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Image encryption based on computer generated hologram and Rossler chaotic system

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Abstract

In this work, the image is encoded using computer generated hologram (CGH) technology with the presence of a Rossler chaotic system. Using Matlab, it was observed through the results that the images created using holograms can be easily recognized, distinguished, and traced back to their origin, but via using Rossler-chaos theory. We can encryption the images and the disappearance of their features and thus the possibility of using this method via image encryption.

Keywords Images encryption · Computer Generated Hologram · Chaos · Rossler

1 Introduction

Marsden et al. (2003) Bechtold et al. (2006).

Encryption is one of the processes currently available to modify data or information from one to another. In the last decade, the issue of encoding images has become the focus of attention of many researchers and has become an important and interesting topic (Jamal 2008a, b). There is a great deal of literature on this topic and many types of encryption techniques are used. One of these literatures had used image encryption algorithms with the help of chaos systems because of its large and complex data, which secures the transfer of data from one place to another without being hacked (Jamal and Kafi 2016, 2019; Jamal and Hassan Mohamed 2017). Previously, a variety of encryption methods were used, such as circular mapping using a pair of keys and a chaotic logistics map (Yousif et al. 2020). When comparing the previous methods used in encryption with the current method used in this research, where the computer-generated Fourier transform data for holographic images of a particular image is collected with the Fourier transform data for any chaotic system, it was noted that the obtained images are encrypted and have a high level of security where it cannot be identifiable and cannot be deciphered. With the development of various means of communication, preserving data security has become an urgent matter that must be solved.

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Currently, any traditional encryption model can be easily hacked, and this leads to significant security risks in breaching the country's system. Therefore, it is absolutely necessary to improve and develop outdated techniques and methods to preserve the confidentiality of the information of the state and its institutions, as the images can be easily decrypted without changing the features of the images.

Due to the characteristic of long-term unpredictability and extreme sensitivity to elementary values, a chaotic system of secure communications and encryption has been proposed. Currently, researchers are working on finding a way to come up with a chaotic security system, which mainly focuses the research on two aspects: the first is to find a more secure communication system, such as hiding ordinary holograms, and secondly, to hide computer holograms. The Matlab program was used to simulate the results. In this work, Fourier hologram images were creating by computer simulation method using the Fraunhofer diffraction formula. In this work, filtering techniques were used to construct a computer hologram. The MATLAB algorithm for making computer holograms is discussed in detail in this work. With this procedure, computer-generated holograms can be easily generated on a computer using this method, even at university level.

2 Constriction hologram

When the recorded fringe pattern is illuminated with the same reference light it will create the object's wavefront (3D image-hologram). Computationally, in computer-simulated holographic imaging, the entire recording and reconstruction processes are simulated using the diffraction formula. A.W. Lohmann and D.P. Paris created the first computer generated hologram (CGH) in 1967 (Lohmann and Paris 1967). Following then, this method has experienced many advancements and modifications, both in terms of process and computing techniques.

Calculating the far field amplitude using the Fraunhofer diffraction formula requires the use of the Fourier transform. The Fourier transform integral is numerically solved via using the Discrete Fourier Transform (DFT) method. However, computing the DFT increases the time required, which is directly proportional to N^2. Cooley-Turkey method, also known as Fast Fourier Transform algorithm, increases the speed of computation for previous mathematical operations. The time required to perform this FFT computation is proportional to N log N. This function is used to determine the far-field amplitude U(x, y). If the function G(x, y) is a Fourier transform of F(ξ, η), then (A. W. Lohmann et. al. 1967).

$$G(x+pN, y+qN)\frac{1}{N}\sum_{\xi=0}^{N-1}\sum_{\eta=0}^{N-1}F(\xi,\eta)\exp\left\{\frac{-2\pi i}{N}(x\xi+y\eta\right\}\times\exp\{-2\pi i(p\xi+q\eta\} \tag{1}$$

For $x, y=0,1,\ldots\ldots N-1$.

