameter vector $v = (\zeta, \tau)^T$, when τ is known and the shape parameter ζ_1 is unknown. Then, the likelihood function $d(y_i, \zeta_1, \tau)$ in equation (2) is

$$L = \prod_{i=1}^{n1} d(y_i, \zeta_1, \tau)$$

$$= \prod_{i=1}^{n1} \frac{\zeta_1}{\tau^{\zeta_1}} {y_i}^{\zeta_1-1} e^{-\left(\frac{y_i}{\tau}\right)^{\zeta_1}}$$

$$= \frac{{\zeta_1}^{n1}}{\tau^{n1\zeta_1}} \left(\prod_{i=1}^{n1} y_i^{\zeta_1-1}\right) e^{-\sum_{i=1}^{n1}\left(\frac{y_i}{\tau}\right)^{\zeta_1}} \tag{21}$$

The natural logarithm in equation (21) is presented by

$$\ln L = \ln \frac{{\zeta_1}^{n1}}{\tau^{n1\zeta_1}} + \sum_{i=1}^{n1} ln\, y_i^{\zeta_1-1} + \ln e^{-\sum_{i=1}^{n1}\left(\frac{y_i}{\tau}\right)^{\zeta_1}}$$

$$= n1 \ln \zeta_1 - n1\zeta_1 \ln \tau + (\zeta_1 - 1)\sum_{i=1}^{n1} \ln y_i - \sum_{i=1}^{n1}\left(\frac{y_i}{\tau}\right)^{\zeta_1} \tag{22}$$

The partial derivative for equation (22) with respect to ζ, which equates the results to zero, is presented by $\frac{\partial \ln L}{\partial \zeta_1} = \frac{n1}{\zeta_1} - n1 \ln \tau + \sum_{i=1}^{n1} \ln y_i - \sum_{i=1}^{n1}\left(\frac{y_i}{\tau}\right)^{\zeta_1} \ln\left(\frac{y_i}{\tau}\right) = 0$. We find ζ_1 by using numerical methods. The MLE method for the shape parameter ζ_1 is presented by

$$\hat{\zeta}_{1_{ml}} = \frac{n_1}{n_1 \ln \tau - \sum_{i=1}^{n_1} ln\, y_i + \sum_{i=1}^{n}\left(\frac{y_i}{\tau}\right)^{\zeta_1} \ln\left(\frac{y_i}{\tau}\right)} \tag{23}$$

In the same way, let $x_1, x_2, \dots, x_{n2}$ be a random sample of the stress X, which is distributed as Raly. – Par. distribution, when τ is known and the shape parameter ζ_2 is unknown. Then, the likelihood function $d(x_i, \zeta_2, \tau)$ in equation (2), for the MLE method, is presented by

$$\hat{\zeta}_{2_{ml}} = \frac{n_2}{n_2 \ln \tau - \sum_{i=1}^{n_2} ln\, x_i + \sum_{i=1}^{n} \left(\frac{x_i}{\tau}\right)^{\zeta_2} ln\left(\frac{x_i}{\tau}\right)} \tag{24}$$

where $\hat{\tau}_1 = \sqrt[\zeta_1]{\frac{\sum_{i=1}^{n1} y_i}{n1}}$ and $\hat{\tau}_2 = \sqrt[\zeta_2]{\frac{\sum_{i=1}^{n2} x_i}{n2}}$

where n_1 and n_2 are the sizes of Y and X samples, respectively. Then, the corresponding maximum likelihood estimator of system reliability (R) in the (S – S) model of single component is given as

$$\widehat{\mathrm{R}}_{ml} = \frac{\hat{\zeta}_{2_{ml}}}{\hat{\zeta}_{1_{ml}} + \hat{\zeta}_{2_{ml}}} \tag{25}$$

3-2- Least squares (LS) method [13]

The least squares method's estimators can be created by minimizing the sum of square error between the value and its expected value. This estimation method is very popular for model fitting, especially in linear and non-linear regressions**.**

$$s_1 = \sum_{i=1}^{n_1}(D(y_i) - E(D(y_i)))^2 \text{ and } s_2 = \sum_{j=1}^{n_2}\left(D(x_j) - E\left(D\left(x_j\right)\right)\right)^2 \tag{26}$$

where E (D(y_i))) and E(D(x_j))) are equal to Li, Lj is the plotting location, where $\mathrm{L_i} = \frac{\mathrm{i}}{\mathrm{n_1}+1}$, where $\mathrm{i} = 1,2, \dots, \mathrm{n}_1$ and $\mathrm{L_j} = \frac{\mathrm{j}}{\mathrm{n_2}+1}$, for $\mathrm{j} = 1,2, \dots, \mathrm{n}_2$. Suppose that $y_1, y_2, \dots, y_{n1}$ is a random sample, where Yi is the strength random variable of Raly. – Par. distribution with sample size n_1 and X is the stress random variable of Raly. – Par. distribution with sample size n_2. From the distribution function (1), we get

