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A Multi-Swarm Structure for Particle Swarm Optimization: Solving the Welded Beam Design Problem

Ahmed T. Kamil¹, Hadeel M. Saleh², Israa Hussain ABD-ALLA³.

¹Computer Engineering Department, Engineering College, Aliraqia University, Baghdad, Iraq

²College of Education for Women, University of Anbar, Ramadi, Iraq

³Ministry of Education / Highschool, Baghdad , Iraq

ahmedscofeld8@gmail.com

Abstract. Several studies exist in the literature that utilized metaheuristics and nature-inspired algorithms to solve engineering problems. Particle Swarm Optimization (PSO) is a well-known nature-inspired algorithm that has been used for different optimization problems due to its simplicity and ability to find near-optimal solutions. However, PSO suffers from a problem of balancing between the global search and local search abilities when applied to engineering problems. Recently, a new variant of PSO based on a novel multi-swarm architecture called Multi-swarm Particle Swarm Optimization (MPSO) was proposed. The proposed MPSO was evaluated on solving normal and large-scale problems. This study evaluated the possibility of using MPSO to simulate Welded Beam Design (WDB) problem which is a mechanical engineering problem. Several simulations were performed using the proposed approach and from the results, MPSO model was observed to simulate WBD problem with better optimal solution compared to the standalone PSO. The outcome of this study further confirmed the robustness of the MPSO over the other known metaheuristics. Generally, MPSO achieved an excellent optimization performance with a fast convergence learning process.

Keywords. Welded Beam Design, Optimization, Metaheuristics, Multi-Swarm

1. Introduction

As a field of computational science, optimization primarily focuses on finding the best achievable solutions to any form of problem [1], [2]; the optimization process ensures that such solutions are representable in the form of numerical values with respect to the given problem. Therefore, the aim of optimization processes is to select the best solution to any problem from the numerous available solutions. Computational science has recently found application in several scientific and engineering fields, including aircrafts designs where light weight is desired, optimal refining of petroleum, profitable business activities, optimal missile trajectories, all forms of science and engineering, etc. [3], [4]. Several algorithms and techniques are available for solving optimization problems [5]–[10].

Life, in all its forms, including the stellar, planetary, and galactic systems are accountable to nature and a major attribute of nature is that it can maintain equilibrium via both known & unknown means; this is exemplified in the concept of achieving optimum in nature. One of the basic concepts of life is



attaining optimum [11]–[14] and in striving to attain the optimum, certain constraints and goals must be addressed and met [15]–[18]. The process of finding an optimum can be considered an optimization task [19]–[21] since it can be seen as the establishment of an optimum solution with respect to a performance matrix (normally called an OF which is problem-specified) in certain application fields [22]–[26].

The advantages of heuristic optimization algorithms have made them attractive recently [27], [28] as they have been deployed for the simulation of numerous natural events, as well as to learn the general pattern of a population. Different types of metaheuristics have been proposed and developed over the years and used to solve several engineering design problems. Among the developed metaheuristics are Fruit Fly Optimization Algorithm (FFO), Harmony Search Algorithm (HSA) Cuckoo Search Algorithm (CSA), and Firefly Algorithm (FA) [29]–[32].

The first proposal for PSO was presented by [33], [34] and several modifications have been made to the original PSO after its first introduction; this has given rise to the emergence of various variants of PSO. The aim of these modifications to the original PSO is to make it more capable of achieving better solutions to certain optimization problems. For instance, the “Meeting Room Approach (MRA)”, a multiswarm approach, has been projected by [35]–[37] to assist in achieving a good balance between exploration and exploitation capabilities of PSO. The performance of the proposed “Multi-Swarm Particle Swarm Optimization (MPSO)” was evaluated on both large scale and normal problems.

In this work, an effort is made towards investigating the feasibility of the newly proposed MPSO to simulate “Welded Beam Design (WBD)” problem which is an optimization problem (constrained). The optimization design in WBD mainly aims at the determination of the lowest cost of operation. The handling of the variables at the design phase is often a problem and as a limitation of most conventional optimization algorithms, they cannot reliably solve discontinuous and non-differentiable problems. For this reason, metaheuristics have been widely used to solve discontinuous and non-differentiable problems [38][39]. To our knowledge, the proposed approach in this study represents a strong and dependable intelligent system for solving the WBD problem in engineering fields.