For all integer values of x, y, ξ, η, p & q, the second exponential term is 1. Thus

$$G(x+pN, y+qN) = G(x,y) \tag{2}$$

The origin of the transformation field lies in the geometric center, in the case of continuous signals. Similarly to Fraunhofer diffraction, the zero frequency term is shown in the display's center. By rearranging the transfer transactions in DFT, the original can be transferred to the center. The matrix is reordered via multiplying it via $(-1)^{\xi+\eta}$.

For $p = q = 1/2$, So,

$$G(x + N/2, y + N/2) = G(x + pN, y + qN)x(-1)^{\xi+\eta} \tag{3}$$

Fourier transforms for reference waves and objects have invisible complex values. For this, the square of the absolute values of the amplitudes (density) is calculated. The Fourier transform is just a representation of the Fourier series of a two-dimensional object, when estimated by periodic sampling. In our work the object is represented by a real matrix. The resulting real matrix contains elements whose value is the square of the value N. When the matrix undergoes Fourier transformations, it becomes complex with $2N^2$ elements. The dimension seems to have grown, but it has not, since the Fourier transform shows the feature of conjugate symmetry.

$$G * (x, y) = G(-x, -y) \tag{4}$$

As a consequence, about half of the samples are unnecessary and may be produced from other samples. This is the characteristic that results in the appearance of duplicate pictures during reconstruction.

The Fraunhofer diffraction formula is utilized to simulate our work. Holograms produced in this manner using the diffraction formula may be utilized for optical reconstruction. To calculate the Fraunhofer diffraction, we need four steps, as shown in Fig. 1 as follows (Anand 2012), while the scheme for working reconstruction computer generated hologram (RCGH) as shown in Fig. 2.

2.1 Step 1

To process FFT values using different computers the matrix size is n^2. These restrictive conditions are excluded if Matlab is used. With all this, a 1024×1024 matrix size

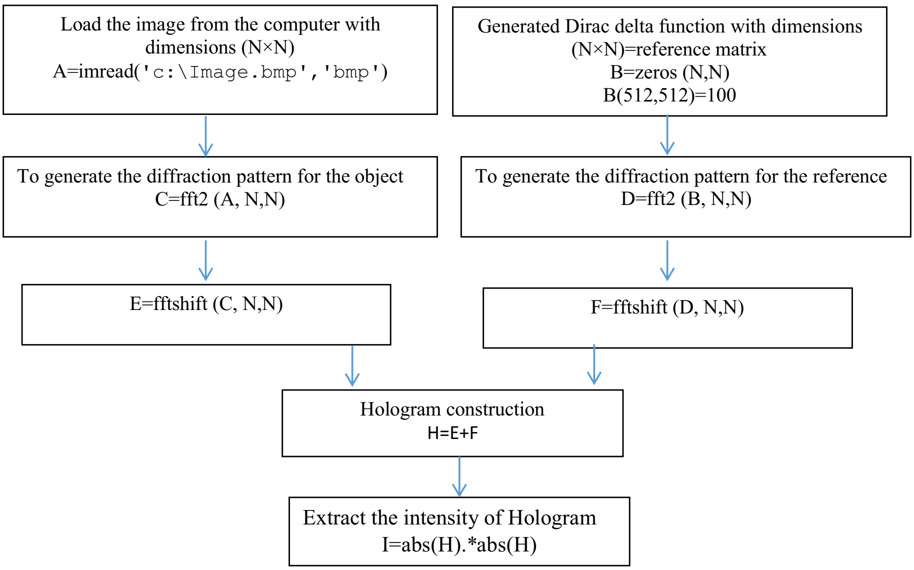


Fig. 1 Scheme for working computer generated hologram (CGH)