$$\mathrm{D}(y_i) = 1 - \mathrm{e}^{-(\frac{y_i}{\tau})^{\zeta_1}} \text{ and } D\left(x_j\right) = 1 - e^{-(\frac{x_j}{\tau})^{\zeta_2}}$$

$$\mathrm{e}^{-(\frac{y_i}{\tau})^{\zeta_1}} = 1 - \mathrm{D}(y_i) \text{ and } e^{-(\frac{x_j}{\tau})^{\zeta_2}} = 1 - D\left(x_j\right)$$

Oversimplification and shifting of $\mathrm{D}(y_i)$ and $D\left(x_j\right)$ by plotting the location (Li, Lj) and equating to zero, yields

$$\ln(e^{-(\frac{y_i}{\tau})^{\zeta_1}}) = ln(1 - Li) \text{ and } \ln(e^{-(\frac{x_j}{\tau})^{\zeta_2}}) = ln(1 - Lj)$$

$$\zeta_1 \ln\left(\frac{y_i}{\tau}\right) - [\ln - \ln(1 - Li)] = 0 \text{, and } \zeta_2\, ln\left(\frac{x_i}{\tau}\right) - [ln - ln(1 - Lj)] = 0 \tag{27}$$

By the substitution of (27) in (26), taking the first derivative with respect to the unknown shape parameters ζ_1 and ζ_2, and equating the results to zero, we get

$$\hat{\zeta}_{1_{ls}} = \frac{\sum_{i=1}^{n_1}[ln - ln(1-Li)]\, ln\left(\frac{y_i}{\tau}\right)}{\sum_{i=1}^{n_1}\left(ln\left(\frac{y_i}{\tau}\right)\right)^2} \tag{27a}$$

and

$$\hat{\zeta}_{2_{ls}} = \frac{\sum_{j=1}^{n_2}[ln - ln(1-Lj)]\, ln\left(\frac{x_i}{\tau}\right)}{\sum_{j=1}^{n_2}\left(ln\left(\frac{x_i}{\tau}\right)\right)^2}. \tag{27b}$$

By replacing $\hat{\zeta}_{1_{ls}}$ and $\hat{\zeta}_{2_{ls}}$ in equation (20), the corresponding least squares estimator of system reliability (R) in the (S – S) model of single component becomes as in the following

$$\widehat{\mathrm{R}}_{ls} = \frac{\hat{\zeta}_{2ls}}{\hat{\zeta}_{1ls} + \hat{\zeta}_{2ls}} \tag{28}$$

3-3- Shrinkage Estimation Method

The method of shrinkage estimation is the line of Bayesian subject on prior information concerning the value of the explicit parameter ζ from former researchers or previous studies.

Yet, in certain situations, an initial guess value (a natural origin) ζ_o of ζ becomes a prior information. It is natural to begin with $\hat{\zeta}$ (e.g., MLE) and adjust it by moving ζ_o. Thompson has suggested the problem of shrinking an unbiased estimator $\hat{\zeta}$ of the parameter ζ toward a prior information (a natural origin) ζ_o by the shrinkage estimator $\psi(\hat{\zeta})\,\hat{\zeta}+(1-\psi(\hat{\zeta}))\,\zeta_o$, $0\leq\psi(\hat{\zeta})\leq1$, which is more efficient than $\hat{\zeta}$ if ζ_o is close to ζ and less efficient than $\hat{\zeta}$ otherwise. Al – Joboori, in 2014, explained this method as Thompson described; the prior information ζ_o is a natural origin and, as such, may arise from any one of a number of reasons. For example we are estimating ζ and: (a) we believe that ζ_o is close to the true value of ζ, or (b) we are concerned that ζ_o may be near the true value of ζ. That is, a negative impact could happen if $\zeta_o=\zeta$, but we were unable to detect it. That is, a negative impact could happen if $\zeta_o\approx\zeta$ and we do not use ζ_o. In this paper. we may assume the regions as follows:

$(R) = [\zeta_o-\epsilon, \zeta_o+\epsilon]$, $\epsilon = 0.01$ [14]

