# Delta robot joints control-based linear MPC controller

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**Abstract:** Controlling the joints' angles of a robot is an important step which lead to controlling the robot end effector position and/or speed. Thus, it has been a vast area of interest in research which has good investigating potentials using several control types such as classical, modern and optimal control methods. In this work, a linear model predictive control MPC technique was proposed to control the joints' angle of a three degree of freedom delta robot. The inverse kinematics, direct kinematics, and dynamic model of the robot were analysed. Then, the dynamic model represented in a linearised around an operating point state space model. In order to investigate the performance of the proposed MPC controller a simulator-based MATLAB program was implemented. The simulation results have showed the efficiency of the proposed controller in the joints' angles control problem. This illustrates that the MPC controller can derive the joints' angles to track the desired angles with invisible steady state error.

**Keywords:** MPC technique; delta robot; simulation; parallel manipulator.

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## 1 Introduction

Optimal control theory has high attention in robot control design, such as $H_\infty$ (Jasim and Gu, 2018; Jasim and Yassen, 2017), linear quadratic regulator (LQR) (Jasim and Gu, 2019), and MPC controllers. Designing an optimal controller for a robot joint requires an optimality criterion to be achieved. It is problem has a cost function includes the system states and the control parameters. The cost function is minimised based on the tracks of the control parameters. Model predictive control is a developed technique of optimal control techniques that designed to control the systems via control inputs calculation by satisfying a set of constraints over a finite time horizon.

The parallel robots has many advantage over the serial robots such as, less moving inertia, high accuracy, high velocity and high stiffness to be used in industrial applications (Kuo and Huang, 2017; Poppeova et al., 2012). According to these advantages, different types of parallel robots were designed with different degree of freedom. Even these robots had these advantages but it's still suffer that the workspace is relatively small when the degree of freedom was be less (Lui et al., 2004). The structure of the parallel kinematic is a closed loop chain mechanism ending by an end effector (Poppeova et al., 2012).

Delta robot is one of the most popular types of the parallel robots. It consists of connected arms with joints on a base. A parallelogram enables a yield connect to stay at a fixed introduction regarding an info interface. The utilisation of three such parallelograms limits totally the introduction of the portable stage which stays just with three absolutely translational degrees of opportunity. The information connections of the three parallelograms are mounted on pivoting switches by means of revolute joints. The revolute joints of the turning switches are incited in two diverse routes: with rotational (DC or air conditioning servo) engines or with direct actuators. At long last, a system is utilised to transmit turning movement from the base to an end effector built on the platform (Poppeova et al., 2012). In this paper, MPC controller was implemented to control the delta robot joints.

Nowadays, researchers discussed several control techniques to control the joints of the three degree of freedom parallel mechanism delta robot. Control of the delta robot based on it is kinematic and dynamic model was proposed in Bortoff (2018). The dynamic model was derived using the Hamiltonian or Lagrangian with the Baumgarte's techniques. The results prove that the open loop control system was unstable and its effects on the closed loop control system. An adaptive control technique was addressed in Cuong et al. (2013) to control the delta robot based on its inverse kinematic and nonlinear dynamic model with parameters uncertainties consideration. The robot performance based on the dynamic model was guaranteed. A novel control technique-based 5th order polynomial path generation was introduced in Oberhauser (2016) for linear delta robot. This control technique was overcome the problem of infinite shake spikes. The proposed method was tested practically and the results were satisfied. An independent delta robot joint control method was addressed in Abu-Alkebash et al. (2017). The robot was modelled firstly, and then its motors were controlled via several control methods. A model-based approach control was proposed for delta robot trajectory tracking in Kuo and Huang (2017). In which the proposed approach calculates the robot motors torques using the kinematics and the dynamics of the robot in the first step. Then, in the second step, the torques were used in the outer loop control to complete the feedback control system. In the inner loop only the torques or the sensors current was used. The experimental results were validates the simulation results.

The paper has four different sections; the main delta robot model derivations – inverse kinematics, forward kinematics, dynamic equations, and a linearised model – were illustrated in Section 2. The proposed MPC technique equations were introduced in Section 3. Section 4 provides the simulation results with its discussion. The work was concluded in Section 5.

## 2 Materials and methods

The dynamic system to be controlled is the 3DOF nonlinear delta robot shown in Figure 1. Derivation of it is dynamic model depends on the inverse kinematic transformed to a state space form and linearised around an operating point as illustrated in the next subsections.

**Figure 1** Delta robot (see online version for colours)


### *2.1 Inverse kinematics*

Inverse kinematics solution is to find the joint's angles from the end effector positions. Several methods were conducted to find the delta robot inverse kinematics. The most popular techniques were based on the geometric relationships or Cartesian coordinates. According to the Cartesian coordinates analysis method describes in Can et al. (2018), the three joint's angles $\Phi = [\varphi_1, \varphi_2, \varphi_3]$ can be described by taking a plane projection for each angle, then by choosing a suitable frame as shown in Figure 2. For example, the first angle can be calculated by choosing the joint frame $K_1J_1$ in YZ plane.