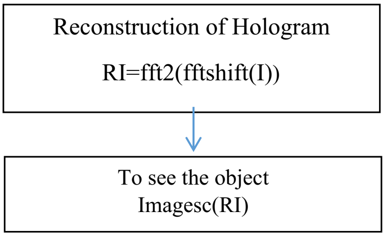


Fig. 2 Scheme for working reconstruction computer generated hologram (RCGH)

Fig. 3 The Object (Lena)

is chosen. Any computer image as shown in Fig. 3, which represents the Lena (Lena is a standard test image widely used in the field of image processing since 1973), is chosen for the purpose of verifying this study. The image taken from the computer is saved in JPG format and is loaded into the Matlab program using the command "imread".

2.2 Step 2

A matrix of 1024 × 1024 elements which represents the Dirac delta function is created via making only 1 element out of a total of 1024 × 1024 elements constant and the other of the elements becoming zero this represent the matrix of reference. The range taken for intensity is within the range [0–100].

2.3 Step 3

The "fft2" function in Matlab is used to compute the Fourier Transform of an image and its reference matrix. After all "fft2" procedures, the "fftshift" command is used to center the diffraction patterns.

2.4 Step 4

Matrix summation is used to combine the far field matrices of the object and reference. The matrix that results is referred to as the hologram matrix. The matrix’s square (each

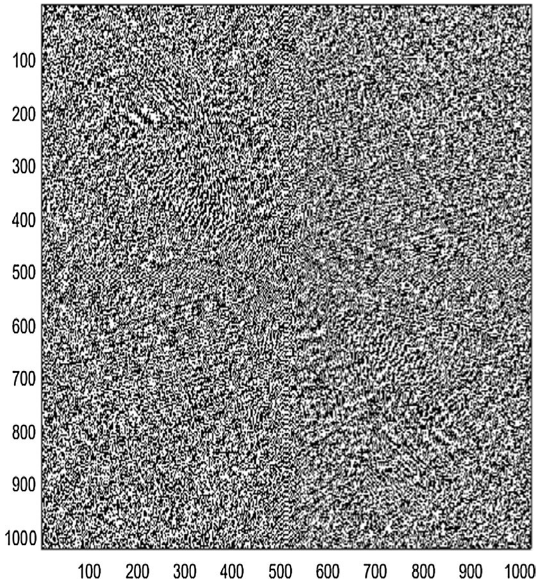


Fig. 4 CGH of the object

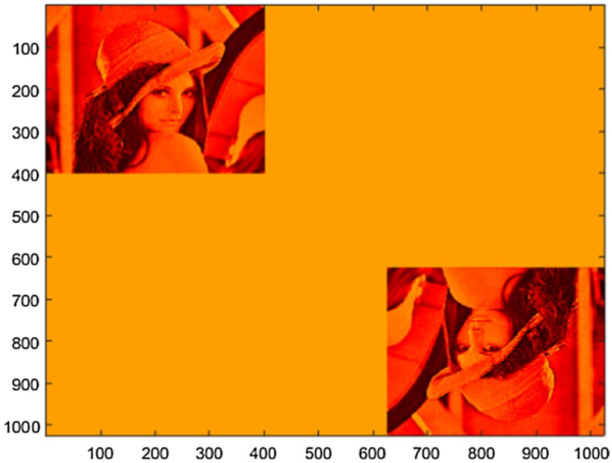


Fig. 5 RCGH of object

element multiplied via itself, rather than the normal matrix multiplication) yields the intensity values throughout the hologram plane. The (.*) command is used to do this kind of multiplication. The "imagesc" command is used to image the matrix. This procedure scales the matrix's pixel values and outputs the image. The hologram and its reconstruction are shown in Figs. 4 and 5.