2. Welded Beam Design (WDB) Problem

Achieving the optima in most real-world optimization problems is a computationally-intensive task. This expensive nature of optimization computation methods has resulted in some research restrictions such as project time and resource constraints; thus, it is necessary to speed up the optimization process and make them less complicated. In most standard optimization algorithms, several function evaluations are required, and their results are often satisfactory due to their inbuilt special mechanism for transfer of information; such mechanisms require the utilization of several first-choice solutions within a specified range of fitness evaluations. Being that each candidate resolution must be evaluated, these frameworks demand much execution time and computing resources. Consequently, studies have focused on the development of optimization frameworks which are computationally efficient for the evaluation of several functions. Some of the approaches developed in recent times achieve satisfactory performances even with fewer function evaluations[40], [41].

The WBD problem was conceptualized by Rao (1996); it considered several design parameters, including design for minimum cost based on shear stress constraints (τ), beams’ end deflection (δ), bending stress in the beam (θ), buckling load on the bar (P_c), and side constraints. WBD considered 4 design parameters (i.e., $h(x_1)$, $l(x_2)$, $t(x_3)$, and $b(x_4)$) as shown in Figure (X); the major aim of this process is to design a welded beam with the least cost input. The WBD problem can be mathematically represented thus:

$$\text{Min } f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 - x_2) \quad (1)$$

S.T.

$$g_1(x) = \tau(x) - 13000 \leq 0$$

$$g_2(x) = \sigma(x) - 30000 \leq 0$$

$$g_3(x) = x_1 - x_4 \leq 0$$

$$g_4(x) = 0.1047x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0$$

$$g_5(x) = 0.125 - x_1 \leq 0$$

$$x_6(x) = \delta(x) - 0.25 \leq 0$$

$$g_7(x) = 6000 - P_c(x) \leq 0$$

where:

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau' \tau'' \frac{x_2}{2R} + (\tau'')^2}$$

$$\tau' = \frac{6000}{\sqrt{2}x_1x_2}$$

$$\tau'' = \frac{6000}{MR}$$

$$M = 6000(14 + \frac{x_2}{2})$$

$$R = \sqrt{\frac{x_2^2}{4} + (\frac{x_1+x_3}{2})^2}$$

$$J = 2 \left\{ \sqrt{2} x_1 x_2 \left[\frac{x_2^2}{12} + \left(\frac{x_1+x_3}{2} \right)^2 \right] \right\}$$

$$\sigma(x) = \frac{504000}{x_4x_3^2}$$

$$\delta(x) = \frac{2.1952}{x_3^3x_4}$$

$$P_c(x) = 64746.022 (1 - 0.0282346X_3)X_3X_4^3$$

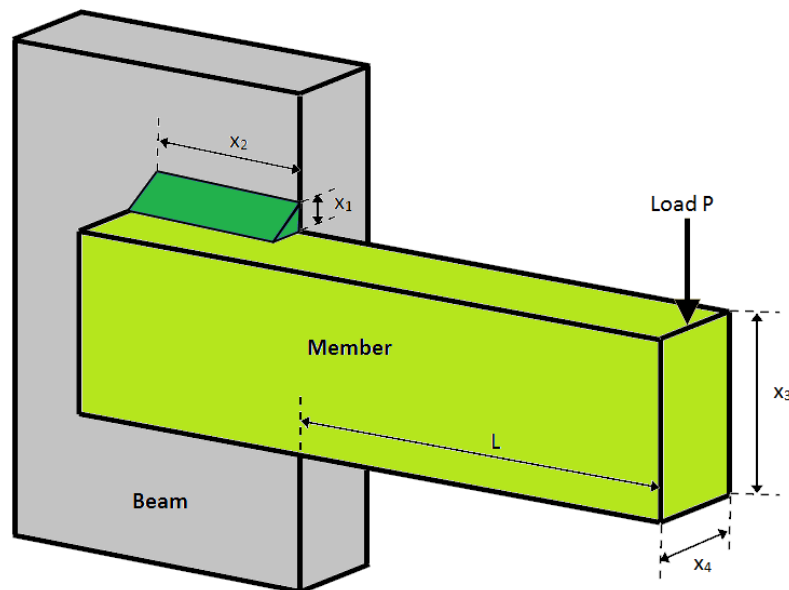


Figure 1. Welded Beam Design

3. Particle Swarm Optimization (PSO)

The PSO was first developed as a nature-inspired metaheuristic based on inspiration from the birds' flocking behavior. In the PSO, the flock of birds is considered as randomly distributed within an area with just one food source (Figure 2). In Figure 2, the only piece of food available to the birds is represented as a dot on the tree. The position of this piece of food is not known to the birds despite being placed at a specified distance from each bird. The bird that is most proximal to the food piece can send a signal to the other birds, facilitating them to flock towards the food piece. Hence, the food piece is considered the optimal value the particles are represented by the bird in the flock.

