

# Phase-Coupled Synchronization with Optoelectronic Feedback

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**Abstract**—In this research work, a phase-coupled scheme containing two chaotic lasers was evaluated using the theoretical dimensionless model. It is possible to significantly increase the perturbation in laser behaviour with the incorporation of optoelectronic feedback. The study presented a novel impact study pertaining to synchronization of the phase of sustained chaotic oscillators with weakly coupled. For coupled chaotic lasers, especially in mix-mode case of synchronous system, the phases remain safe. Even though the amplitudes vary untidily and are almost without correlated. In the study, the authors identified a novel synchronization model when coupled with a chaotic oscillator. This occurred in the presence of entraining the frequencies whereas the phase difference remained unbounded.

**Keywords**— *optoelectronic feedback, phase-coupled, phase-looking.*

## I. INTRODUCTION

Control of chaotic systems is one of the important research domains that distinguishes the control theory and dynamic systems. This domain studies about the methods to keep the disordered systems under control from exhibiting chaotic behaviour [1-3]. The system becomes instable if there is a small change brought in the input of the system since the inputs are high sensitive in nature. The changes bring a huge difference in the behavior of system. Most light sources exhibit intensity and phase fluctuations due to the nature of quantum transition [4-6]. Chaotic behavior appears in a variety of systems including electrical circuits, laser dynamics, mechanical interactions, mechanical and magnetic devices. Complex dynamics is a type of chaotic behaviour, for instance, mixed-mode oscillations and non-uniform levitation by high-dimensional systems [7].

The dynamic chaos can be defined as an irregular oscillation that occurs in the temporal evolution of non-linear dynamic system. This irregular oscillation can be observed as the system output in a deterministic fashion. It differs from random processes. Further, the quantum dot semiconductor laser has been established to have a direct effect on dynamic systems [8-10].

Chaos can also be defined as irregular movements of a dynamic system. Being a long-term irregular development, it

meets certain mathematical criteria in the occurrence of a non-linear system [11]. Lorenz defined that chaos can be characterized as the dynamic system in which most of the orbits appear to be sensitive [12]. Chaos can be controlled by relying on nonlinear differential equations. Whenever there occurs external disturbances, it tend to impact the chaos generation as well as nonlinear dynamics. This is because the non-linear interactions of the light with laser medium exert a stabilizing or destabilizing effect [13, 14]. Chaotic dynamic behaviour has many characteristics for instance, complex behavior dynamics of the spread spectrum such as noise which is used to encode the data [15]. The chaotic waveform is used as a carrier of information and it plays a vital role in safe communication [16]. Recently, various techniques have been proposed to control the chaos and to suppress it [4]. Among such studies, two classes such as abroad class and major class, are defined by the dissipative nonlinear oscillators. As per the literature [4], the general equation of the motion is as follows

$$\ddot{x} + \delta \dot{x} + \frac{dv}{dx} = F \cos(\omega t) \quad (1)$$

In the above equation, the function of potential is  $v(x)$  and is accountable for restoring the system influential force. Here, the damping coefficient is  $\delta$  whereas the periodic external forcing is  $F \cos(\omega t)$ . Phase difference i.e.,  $\phi$  that exists between two values such as primary driving and small harmonic perturbation remains the control parameter during the phase control of chaos parameter. This value is introduced into the system either as parametric or additional forcing. As per the literature [11], this value is proved either through numbers or through experimental procedure. The phase difference  $\phi$  must be appropriately selected since it has a vital role to stabilize the system dynamics. The primary aim of the phase control in chaos technique is to induce a periodic behaviour out of the chaotic nonlinear system with the help of weak harmonic perturbation and accurate value of the phase [12]. The current study analyzed the phase synchronization of the chaotic oscillators. Since the study intended to exhibit the role of phase difference in laser dynamics, the authors used Kais Al Naimee model for optoelectronic feedback of semiconductor laser.