2.4.1 Rossler model

The Rössler attractor system is a system of three nonlinear ordinary differential equations that Otto Rössler investigated in the 1970's (Rössler 1976, 1979). These differential equations denote a continuous-time dynamical system with chaotic dynamics due to the attractor's fractal properties (Peitgen et al. 2004). In this system, it is observed that the orbit inside the attractor takes a spiral path and in the outward direction is close to the x and y plane around a unstable fixed point. Once the graph spirals sufficiently far out, a second fixed point exerts effect on it, resulting in a rise and twist in the z-dimension. When studying the behavior of this system, especially in the time domain, it was observed that the oscillations behave chaotically, although each variable oscillates within a fixed and specified range of values. While this attractor has some resemblance to the Lorenz attractor, it is simpler and contains just one manifold. Otto Rossler laid out the basics of Rossler's attractor equations in Rössler (1976), but the basic theoretical equations later proved useful for simulating the equilibrium of a chemical reaction. The Rossler electronic circuit consists of a single nonlinear element and is a piecewise linear function made of operation amplifier type U4A with diode, three resistors and a diode. The following equations (Caroll 1995) describe the Rossler electrical circuits

$$\frac{dx}{dt} = -(y + z) \tag{5}$$

$$\frac{dy}{dt} = x + ay \tag{6}$$

$$\frac{dz}{dt} = b + xz - cz \tag{7}$$

whereas, the parameters a, b and c studied by Rosslar have real values, and their values are $a = b = 0.2$ and $c = 5.7$, while the values of x, y and z are the three variables that evolve with time. The first and second equations have linear terms that create fluctuations in the variable x and y while the last equation contains one nonlinear term (xz) so the chaotic behavior expected of that system appears. Many uses have been developed for chaotic systems, most notably in image encryption (Lopez et al. 2005; Zaher 2009; Van den Hoven, 2007).

3 Results and discussion

Figure 1 illustrates the main steps in creating an image encoder using a computer-generated hologram. To perform the encoding process, two-dimensional images of the personal computer are selected with dimensions (N N) equal to (1024 1024), where the image of the famous supermodel Lena was chosen in this study and as shown in Fig. 3.

The Matlab program is applied to create a computer generated hologram of the image of Lena via calculating the Fast Fourier Transform (FFT), as shown in Fig. 4. One of the advantages of the obtained image (computer generated hologram) is that it has an inhomogeneous point distribution over the area of the binary image and a gray-color format. To perform the process of reconstructing the computer hologram, we get the image shown in Fig. 5, where we notice the reformulation of the original image in two orders, and this is one of the characteristics of the Fourier transforms. To perform the computer hologram reconstruction process, the image shown in Fig. 5 was obtained, where it was observed that the original image was reformatted into two orders and clearly without defect, and this is one of the characteristics of Fourier transforms.

On the other hand, the Rossler system's behavior was investigated numerically via programming the Rossler system's three differential equations (Eqs. 5–7) using the Matlab