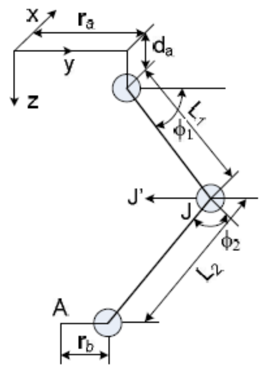
$$\varphi_1 = \tan^{-1}\frac{Z_{J1}}{Y_{K1} - Y_{J1}} \tag{1}$$

By exploit the above advantage using another two planes and rotating the XY plane a (120°) counterclockwise around the $Z$ axis, a new reference frame $\overline{X}\overline{Y}\overline{Z}$ was produced as in equation (2) (Cuong et al., 2013).

$$\begin{cases} \overline{X} = X\cos(\pm 120) + Y\sin(\pm 120) \\ \overline{Y} = -X\sin(\pm 120) + Y\cos(\pm 120) \\ \overline{Z} = Z \end{cases} \tag{2}$$

Now, from the new frame illustrated in equation (2) and using the same procedure described to find the first angle the other two angles can be calculated.

**Figure 2** XY plane projection




## 2.2 Direct kinematics

As the three joint's angles were obtained in the above Sub-section (3.1), the end effector position can be obtained using the following procedure; firstly, based on the Y-axis displacement, then the following vector equation can be obtained.

$$\overrightarrow{OJ_1} = \overrightarrow{OK_1} + \overrightarrow{K_1J_1} + \overrightarrow{J_1J_1} \tag{3}$$

Then the distance from the pivot point to the original quadrant will be:

$$OK_1 = OK_2 = OK_3 = \sqrt{d_A^2 + r_A^2} \tag{4}$$

And the distance from the intersection of the three spheres at the centre base is:

$$J_1\bar{J}_1 = J_2\bar{J}_2 = J_3\bar{J}_3 = r_B \tag{5}$$

Next, if the sphere's radius is $L_2$, then:

$$\begin{cases} K_1J_1 = L_2\cos(\varphi_1) \\ K_2J_2 = L_2\cos(\varphi_2) \\ K_3J_3 = L_2\cos(\varphi_3) \end{cases} \tag{6}$$

and

$$R = \sqrt{d_A^2 + r_A^2 - r_B} \tag{7}$$

$$O\bar{J}_1 = OK_1 + K_1J_1 - J_1\bar{J}_1 \tag{8}$$

where $\bar{J}_1, \bar{J}_2, \bar{J}_3$ are the spheres' centres and their coordinate is ($X$, $Y$, $Z$), then the $\bar{J}_1$ coordinate obtained as:

$$\begin{bmatrix} 0 & R + L_2\cos(\varphi_1) & L_2\sin(\varphi_1) + d_A \end{bmatrix}^T = \begin{bmatrix} X_1 & Y_1 & Z_1 \end{bmatrix}^T \tag{9}$$

Using the same above procedure the coordinates of the $\bar{J}_2$ and $\bar{J}_3$ can be obtained respectively as in the following equations:

$$\begin{bmatrix} \left(R+L_2\cos(\varphi_2)\right)\cos\left(\frac{\pi}{6}\right) & \left(R+L_2\cos(\varphi_2)\right)\sin\left(\frac{\pi}{6}\right) & L_2\sin(\varphi_2)+d_A \end{bmatrix}^T \\ = \begin{bmatrix} X_2 & Y_2 & Z_2 \end{bmatrix}^T \tag{10}$$

$$\begin{bmatrix} -\left(R+L_2\cos(\varphi_3)\right)\cos\left(\frac{\pi}{6}\right) & \left(R+L_2\cos(\varphi_3)\right)\sin\left(\frac{\pi}{6}\right) & L_2\sin(\varphi_3)+d_A \end{bmatrix}^T \\ = \begin{bmatrix} X_3 & Y_3 & Z_3 \end{bmatrix}^T \tag{11}$$

Now, the three spheres intersection can be obtained as:

$$\begin{cases} (X-X_1)^2+(Y-Y_1)^2+(Z-Z_1)^2 = L_1^2 \\ (X-X_2)^2+(Y-Y_2)^2+(Z-Z_2)^2 = L_1^2 \\ (X-X_3)^2+(Y-Y_3)^2+(Z-Z_3)^2 = L_1^2 \end{cases} \tag{12}$$

Then the solution is:

$$\begin{cases} X = \dfrac{a_1 Z + b_1}{d} \\ Y = \dfrac{a_2 Z + b_2}{d} \\ Z = \dfrac{-b \pm \sqrt{\beta}}{2a} \end{cases} \tag{13}$$

According to equation (13), there are two possible solutions for the spheres intersection.