## II. LASER MODEL WITH OEFB

Two equations such as electric field and the population inversion decode the dynamics of semiconductor laser. B lasers, the name allotted for this laser since it has cancelled the polarization term. The semiconductor laser's dynamics and optoelectronic feedback, which is featured by numerical model, has a total of three equations. K.A.AL-Naimee et al [13] amended these equations in order to be inclusive of the optoelectronic feedback.

$$\dot{I} = -\gamma_f I + \kappa \dot{S} \quad (2)$$

$$\dot{N} = \frac{I_0 + f_F(I)}{eV} - \gamma_C N - g(N - N_t)S \quad (3)$$

$$\dot{S} = [g(N - N_t) - \gamma_0]S \quad (4)$$

here  $I$  is the high-pass filtered feedback current, the bias current is  $I_0$ , function  $f_F(I) \equiv AI/(1+sI)$  corresponds to function of the feedback amplifier,  $e$  denotes the elementary electron charge,  $V$  denotes the volume of the active layer,  $g$  denotes the gain,  $\gamma_C$  represents the population relaxation rate, photon damping in active layer is  $\gamma_0$ ,  $\kappa$  denotes a proportional to responsivity of the photo detector,  $N_t$  corresponds to carrier transparency,  $S$  corresponds to the dynamics of the photons and  $N$  corresponds to the carrier density,  $\gamma_f$  is high-pass filter of cutoff frequency of. It is useful to reneo the equations (2, 3 & 4) in form of dimensionless for facilitate numerical operations. To these equations, with new variables as follows [13]:

$$x = \frac{g}{\gamma_C} S, y = \frac{g}{\gamma_0} (N - N_t), w = \frac{g}{\kappa \gamma_C} I - x, \gamma = \frac{\gamma_C}{\gamma_0}$$

the scale of time  $\dot{t} = \gamma_0 t$ . Now, the equations (2,3 and 4) become

$$\dot{w} = -\varepsilon(w + x) \quad (5)$$

$$\dot{y} = \gamma(\delta_0 - y + f(w + x) - xy) \quad (6)$$

$$\dot{x} = x(y - 1) \quad (7)$$

here  $f(w + x) \equiv \alpha \frac{w+x}{1+s(w+x)}$ ,  $s = \gamma_C \dot{s} \kappa / g$  denotes the coefficient of saturation,  $\delta_0 = (I_0 - I_t)/(I_{th} - I_t)$  and  $(I_{th} = eV\gamma_C(\frac{\gamma_0}{g} + N_t))$  are the current of bias and solitary laser threshold, respectively,  $\alpha = \kappa / (eV\gamma_0)$  is the feedback strength and the bandwidth at resonant frequency is  $\varepsilon = \frac{w_0}{\gamma_0}$ .

For the intent of facilitation, equations of dimensionless, let us consider  $z = w + x$ , thus the equations. (5, 6 & 7) can be rewritten as follows

$$\dot{z} = -\varepsilon z + \dot{x} \quad (8)$$

$$\dot{y} = \gamma(\delta_0 - y + \alpha \frac{z}{1+sz} - xy) \quad (9)$$

$$\dot{x} = x(y - 1) \quad (10)$$

Phase-coupled chaotic system has major advantages and utility areas in optical and electronic applications such as any or all of the sciences, such as neurochemistry and encryption of communication. Newly, based on the stability theory, coupled systems at chaotic state were exceedingly classified and estimated using phase-coupling based on its suitability. Therefore, dynamics is at most scrupulous in periodic (regular) and irregular (chaotic or complex) with feedback based lasers in nonlinear systems.

The current discuss study speak, about dynamic oscillations of laser in the OEFB. In this paper, the behavior was studied of coupling unidirectional system under OEFB. In method of OEFB, the rate for carrier's equations and number of photons equations remained enough for clarify the model under study. Laser system in OEFB has wonderful benefits compared to other dynamical systems. The carrier time scale is three times higher than the photon lifetime [8]. Phase-coupling is briefly studied in optical feedback [19]. No research investigations have been conducted so far in this background with optoelectronic feedback. Further, a phase-coupled term is also added to the third equation and finally, the formulas for two corresponding systems are shown in "Fig. 1" and are written below.

$$\dot{z}_2 = -\varepsilon_2 z_2 + \dot{x}_2 + \kappa \dot{x}_1 \quad (11)$$

Where 1 and 2 denote the master and slave in phase-coupled optoelectronic loop.