The $(\hat{\zeta})$ is called a shrinkage (weight) coefficient when $0 \leq (\hat{\zeta}) \leq 1$, which denotes the confidence of $\hat{\zeta}$, and (1- $\psi(\hat{\zeta})$) denotes the confidence of ζ_o. Thompson remarked that the shrinkage coefficient may possibly be a function of $\hat{\zeta}$ or may be constant. Now , it is possible to use the shrinkage estimation method (Sh) to estimate the parameter in the Rayl.-Par. distribution, by applying the shrinkage (weight) coefficient $\psi\left(\hat{\zeta}_i\right) = K$ in

$$\hat{\zeta}_{ish} = \psi(\hat{\zeta}_i)\hat{\zeta}_{iML} + (1 - \psi(\hat{\zeta}_i))\zeta_{io} \quad , \quad i=1,2 \tag{29}$$

3 – 3 – 1 Shrinkage weight function estimation method (tsh) [15]

In this subsection , the shrinkage weight factor will be considered as a function of size n_1 and n_2,

respectively, taking the forms below, as $\psi(\hat{\zeta}_1) = \left|\frac{\tan n_1}{n_1}\right|$, $\psi(\hat{\zeta}_2) = \left|\frac{\tan n_2}{n_2}\right|$, and $0 \leq \psi(\hat{\zeta}) \leq 1$, where ,$n_1$ and n_2 refer to the sample size of Y and X, respectively.

Therefore, the shrinkage estimator, using shrinkage weight functions of $\hat{\zeta}_1$ and $\hat{\zeta}_2$ defined in equation (29), will be

$$\hat{\zeta}_{i_{tsh}} = \left|\frac{\tan n_i}{n_i}\right| \hat{\zeta}_{i_{ML}} + \left(1 - \left|\frac{\tan n_i}{n_i}\right|\right) \zeta_{i_o} \tag{30}$$

$$\hat{\zeta}_{1_{tsh}} = \left|\frac{\tan n_1}{n_1}\right| \hat{\zeta}_{1_{ML}} + \left(1 - \left|\frac{\tan n_1}{n_1}\right|\right) \zeta_{1_o} \tag{31}$$

$$\hat{\zeta}_{2_{tsh}} = \left|\frac{\tan n_2}{n_2}\right| \hat{\zeta}_{2_{ML}} + \left(1 - \left|\frac{\tan n_2}{n_2}\right|\right) \zeta_{2_o} \tag{32}$$

By substituting $\hat{\zeta}_{1_{tsh)}}$ and $\hat{\zeta}_{2_{tsh)}}$ in equation (20), the reliability estimation of the stress – strength model, using the shrinkage function estimator of single component, will become

$$\hat{R}_{\mathbf{tsh}} = \frac{\hat{\zeta}_{2_{\mathbf{tsh}}}}{\hat{\zeta}_{1_{\mathbf{tsh}}} + \hat{\zeta}_{2_{\mathbf{tsh}}}} \tag{33}$$

3 – 3 – 2 Beta Shrinkage Factor (bsh) [16]

In this subsection, the hypothesis of $(\hat{\zeta}) = \beta(1,n_i)$ for the Beta shrinkage (weight) coefficient has been taken as $\psi(\hat{\zeta}_1) = \beta(1,n_1)$ and $\psi(\hat{\zeta}_2) = \beta(1,n_2)$. This suggests the following shrinkage estimators:

where , $\beta(1, n_i) = (1 - y)^{n_i-1}$, $\quad 0 < y < 0$ (34-c)

$$\hat{\zeta}_{1_{bsh}} = (1, n_1)\,\hat{\zeta}_{1_{ML}} + (1-\beta(1, n_1))\,\hat{\zeta}_{1_o} \tag{34}$$

$$\hat{\zeta}_{2_{bsh}} = (1, n_2)\,\hat{\zeta}_{2_{ML}} + (1-\beta(1, n_2))\,\hat{\zeta}_{1_o} \tag{35}$$

By substituting $\hat{\zeta}_{1_{bsh)}}$ and $\hat{\zeta}_{2_{bsh)}}$ in equation (20), the reliability estimation of the stress – strength model, using Beta shrinkage (weight) coefficient, will become:

$$\hat{R}_{bsh} = \frac{\hat{\zeta}_{2_{bsh}}}{\hat{\zeta}_{1_{bsh}} + \hat{\zeta}_{2_{bsh}}}. \quad (36)$$

3 – 3 – 3 – The proposed modified shrinkage weight function

We applied the modified shrinkage coefficient $\psi\left(\hat{\zeta}_i\right) = \frac{e^{\left(\frac{\sin(n_1)}{n_2}\right)}}{n_1 n_2}$, for $i = 1, 2$, in

$$\hat{\zeta}_{imsh} = \psi(\hat{\zeta}_i)\hat{\zeta}_{iML} + \left(1 - \psi(\hat{\zeta}_i)\right)\zeta_{io}, \quad i=1,2 \quad (37)$$

to obtain the constant shrinkage estimators of ζ_1 and ζ_2, as below

$$\hat{\zeta}_{1_{msh}} = \psi(\hat{\zeta}_1)\hat{\zeta}_{1_{ML}} + ((1 - \psi(\hat{\zeta}_1))\zeta_{1_o} \quad (38)$$

$$\hat{\zeta}_{2_{msh}} = \psi(\hat{\zeta}_2)\hat{\zeta}_{2_{ML}} + \left(1 - \psi(\hat{\zeta}_2)\right)\zeta_{2_o} \quad (39)$$