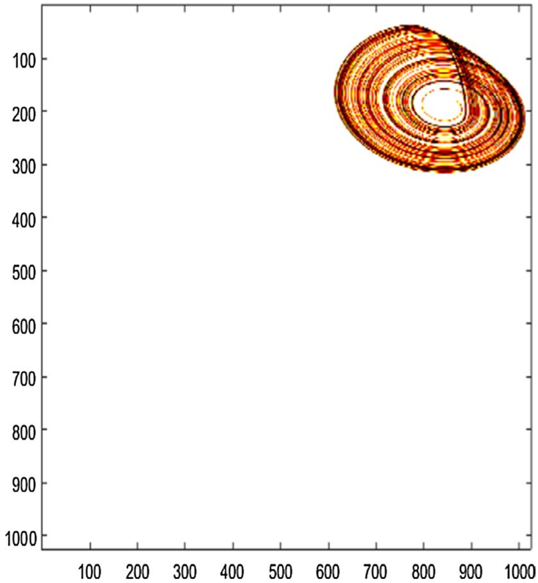


Fig. 6 (y–x) attractor of the Rossler system

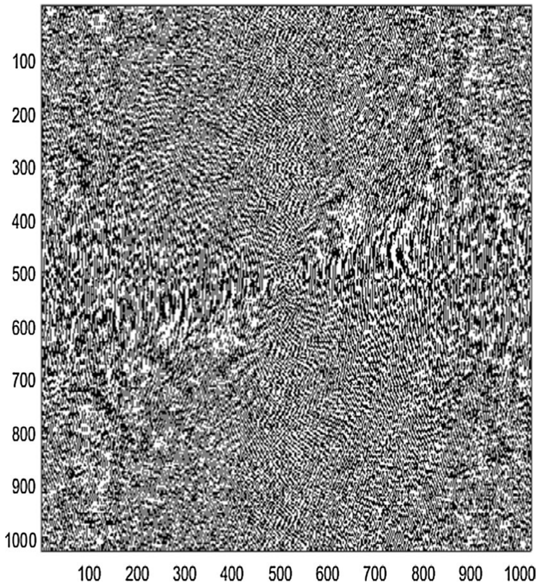


Fig. 7 CGH of the attractor

software, where the differential equations were solved using fourth-degree Runga–Kutta integration. The values of a, b, and c were fixed a = 0.2, b = 0 and c = 5.7 respectively and, noting that the values of the initial conditions of the system were as $x, y,$ and z equal 1,1 and 0 respectively. Figure 6 shows the dynamic characteristic attractor of (z–y) of the Rossler system.

The system becomes a hyper-chaotic at these constant values and it is showing a single scroll attractor. In the same way, a computer generated hologram is constructed for the