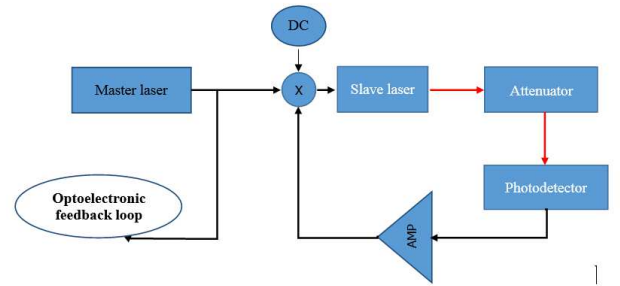


Fig. 1. The configuration of optoelectronic feedback system for both master and slave lasers.

## III. RESULTS OF PHASE-COUPLED SYNCHRONIZATION

In two lasers with OEFB, the understanding of phase-coupling lets the researchers to employ the illustration of behaviors as a region that one set at the master dynamics by several processes for instance, frequency of phase. When the output of master is controlled, it is an incorporation of the output of both master and the slave. The output of the master dynamics can be predicted and can be calculated for the slave hand. Due to the conforming phase looking from the master output, the expectable output can be exploited by the slave. The presentations of a coupling arrangement have been calculated for a wide range of factors. Some appropriate behaviors were found in the systems of communication applications.

For the laser time series, because of its simple form, the phase can be introduced in a more straightforward way,

additionally the corresponding interspike interval (ISI) probability distribution, correlation map and attractor in sometime. Before dividing the work into two cases in order to achieve the phase synchronization state for each of the two conjugated systems, we choose the complete synchronization state as in “Fig. 2.a” to be the starting from this initial state. This is clear into correlation map “Fig. 2.b” and ISI as seen in “Fig. 2.c”, with  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = \epsilon_2 = 0.9 \times 10^{-5}$  and  $k = 0$  parameters are used.