The distance of each bird from the food piece is represented by the value of the OF; hence, the flocking pattern of the birds can be modeled as an optimization process. Considering Figure 1, the most proximal bird to the piece of food (goal) is designated as X_i ; hence, it is the existing global best. The position of the global best particle from the optimum is given as $Nbest_i$ [34], [42]. The PSO concept relies on the notion that each particles' velocity and position are specified when exploring for the best solution to an NP-hard problem. The particles' position is updated iteratively based on the currently achieved local and global optima; thus, the updating of the position of each particle (e.g., particle i) is expressed thus:

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (2)$$

where t represent the current status, $t+1$ is the status after updating, $X_i(t+1)$ is the velocity of the new particles. Observe that the time difference $\Delta t = (t+1) - 1$ is a time unit and the velocity of particle i is represented as:

$$V_i(i+t) = \omega V_i(t) + c_1 r_1 (X_i^P - X_i(t)) + c_2 r_2 (X^G - X_i(t)) \quad (3)$$

where $v_i(t)$ is the current velocity of the particle, X_i^P and X^G are the particles' local best and global best position respectively, at the swarm level, while ω , c_1 , & c_2 represent the constants that determine the significance of each velocity component; r_1 & r_2 are randomly selected values in the interval of $[0, 1]$.

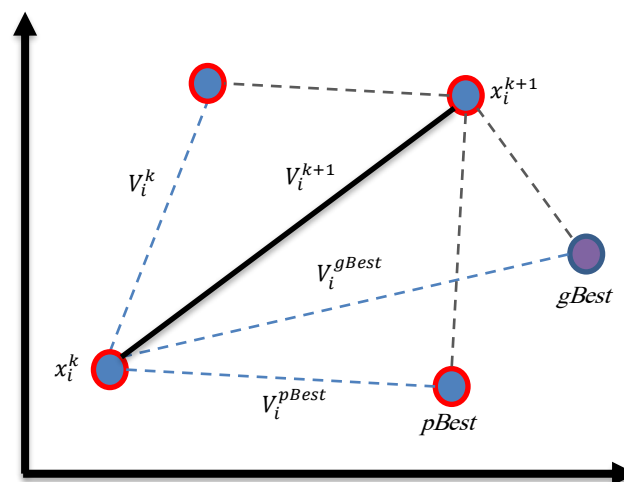


Figure 2. A depiction of the PSO

4. Multi-Swarm Particle Swarm Optimization (MPSO)

Multi-swarms are conceptualized on the interaction between a group of organisms when exploiting a common solution to a given problem. Several multi-swarm approaches have been proposed and developed, with each of them getting inspired by natural events. In this study, a new interactive multi-swarm approach that was inspired by the social life of humans when interacting with their leaders was presented. The proposed multi-swarm in this study was inspired by the interaction between groups of humans (referred to as Clans) and the clan leaders. This scheme is composed of numerous clans and each clan is made up of several members (representing a set of solutions). In each clan, the best member is selected as the clan leader and is empowered to regulate the other clan members in terms of their settlement, movement, and migration [22], [26].

The clan leaders (in each generation) periodically assemble in a room to elect an overall best leader; upon the selection of an overall best leader, the positional information of the elected leader will be shared to the ordinary clan leaders (using Eq. 3, 4, and 5) for positional updates. This positional information dissemination strategy from the overall best leader to the ordinary leaders balances the exploration & exploitation phases of the PSO. Figure 3 presents the proposed model of the proposed MRA. From the figure, it is clear that each clan is only permitted to perform a single PSO search per generation and this

covers both the velocity and positional updating. The selection of new clan leaders is only done after new populations have been generated; after their selection, they are once again sent to the meeting room to select an overall best leader. The inertia weight and position of each normal are updated leader in the meeting room with respect to the positional information of the overall best leader using the relations:

$$w^{Ln} = \left(\frac{w^{Lg} - w^{Ln}}{Itr} \right) \times rand() \quad (4)$$

$$v_i^{Ln}(t+1) = w^{Ln} \times v_i^{Ln}(t) + rc \left(P_g^L - P_n^L(t) \right) \quad (5)$$