## *2.3 Dynamic equations*

In this section, we are going to perform the inverse dynamic modelling of the parallel manipulator-based mostly upon the principle of virtual work. The inverse dynamics problem is to seek out the mechanism torques and/or forces needed to come up with a desired mechanical phenomenon of the manipulator.

It is typically suitable to specific the manipulator dynamic equations in a very one equation that hides the several little details, however it presents various equations' structure. The state-space equation is obtained by solving the Newton Euler equations symbolically for any robot manipulator, then the dynamic equation was obtained. This dynamic equation can be written as in the following form:

$$T = M(\varphi)\ddot{\varphi} + P(\varphi,\dot{\varphi})\dot{\varphi} + \Gamma(\varphi) \tag{14}$$

where $T$ is the torque, $M(\varphi)$ is a square robot mass matrix, $P(\varphi,\dot{\varphi})$ is the centrifugal and Coriolis vector, and $\Gamma(\varphi)$ is the gravity vector.

### 2.4 Linearised dynamic model

In this work, the linear discrete MPC was implemented to a linearised delta robot dynamic system. The linearised model around an operating point described in Rachedi et al. (2012) is written in a state space form as:

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX + DU \end{cases} \tag{15}$$

where $X = [\varphi_1, \varphi_2, \varphi_3, \dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3]^T$ is the states, $Y = [\varphi_1, \varphi_2, \varphi_3]^T$ is the output vector, $U = [T_1, T_1, T_1]^T$ is the input vector, and $A_{6\times6}$, $B_{6\times3}$, $C_{3\times6}$, $D_{3\times3}$, are the state space matrices. The state space linear matrices are:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.66 & -4.64 & 10.55 & 0 & 0 & 0 \\ 0.17 & -2.50 & 0.82 & 0 & 0 & 0 \\ 3.05 & -4.95 & 9.15 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.81 & 15.58 & -0.81 \\ -3.70 & 3.20 & 6.90 \\ -8.68 & 4.51 & -4.015 \end{bmatrix}$$

$$C = \begin{bmatrix} 30.9897 & -40.9965 & -88.2854 & 0 & 0 & 0 \\ 57.2958 & 0 & 0 & 0 & 0 & 0 \\ -10.0065 & 100.6680 & -47.2894 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

## 3 Model predictive control design

The MPC computations depends on the immediate measurements and the coming estimations values of the outputs. The goal of the MPC control computations is to calculate the control action sequences decide a grouping of control moves which manipulates the suitable changes in the inputs with the goal that the estimated reaction track the set point in an optimal way. The MPC idea is to compute a set of input values at the immediate sampling instant based on the real output, estimated output and the manipulated input. According to its principle as a calculation technique to improve the control performance in process and petrochemical applications, predictive control has undoubtedly become the most widely used advanced control methodology currently in use in industry. MPC now has a sound theoretical basis and its properties are well understood in terms of stability, optimality and robustness (Seborg et al., 2016).

Figure 3 shows the main MPC structure to be implemented to control the delta robot. In which, the future system outputs was estimated depends on the immediate and previous values and the future control moves. The optimiser was calculating these control moves with cost function and constraints consideration to introduce the future inputs to the system.

The linear MPC system in discrete form is:

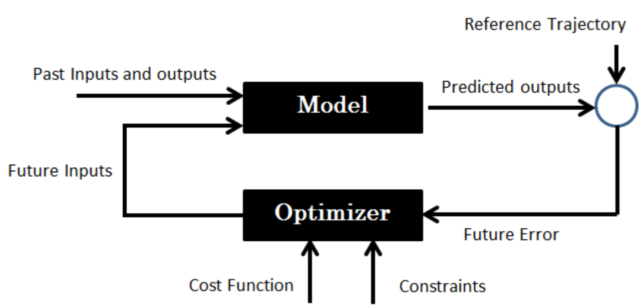
$$\begin{cases} X(k+1) = AX(k) + B_U U(k) + B_v v(k) + B_d d(k) \\ Y(k) = CX(k) + D_v v(k) + D_d d(k) \end{cases} \tag{16}$$

where $v(k)$ is the actual disturbances vector and $d(k)$ is the unmeasured disturbances which modelled as:

$$\begin{cases} X_d(k+1) = ZX_d(k) + VN(k) \\ d(k) = LX_d(k) + SN(k) \end{cases} \tag{17}$$

The proposed MPC controller chooses the $U(k)$ series (Bemporad et al., 2000).