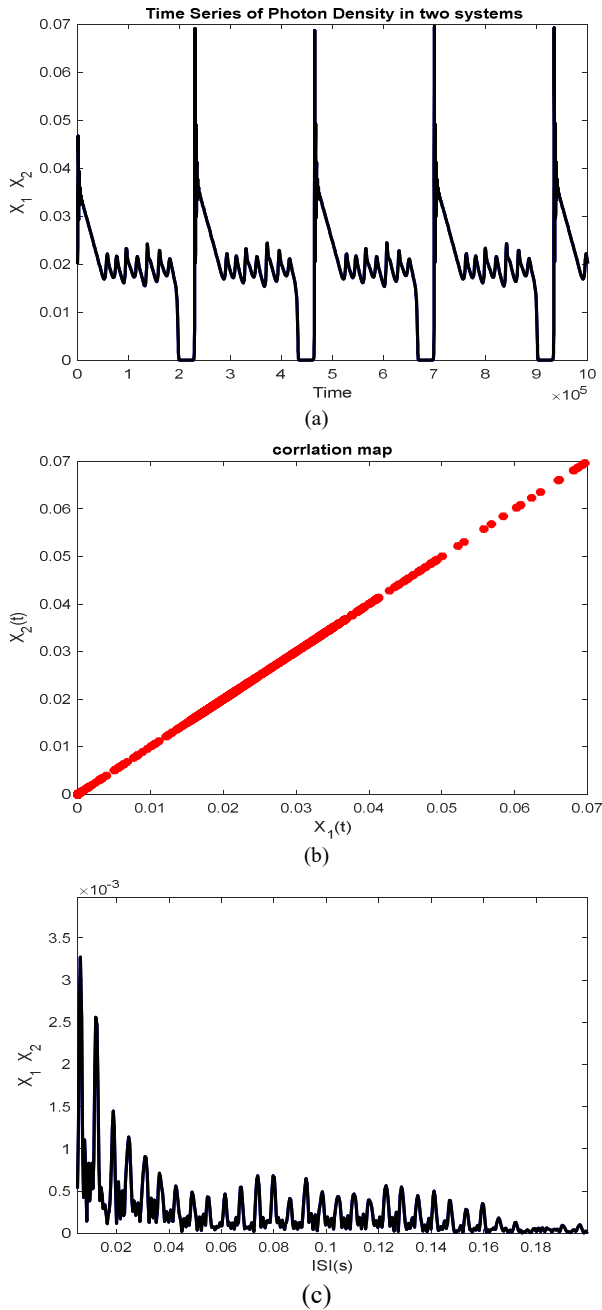


Fig. 2. (a) The time series of photon number (b) correlation map (c) ISI for both systems in phase-coupling with optoelectronic feedback loops and for master ( $x_1$ ) blue line and slave ( $x_2$ ) black line, respectively.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$  and  $k = 0$  parameters are used.

**Case 1**, at a relatively large value of the coupling strength value  $k = 0.02$  we notice that the steady-state behavior is dominant as it is observed in “Fig. 3. a and b” after its

behavior in “Fig. 2” is a mix-mode. We now reduce the value of the coupling strength by  $k = 2 \times 10^{-5}$ . We see weakly and precisely the phase coupling to match the master’s behavior as in the “Fig. 4.a”. While “Fig. 4.b” shows the periodic change in the behavior of the slave in accordance with the behavior of the master. “Fig. 5” represents the convergence of the behavior of each of the two systems and the phase coincidence greatly, where the time series “Fig. 5.a” shows a great synchronization of the phase, and “Fig. 5.b” shows the symmetry arising from phase coupling, while “Fig. 5.c” also shows the approach of the periods more and more, and that happens when the value of  $k$  reaches  $2 \times 10^{-8}$ . Continuing in this manner eventually leads to the state shown in the “Fig. 6.a”, i.e. at  $k = 2 \times 10^{-11}$ , where the phase synchronization state increases, and the symmetry state is also clear from the “Fig. 6.b and c”.

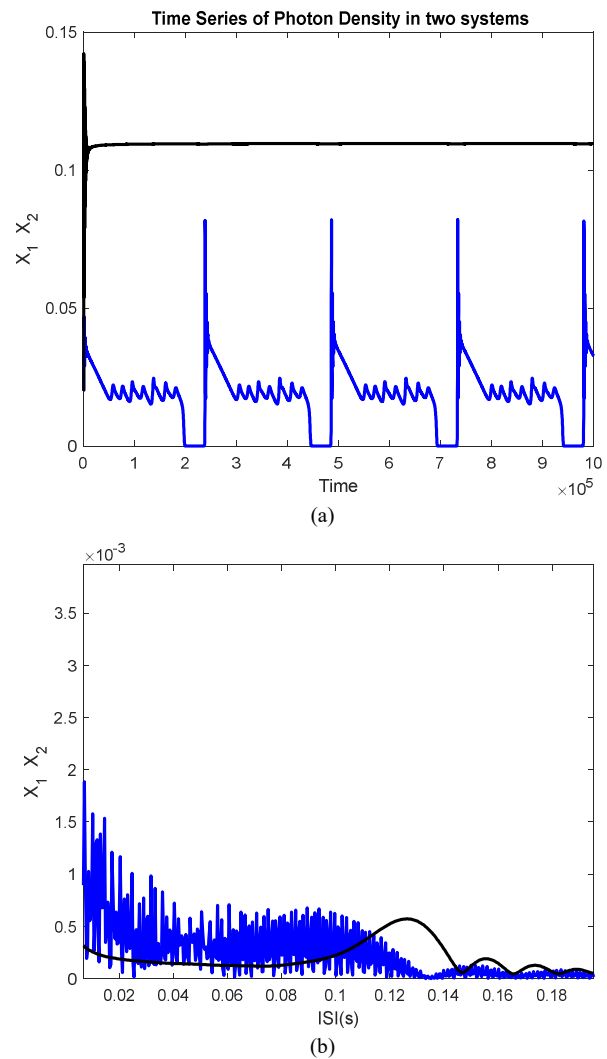


Fig. 3. (a) The time series and (b) ISI of photon number for both systems in phase-coupling with optoelectronic feedback loops.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$  and  $k = 0.02$  parameters are used.