$$x_i^{Ln}(t+1) = x_i^{Ln}(t) + v_i^{Ln}(t) \quad (6)$$

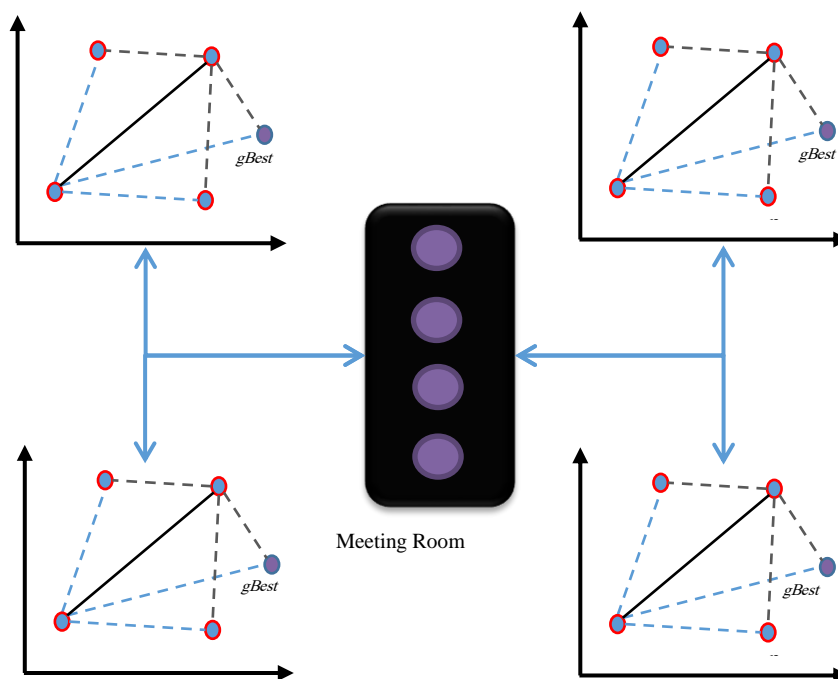


Figure 3. The multiple PSO structure

where Ln and Lg represent the normal leaders and the overall best leader respectively, x_i^L represent the normal leaders' position, v_i^{Ln} is the normal leaders' velocity, w^{Lg} & w^{Ln} are the inertia weights of the overall best leader and the normal leaders. Owing to the changes that follow the updating of the position of the clan leaders after the generation of each population, new clan leaders are always elected and sent to the meeting room after each generation. Eq (4) calculates the new w or the inertia weight, which controls the PSO exploration capability. Figure 3 presents the pseudocode of the proposed MPSO.

5. Results and Discussion

The WBD, as a complex constrained problem, is saddled with the obstacle of achieving a trade-off between the main OF and its associated constraints. This study, therefore, aims to establish an appropriate computational solution to the WBD by adopting a penalty function method to solve a constrained WBD problem. An MPSO approach was developed and used to solve the WBD problem and from the achieved modeling solution, the optimum MPSO control parameters were achieved as $c_1 = 1.42$, $c_2 = 1.42$, $w = 0.75$. The optimum number and size of the swarms were 5 and 10, respectively. From the engineering perspective, newly propose intelligent optimization models are meant to be validated with other relevant published researches. Fortunately, the problem addressed in

this study has been previously solved in the previous studies using other optimization techniques; this simplified the process of validating the current study with a dependable benchmark. The results of the MPSO were tabulated and compared with several related works when solving WBD problem as in Table 1.

The solution achieved by MPSO was $(\mathbf{0.2055} \ \& \ \mathbf{3.0.4703} \ \& \ \mathbf{9.0378} \ \& \ \mathbf{0.2.056}) = \mathbf{1.7251}$ with the constraint condition solution of: $\mathbf{g} = (-2.19834, -18.3376, -0.000004, -3.43257, -0.08071, -0.2283, -0.11929)$. In the WDB problem, several parameter variables are involved in the 6 conditions among the 7 constraint conditions. This implies that only when the parameters have changed can the range of correlative parameters be determined. If the blindfold search scale is beyond the parameter range, the pseudo value will be generated, and this will result in a failed search process. The outcome of the comparison between the proposed MPSO and the other existing metaheuristic is tabulated in Table 1 and Figure 4. However, Table 2 projects the superiority of the proposed MPSO over those earlier reported by [7, 12, 16, 27-29]. Table 3 showed the stability of the proposed MPSO to be optimal compared to the benchmarked algorithms.

Table 1. The simulation results.