By substituting equations (38) and (39) in equation (20), the reliability of the estimation of the stress – strength model, , using the constant shrinkage coefficient is

$$\hat{R}_{msh} = \frac{\hat{\zeta}_{2_{msh}}}{\hat{\zeta}_{1_{msh}} + \hat{\zeta}_{2_{msh}}} \quad (40)$$

4. Monte Carlo simulation study

Simulation is a numerical technique for conducting experiments on the computer. Monte Carlo simulation is a computer experiment involving random sampling from probability distributions. In order to verify the performance of the proposed estimation method, which is introduced to estimate the reliability of a single component system, a Monte Carlo simulation was used. The proposed estimation method was implemented using a variety of sample numbers (25, 50, 75, 100). The statistical outcomes for every sample were based on mean squared errors criteria with 1000 replicates. The following steps explain Monte Carlo simulation for each model.

Step 1: Initialization and generation of random samples, that follow the continuous uniform distribution defined on the interval (0,1), to find the performance of x (stress) and y (strength) as $p_1, p_2, \ldots, p_{n1}$ and $q_1, q_2, \ldots, q_{n2}$, respectively, as follows:

U~ Uniform (0,1) [17].

Step 2: Transformation of the above uniform random samples to random samples of Rayleigh – Pareto distribution, using the cumulative distribution function (cdf) as $D(x; \tau, \zeta) = 1 - e^{-\left(\frac{x}{\tau}\right)^{\zeta}}$. Then,

$Y_i = \tau \ln[1 - D(u)]^{-\frac{1}{\zeta}}$ for $i = 1,2,\ldots,n_1$ and $X_j = \tau \ln[1 - D(w)]^{-\frac{1}{\zeta}}$, for $j = 1,2,\ldots,n_2$, where u and w are random variables of Uniform (0,1).

Step 3: τ is considered as the known parameter of the mean of sample and ζ is considered as the unknown parameter. The MLE estimators $\hat{\zeta}_{1_{ml}}$ and $\hat{\zeta}_{2_{ml}}$ were calculated, respectively, from equations (23) and (24).

Step 4 : The OLS estimators $\hat{\zeta}_{1_{ls}}$ and $\hat{\zeta}_{2_{ls}}$ were calculated, respectively, from equations (27a) and (27b).

Step 5: The shrinkage weight function estimators $\hat{\zeta}_{1_{tsh}}$ and $\hat{\zeta}_{2_{tsh}}$ were calculated, respectively, from equations (31) and (32).

Step 6 : The Beta shrinkage estimators $\hat{\zeta}_{1_{bsh}}$ and $\hat{\zeta}_{2_{bsh}}$ were calculated , respectively, from equations (34) and (35).

Step 7 : The modified shrinkage weight function estimators $\hat{\zeta}_{1_{msh}}$ and $\hat{\zeta}_{2_{msh}}$ were calculated from equations (38) and (39),respectively

Step 8 : The estimated reliability values ($\hat{R}_{ml}$, $\hat{R}_{ls}$, $\hat{R}_{tsh}$, $\hat{R}_{bsh}$ and $\hat{R}_{msh}$) of the stress – strength model from different types of estimation methods were calculated from equation (25) , (28) , (33) , (36) and (40), respectively. The results in Tables 1, 3, 5, and 7 demonstrate the reliability of estimation ,while the results in Tables 2, 4, 6, and 8 show that the comparison between these methods when the criterion of mean squares error is used. However, the MSE of all estimators depended on the values of the samples size, i.e. n_1 and n_2. The performance of the reliability estimator that depends on the modified shrinkage weight coefficient ($\hat{R}_{msh}$) was better than that of other estimators. However, the performance of the reliability estimators that depend on other shrinkage parameters ($\hat{R}_{tsh}$ and $\hat{R}_{bsh}$), among others, was good, especially when the value of $\hat{\zeta}_{1_{sh}}$ approximates ~ to $\hat{\zeta}_{2_{sh}}$. The performance of other reliability estimators does not vary significantly.

Table 1-Estimated value of R= 0.57143 , when $\tau = 1$, $\hat{\zeta}_1 = 1.5$, and $\hat{\zeta}_2 = 2$.