Fig. 8 RCGH of the attractor

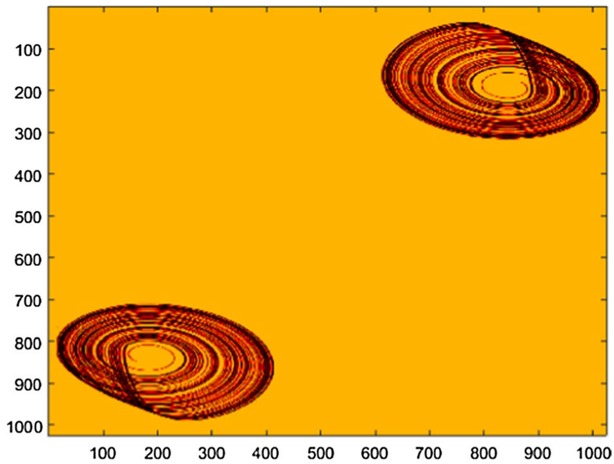


Fig. 9 CGH for summation mathematical operation of the attractor and object

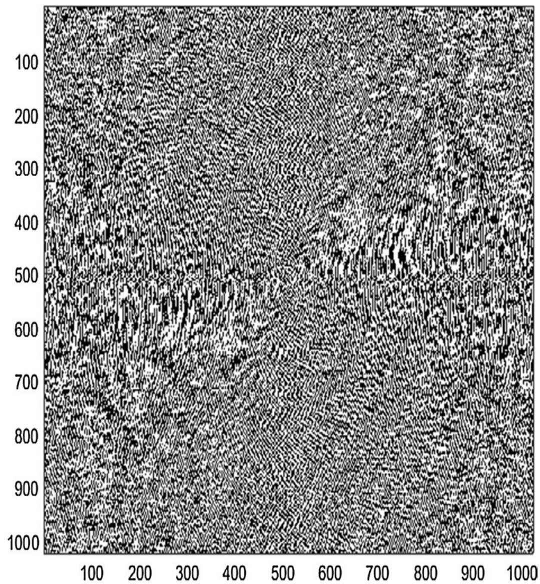


image of the attractant, as shown in Fig. 7, and in fact this image represents the Fourier transform as well, while Fig. 8 represent the reconstruction hologram. Now for the purpose of fully studying the possibility of image encryption, a test is performed of several mathematic operations, which are either summation, multiplication or division of the values of the Fourier transforms of the image (Lena) to be encoded with the values of the Fourier transformations of the image of the chaotic attractor (Rossler type). When using the summation mathematical operation for the values of Fourier transforms for both images, a Fig. 9 is obtained, and when performing the computer hologram reconstructions of the summation mathematical operation, Fig. 10 is obtained, and through the figure it is noticed that the characteristics of the image obtained as a result of this calculation did not provide a complete encoding of the image because the image features it has not completely disappeared, so it is concluded that the summation mathematical operation failed in encryption,

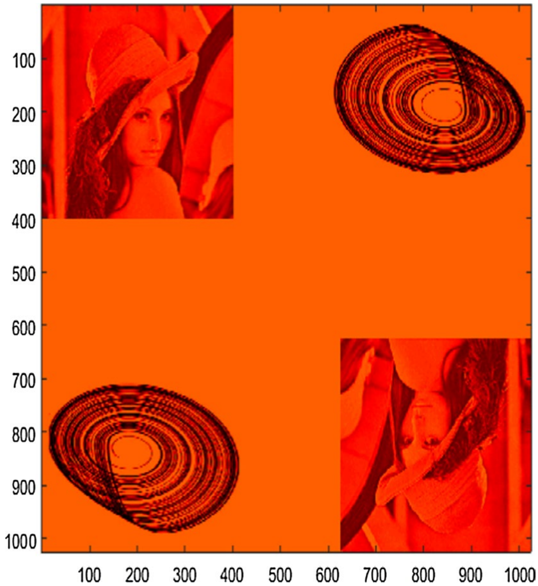


Fig. 10 RCGH of the summation mathematical operation

as it is noticed that the characteristics of the original images are prominent and appear. This is why the summation mathematical operation is considered an unsuccessful process to hide the features of the original images.

As for when using the state of multiplication mathematical operation between the values of the Fourier transforms of the two images, it is clear in Fig. 11, and when performing the computer hologram reconstructions of Fig. 11, that is, for the case of multiplication mathematical operation, Fig. 12 is obtained, which somewhat shows the

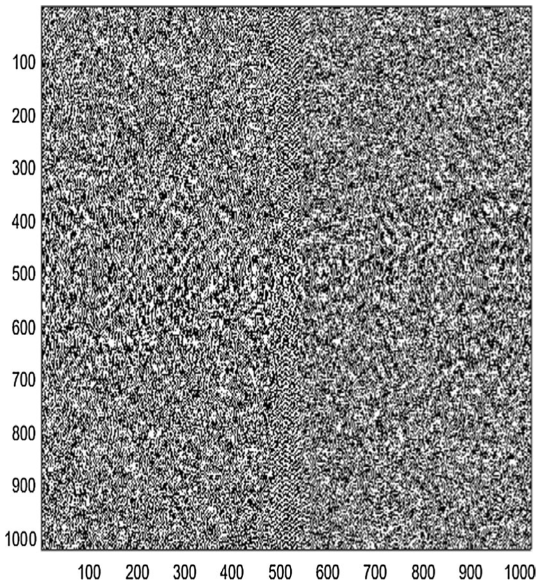


Fig. 11 CGH for multiplication mathematical operation of the attractor and object