Method	Design Variables				Cost
	x_1	x_2	x_3	x_4	
PSO	0.2064	3.5283	8.9884	0.2080	1.7423
CSA	0.2057	3.5194	9.0366	0.2057	1.7315
FA	0.2416	3.6552	8.5071	0.2344	1.8641
FPA	0.2057	3.5195	9.0366	0.2057	1.7315
GSA	0.2195	4.7283	8.5009	0.2715	1.9350
MVO	0.1990	3.6529	9.1144	0.2054	1.7498
MPSO	0.2055	3.4703	9.0378	0.2056	1.7251

Table 2. The simulation results.

Method	Design Variables				Cost
	x_1	x_2	x_3	x_4	
[44]	0.2088	3.4205	8.9975	0.21	1.7483
[45]	0.1997	3.6120	9.0375	0.2060	1.7373
[46]	0.2444	6.2379	8.2885	0.2445	2.3811
[47]	0.2443	6.2175	8.2914	0.2443	2.3809
MPSO	0.2055	3.4703	9.0378	0.2056	1.7251

Table 3. A comparison of all experiments.

Method	Best	Mean	Worst	Std
[44]	1.7483	1.7719	1.7858	0.0112
[45]	1.7373	1.8133	1.9946	0.0705
[46]	2.3854	3.2551	-	0.96
[47]	2.3809	2.3819	-	0.0052
MPSO	1.7251	1.7284	1.7339	0.0082

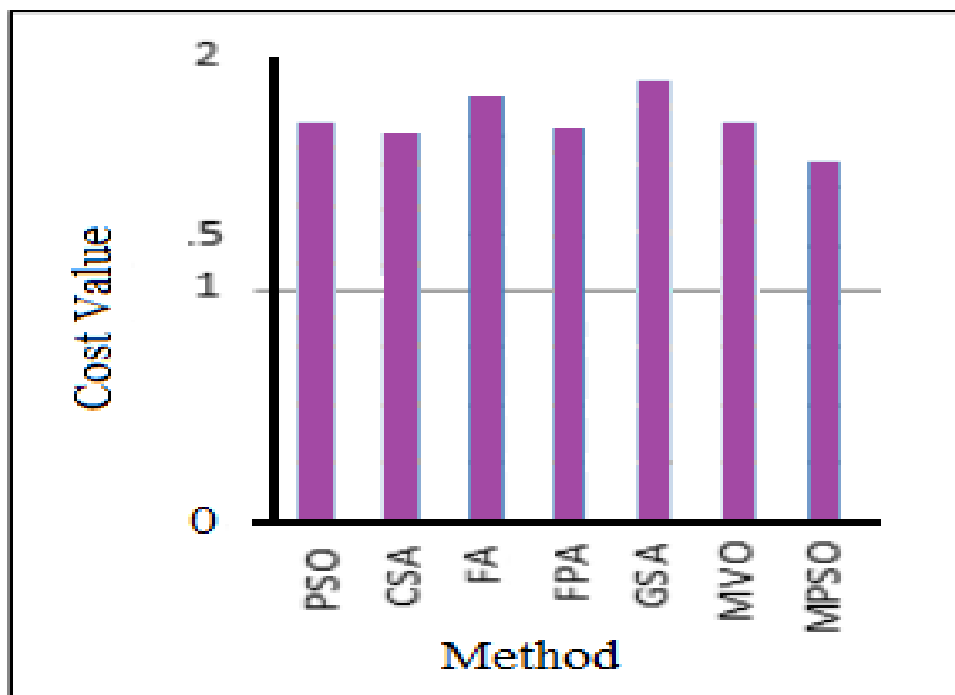


Figure 4. Results comparison between the metaheuristic.

6. Conclusion

Optimization methods or metaheuristics play an important role in computer engineering. PSO remains one of the most popular metaheuristics used in the literature for generally solving optimization problems, as well as for solving design problems in engineering. However, PSO is mainly faced with the problems of executing local search more than the global search; hence, it is always susceptible to local optima entrapment. To address this issue, this study presents a new PSO variant (a multi-swarm particle optimization (MPSO)). The proposed MPSO was applied to the well-known WDB problem and its performance was validated against several metaheuristics, such as original PSO, Firefly algorithm, Flower Pollination Algorithm, Gravitational Search Algorithm, and Multiverse Optimization. From the validation studies, the MPSO achieved a better optimization solution with more accuracy and faster convergence compared to the benchmarked algorithms.

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