Sample size of Y n_1	Sample size of X n_2	$\hat{R}_{ml}$	$\hat{R}_{ls}$	$\hat{R}_{msh}$	$\hat{R}_{tsh}$	$\hat{R}_{bsh}$
25	25	0.56535	0.45276	0.57141	0.56553	0.57131
	50	0.51598	0.63055	0.5713	0.56577	0.57316
	75	0.76148	0.54184	0.57176	0.5657	0.57773
	100	0.38038	0.63132	0.57078	0.56574	0.56967
50	25	0.4974	0.60266	0.57122	0.56574	0.56791
	50	0.5499	0.56453	0.57137	0.56571	0.57122
	75	0.70487	0.61756	0.57169	0.56573	0.57331
	100	0.34385	0.59159	0.57062	0.56572	0.56956
75	25	0.51726	0.52699	0.57126	0.56569	0.56739
	50	0.56867	0.60768	0.5714	0.56573	0.57053
	75	0.48397	0.56985	0.57118	0.56572	0.57082
	100	0.57056	0.62321	0.57146	0.56572	0.57185
100	25	0.5857	0.62521	0.5714	0.56574	0.56787
	50	0.55198	0.61328	0.57133	0.56572	0.57001
	75	0.51347	0.55951	0.57125	0.56571	0.57068
	100	0.47634	0.55152	0.57117	0.56571	0.57097

Table 2-Estimated mean squares error of the ML, LS, MSH , TSH and BSH reliability estimators when R = 0.57143, $\tau = 1$, $\hat{\zeta}_1 = 1.5$ and, $\hat{\zeta}_2 = 2$

Sample size of Y n_1	Sample size of X n_2	ML	LS	MSH	TSH	BSH	Best method
25	25	3.6928e08	1.4082e-05	2.306e-13	3.474e-08	1.4558e-11	MSH
	50	3.0749e06	3.4959e-06	1.6118e11	3.2064e08	3.0064e-09	MSH
	75	3.6118e05	8.7559e07	1.109e-10	3.2798e08	3.9723e-08	MSH
	100	3.6501e05	3.5876e-06	4.2456e10	3.2354e08	3.0837e-09	MSH
50	25	5.4808e06	9.7515e07	4.4047e11	3.232e-08	1.2393e-08	MSH
	50	4.6333e07	4.7549e08	2.9062e12	3.2658e08	4.3339e-11	MSH
	75	1.7807e05	2.1277e06	6.9078e11	3.2496e08	3.5455e09	MSH
	100	5.1794e05	4.0645e07	6.5139e10	3.2594e08	3.4843e09	MSH
75	25	2.934e-06	1.9747e06	2.9515e11	3.2886e08	1.6344e-08	MSH
	50	7.6208e09	1.3141e-06	6.7151e13	3.2526e08	7.9919e-10	MSH
	75	7.6483e06	2.5023e09	6.0435e11	3.2645e08	3.735e-10	MSH
	100	7.486e-10	2.6811e-06	7.417e-13	3.2566e08	1.8074e-10	MSH
100	25	2.0381e07	2.8924e06	5.6585e13	3.2387e08	1.2691e-08	MSH
	50	3.781e-07	1.7514e-06	1.0103e11	3.2546e08	2.0035e09	MSH
	75	3.3588e06	1.4201e-07	3.1776e11	3.2662e08	5.6185e-10	MSH
	100	9.0428e06	3.9623e-07	6.7565e11	3.2669e08	2.1365e-10	MSH

Table 3-Estimated value of R=0.66667, when $\tau = 1$, $\hat{\zeta}_1 = 1.5$, $\widehat{\text{and } \zeta_2} = 3$

Sample size of Y n_1	Sample size of X n_2	$\hat{R}_{ml}$	$\hat{R}_{ls}$	$\hat{R}_{msh}$	$\hat{R}_{tsh}$	$\hat{R}_{bsh}$
25	25	0.40557	0.54277	0.66611	0.65981	0.66225
	50	0.57308	0.61803	0.66652	0.65997	0.66873
	75	0.51915	0.61726	0.66636	0.65998	0.66815
	100	0.45247	0.61652	0.66621	0.65998	0.66751
50	25	0.49716	0.62377	0.6663	0.65997	0.66209
	50	0.51454	0.67195	0.66638	0.66	0.66554
	75	0.52703	0.67952	0.6664	0.66001	0.66662
	100	0.55278	0.63896	0.66648	0.66	0.66741
75	25	0.51448	0.69314	0.66628	0.66002	0.66177
	50	0.5502	0.58278	0.66644	0.65998	0.66505
	75	0.48399	0.65064	0.66627	0.66	0.66567
	100	0.50065	0.66126	0.66631	0.66	0.66624
100	25	0.53164	0.685	0.66633	0.66001	0.66145
	50	0.52436	0.63613	0.66632	0.66	0.66455
	75	0.51832	0.67364	0.6663	0.66	0.66554
	100	0.55328	0.66836	0.66642	0.66	0.66622