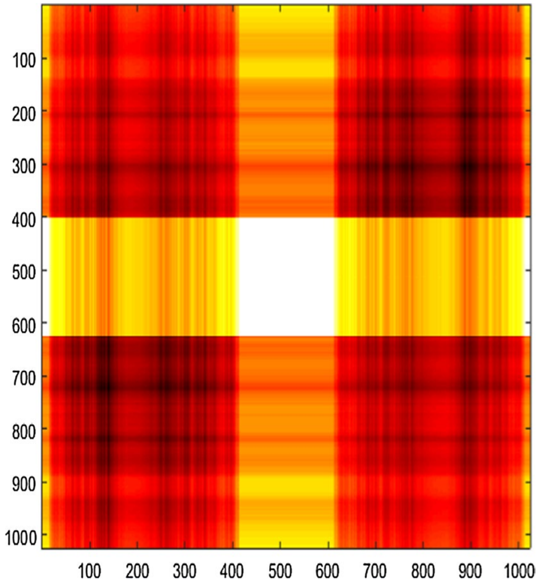


Fig. 12 RCGH of the multiplication mathematical operation

features of the original images, it is also considered somewhat inaccurate to conceal the features of the image permanently.

Finally, when performing the division mathematical operation for the values of the Fourier transforms of both images, Fig. 13 is obtained, and when performing the hologram reconstructions of the above image, as shown in Fig. 14, it is noticed that the image properties obtained as a result of this calculation provide a complete encoding of

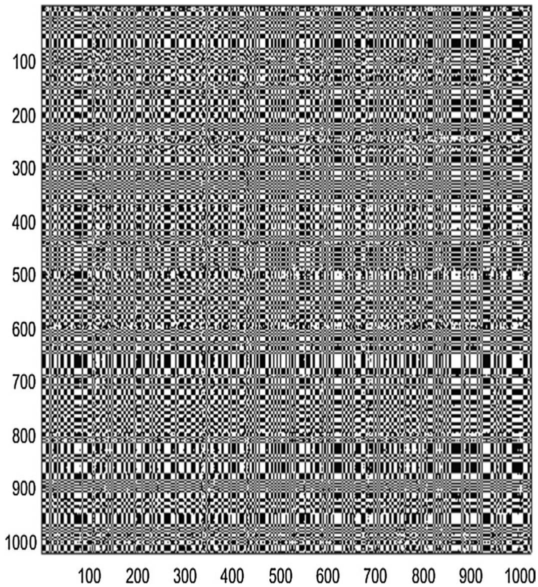
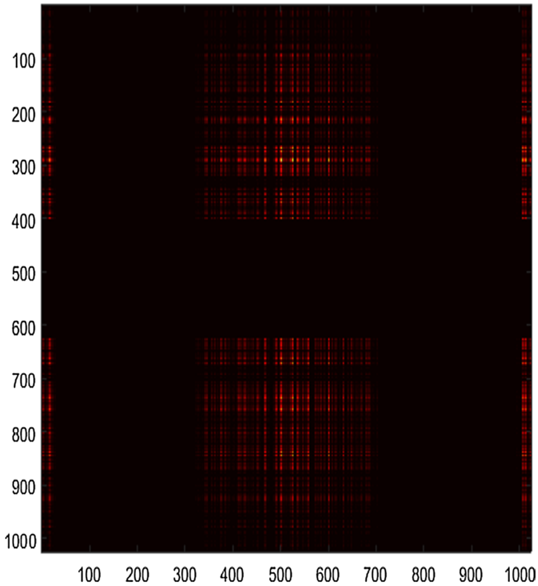


Fig. 13 CGH for division mathematical operation of the attractor and object

Fig. 14 Reconstruction of CGH of object and Chua attractor



the image because the image features have completely disappeared, So we conclude that the division mathematical operation was largely successful in cryptography.

4 Conclusion

In this paper, the principles and basics of the computer hologram generation mechanism and the reconstruction process are discussed. We believe this paper will initiate an active research into computer-generated holograms in academic institutions. Through the obtained results, we conclude that it is possible to use the hologram—chaos method to fully encode the information, especially when using the division mathematical operation for the values of the Fourier transforms of the image to be encoded with the values of the Fourier transforms of the attractor Rossler system. The reason for the disappearance of the original image features when using the division mathematical operation for the values of the Fourier transforms of both images is because of the characteristics of the large and complex data Fourier transform values of the chaotic system, which insures the transfer of data from one place to another without being hacked.

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