**Figure 3** MPC control structure (see online version for colours)




## 4 Simulation results

In this section, the linearised delta robot model was tested based on the proposed linear MPC controller. Three tests were investigated to ensure the optimality of the proposed controller. The first one was to track the desired three angles $[10°, 10°, 10°]^T$, while the second one was to track the desired three angle $[-20°, -20°, -20°]^T$, and the third one was a different angles of [10°, 0°, –20°, 0°] for the three angles.

As shown in Figures 4, 5, and 6, it is clear that the obtained angles were tracks the desired angles successfully with very small error. These simulation results show the stability of the proposed MPC controller especially when it drove the joints' angles tracks the desired different and fast changed joints' angles.

## 5 Conclusions

In order to control the parallel delta robot joints' angles, this work proposed a linear model predictive control technique. A MATLAB simulator was implemented to examine the effectiveness of the proposed linear MPC method in solving the problem of the joints' angles control. The inverse kinematics, direct kinematics and the dynamic model of the delta robot were derived based on the Cartesian derivation technique. The expected simulation results were obtained with a very small and acceptable error. The next step towards this direction is to track the desired delta robot end effector.

**Figure 4** Joints' angles against desired angles of 10° (see online version for colours)

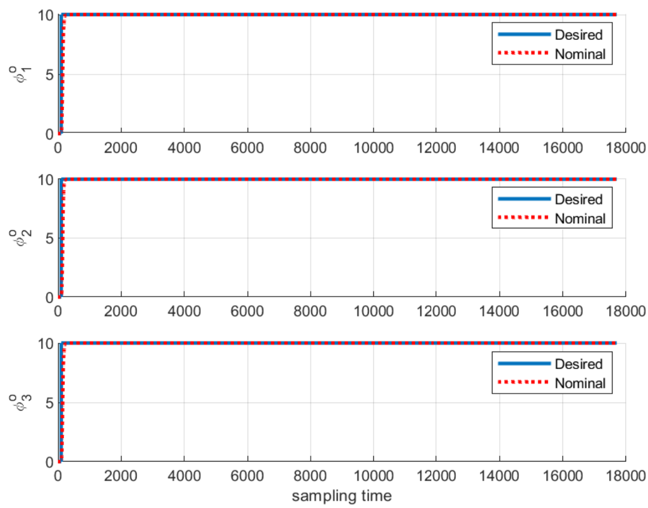



**Figure 5** Joints' angles against desired angles of –20° (see online version for colours)

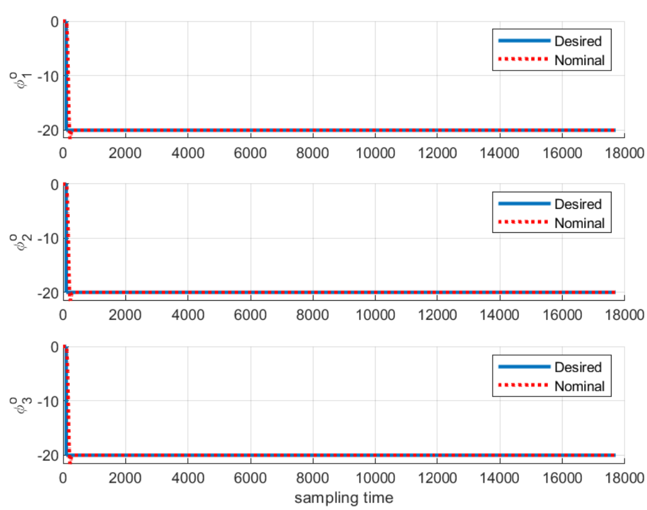
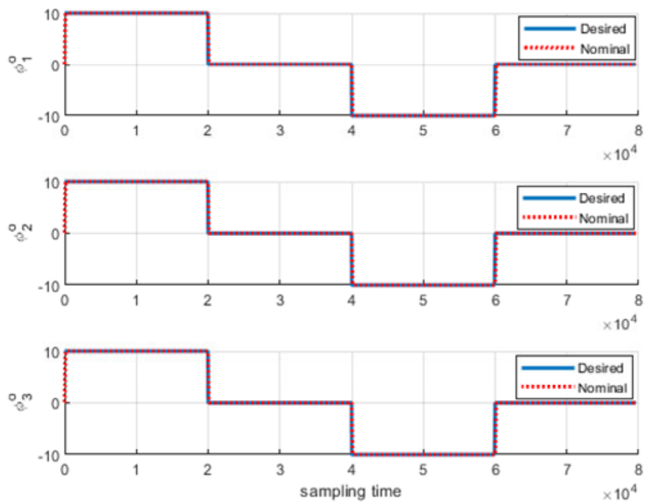


**Figure 6** Joints' angles against different desired angles (see online version for colours)

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