Table 4-Estimated mean squares error of the ML, LS, MSH , TSH, and BSH reliability estimators when R= 0.66667, $\tau = 1$, $\hat{\zeta}_1 = 1.5$, and, $\hat{\zeta}_2 = 3$

Sample size of Y n_1	Sample size of X n_2	ML	LS	MSH	TSH	BSH	Best method
25	25	6.8173e05	1.5351e05	3.0728e10	4.6967e-08	1.9501e08	MSH
	50	8.7592e06	2.3659e06	2.1997e11	4.4892e-08	4.2466e09	MSH
	75	2.1762e05	2.4414e06	9.1527e11	4.4748e-08	2.2062e09	MSH
	100	4.5879e05	2.5147e06	2.0795e10	4.4676e-08	7.1042e10	MSH
50	25	2.8731e05	1.8403e06	1.3165e10	4.4828e-08	2.0905e08	MSH
	50	2.3144e05	2.7928e08	8.4818e11	4.4381e-08	1.2691e09	MSH
	75	1.9499e05	1.6517e07	7.2979e11	4.4375e-08	1.9324e12	MSH
	100	1.2971e05	7.6777e07	3.5042e11	4.45e-08	5.5223e10	MSH
75	25	2.3162e05	7.006e-07	1.466e-10	4.4213e-08	2.3963e08	MSH
	50	1.3565e05	7.037e-06	5.2127e11	4.4717e-08	2.6082e09	MSH
	75	3.3369e05	2.5675e07	1.6123e10	4.4467e-08	9.9824e10	MSH
	100	2.756e-05	2.9222e08	1.2899e10	4.4442e-08	1.8174e10	MSH
100	25	1.8232e05	3.361e-07	1.1182e10	4.4314e-08	2.7227e08	MSH
	50	2.025e-05	9.3229e07	1.2154e10	4.4507e-08	4.4761e09	MSH
	75	2.2007e05	4.8673e08	1.3784e10	4.442e-08	1.2786e09	MSH
	100	1.2857e05	2.8557e09	6.2317e11	4.4433e-08	1.9722e10	MSH

Table 5-Estimated value of R=0.46154, when $\tau = 1$, $\hat{\zeta}_1 = 3.5$, and $\hat{\zeta}_2 = 3$

Sample size of Y n_1	Sample size of X n_2	$\hat{R}_{ml}$	$\hat{R}_{ls}$	$\hat{R}_{msh}$	$\hat{R}_{tsh}$	$\hat{R}_{bsh}$
25	25	0.49308	0.35587	0.46159	0.45676	0.46191
	50	0.49585	0.43866	0.46159	0.45691	0.46549
	75	0.53514	0.48496	0.46167	0.45694	0.46691
	100	0.50549	0.48434	0.46164	0.45693	0.46728

50	25	0.43969	0.33059	0.4615	0.45682	0.45773
	50	0.4865	0.46797	0.46158	0.45693	0.46169
	75	0.54547	0.52501	0.46168	0.45694	0.46314
	100	0.50822	0.47904	0.46164	0.45693	0.46354
75	25	0.50366	0.35642	0.46159	0.45687	0.45717
	50	0.50882	0.41929	0.4616	0.45691	0.46061
	75	0.48486	0.51714	0.46157	0.45693	0.46163
	100	0.52912	0.4805	0.46166	0.45693	0.46236
100	25	0.51181	0.38829	0.46158	0.4569	0.4565
	50	0.50645	0.51464	0.46158	0.45693	0.45996
	75	0.48795	0.49648	0.46156	0.45693	0.46103
	100	0.48467	0.42005	0.46158	0.45692	0.46161

Table 6-Estimated mean squares error of the ML, LS, MSH , TSH, and BSH reliability estimators when R= 0.46154, $\tau = 1$, $\hat{\zeta}_1 = 3.5$, and, $\hat{\zeta}_2 = 3$

Sample size of Y n_1	Sample size of X n_2	ML	LS	MSH	TSH	BSH	Best method
25	25	9.951e-07	1.1166e05	2.213e-12	2.2812e08	1.4169e-10	MSH
	50	1.1771e06	5.2344e07	3.1652e-12	2.1436e08	1.5609e-08	MSH
	75	5.4172e06	5.4879e07	1.7054e-11	2.1164e08	2.8838e-08	MSH
	100	1.932e-06	5.1992e07	1.0788e-11	2.1201e08	3.2992e-08	MSH
50	25	4.7743e07	1.7149e05	1.5572e-12	2.2235e08	1.4503e-08	MSH
	50	6.2303e07	4.1372e08	1.4549e-12	2.1261e08	2.1819e-11	MSH
	75	7.045e-06	4.0284e06	1.9278e-11	2.1135e08	2.5502e-09	MSH
	100	2.179e-06	3.063e-07	1.0067e-11	2.1261e08	4.0152e-09	MSH
75	25	1.7746e06	1.1049e05	2.2246e-12	2.1792e08	1.9108e-08	MSH
	50	2.2351e06	1.7849e06	4.0366e-12	2.1394e08	8.5557e-10	MSH
	75	5.4389e07	3.0914e06	1.2264e-12	2.1204e08	7.6093e-12	MSH
	100	4.5671e06	3.5954e07	1.5939e-11	2.1273e08	6.7455e-10	MSH
100	25	2.5269e06	5.3654e06	1.6693e-12	2.1551e08	2.5414e-08	MSH
	50	2.017e-06	2.8194e06	1.6551e-12	2.1196e08	2.4927e-09	MSH
	75	6.9746e07	1.221e-06	5.5293e-13	2.1253e08	2.5941e-10	MSH
	100	5.3515e07	1.7209e06	1.5007e-12	2.1336e08	4.7534e-12	MSH

Table 7-Estimated value of R=0.35714 when $\tau = 1$, $\hat{\zeta}_1$ =5.4 , $\widehat{\text{and}\ \zeta_2} = 3$

Sample size of Y n_1	Sample size of X n_2	$\hat{R}_{ml}$	$\hat{R}_{ls}$	$\hat{R}_{msh}$	$\hat{R}_{tsh}$	$\hat{R}_{bsh}$
25	25	0.51662	0.3842	0.35733	0.35362	0.35868
	50	0.47926	0.32178	0.35729	0.35355	0.36156
	75	0.49516	0.31778	0.35731	0.35355	0.36276
	100	0.5028	0.27813	0.35736	0.35354	0.36327
50	25	0.56546	0.35819	0.35743	0.35358	0.35565
	50	0.48094	0.39831	0.35729	0.35359	0.35772
	75	0.49839	0.38261	0.35731	0.35358	0.35885
	100	0.47683	0.26283	0.35732	0.35355	0.35931
75	25	0.49222	0.30506	0.35729	0.35355	0.35324
	50	0.49241	0.37674	0.35729	0.35358	0.35646
	75	0.45952	0.33365	0.35726	0.35357	0.35744
	100	0.50402	0.38629	0.35735	0.35358	0.35813
100	25	0.49754	0.21506	0.35728	0.35352	0.35263
	50	0.46287	0.34092	0.35725	0.35357	0.35573
	75	0.4892	0.32788	0.3573	0.35357	0.35693
	100	0.50511	0.39464	0.35734	0.35358	0.35749

Table 6-Estimated mean squares error of the ML, LS, MSH , TSH, and BSH reliability estimators when R= 0.35714, $\tau = 1$, $\hat{\zeta}_1 = 4.5$, and, $\hat{\zeta}_2 = 3$

Sample size of Y n_1	Sample size of X n_2	ML	LS	MSH	TSH	BSH	Best method
25	25	2.5434e05	7.3201e-07	3.6547e11	1.2409e-08	2.3496e-09	MSH
	50	1.4913e05	1.2502e-06	2.1605e11	1.2937e-08	1.9549e-08	MSH
	75	1.9048e05	1.5497e-06	2.9558e11	1.2892e-08	3.1553e-08	MSH
	100	2.1217e05	6.2424e-06	4.5587e11	1.2971e-08	3.7558e-08	MSH
50	25	4.3397e05	1.0976e-09	8.3574e11	1.2729e-08	2.2295e-09	MSH
	50	1.5326e05	1.6949e-06	2.2022e11	1.2628e-08	3.3088e-10	MSH
	75	1.9952e05	6.4866e-07	2.9079e11	1.27e-08	2.899e-09	MSH
	100	1.4326e05	8.8943e06	3.0536e11	1.2885e-08	4.7027e-09	MSH
75	25	1.8245e05	2.7122e06	2.095e-11	1.2938e-08	1.5235e-08	MSH
	50	1.8296e05	3.8417e-07	2.1957e11	1.2711e-08	4.6902e-10	MSH
	75	1.048e-05	5.5209e-07	1.4145e11	1.278e08	8.7849e-11	MSH
	100	2.1573e05	8.497e-07	4.3066e11	1.2724e-08	9.6546e-10	MSH
100	25	1.971e-05	2.0188e-05	1.9689e11	1.3146e-08	2.0344e-08	MSH
	50	1.1178e05	2.6308e-07	1.1141e11	1.2773e-08	1.995e-09	MSH
	75	1.7439e05	8.561e-07	2.4492e11	1.2779e-08	4.4951e-11	MSH
	100	2.1893e05	1.406e-06	3.7293e11	1.2726e-08	1.1819e-10	MSH

5- Conclusions

In this paper, we offered a Rayleigh Pareto distribution. We derived the stress-strength model in the cumulative distribution function (cdf) and probability density function (pdf). We studied some statistical properties, including the entropy, mean, mode, median , variance , the r-th moment about the mean, the r-th moment about the origin, reliability, hazard functions, coefficients of variation, and coefficients of skewness and of kurtosis. We estimated the stress- strength reliability (R) using some estimation methods, such as the maximum likelihood estimation, ordinary least squares, and two types of shrinkage methods. We proposed a new type of shrinkage estimation which is dependent on different shrinkage weight coefficients. Finally, A simulation study was used for different situations to show the superiority of these reliability estimators using MATLAB software.

References

[1] Marshall A. and Olkin I. “A new method for adding a parameter to a family of distribution with application to the exponential and Weibull families ”, *Biometrika*, vol. 48, no. 3, pp. 641 – 52, 1997.

[2] Gupta C., Gupta P., & Gupta D. “Modeling failure time data by Lehmann alternatives”, *Communications in Statistic- Theory and Methods*, vol. 27, no. 4, pp. 887 – 904, 1998.

[3] Nadarajah, S. & Kotz, S. “The Exponentiated Type Distributions”, *Acta Applicandae Mathematica*, vol. 92, pp. 97–111, 2006.

[4] Jyothi, P. “Reliability Computation of System Reliability for the New Rayleigh Pareto Distribution”, *International Journal of Science and Research*, vol. 8, no. 1, pp. 2053 – 2055, 2019.

[5] Ventura, L. and Racugno, W. “Recent advances on Bayesian inference for P(X < Y) ”, *Bayesian Analysis*, vol. 6, no. 3, pp. 411 – 428, 2011.

[6] Enis, P., Geisser, S. “Estimation of the probability that Y< X”, JASA,66, 162 – 168.

[7] Mc Cool, J. 1991. “Inference on P(Y<X) in the Weibull case”, *Communications in Statistic-Simulation and Computations*, vol. 20, pp. 129-148, 1971.

[8] Kundu, D. and Gupta, R. “Estimation of P(Y<X) for the generalized exponential distribution”, *Metrika*, vol. 61, no. 3, pp. 291-308, 2005.

[9] Kundu, D. and Gupta, R. "Estimation of P(Y<X) for Weibull distribution", *IEEE Transactions on Reliability*, vol. 55, no. 2, pp. 270-280, 2006.

[10] Arulmozhi, G. "Direct method for reliability computation of k-out-of-n: G systems", *Applied Mathematics and Computation*, vol. 143, pp. 421–429, 2003.

[11] Jyothi, P. "Reliability Computation of System Reliability of a stress-strength model with New Rayleigh Pareto Distribution", *International Journal of Statistics and Applied Mathematics,* vol. 4, no. 5, pp. 112 – 116, 2019.

[12] Cohen, A. and Helm, F. "Estimation in Exponential Distribution", *Technometrics*, vol. 15, pp. 415 – 418, 1973.

[13] Hassan A. and Basheikh H. "Estimation of Reliability in Multi-Component Stress-Strength Model Following Exponential Pareto Distribution", *The Egyptian Statistical Journal*, Institute Of Statistical Studies and Research, Cairo University, vol. 56, no. 2, pp. 82 – 95, 2012.

[14] Al – Joboori A. , et al. "Single and Double Stage Shrinkage Estimators for the Normal Mean with the Variance Cases", *International Journal of Statistics*, vol. 38, no. 2, pp. 1127 – 1134, 2014.

[15] Salman, A. & Sail, F. "Different Estimation Methods for System Reliability Multi-Components model: Exponentiated Weibull Distribution", *Ibn AL-Haitham Journal For Pure and Applied Science*, pp. 363 – 377, 2018.

[16] Salman, A. and Hamed, A. "Estimating the shape parameter for the power function distribution through shrinkage technique", *International Journal of Science and Research*, pp. 78 – 96, 2007.

[17] Jebeli, M. and Deiri, E. "Estimation Methods for the probability denisity fuction and the cumulative distribution function of the pareto – Ralyiigh distribution", *Statistics A Journal of Theoretical and Applied Statistics*, pp. 1 – 17, 2019.