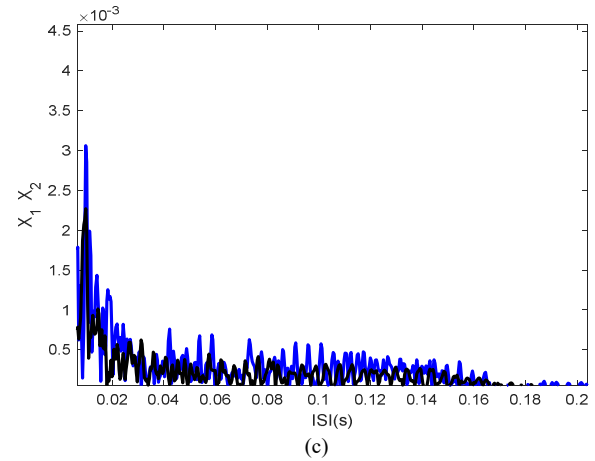
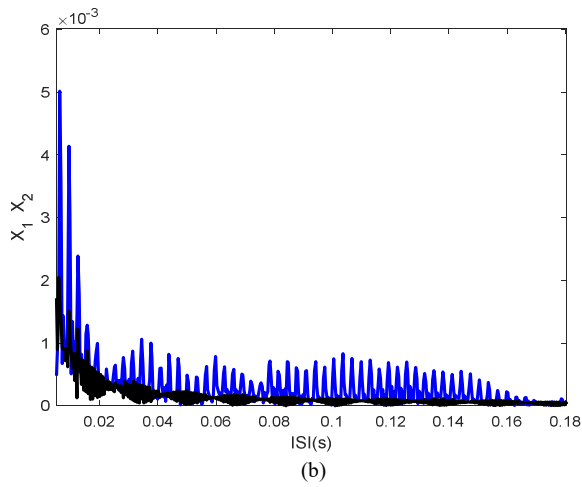
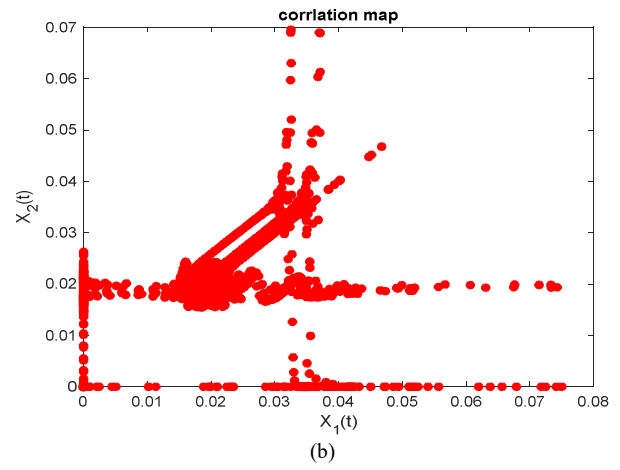
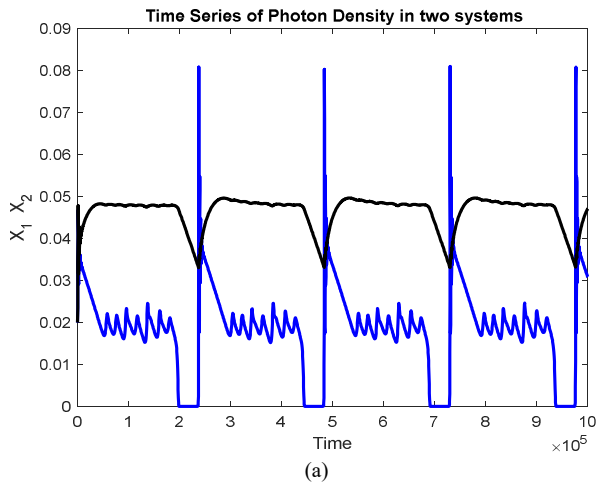
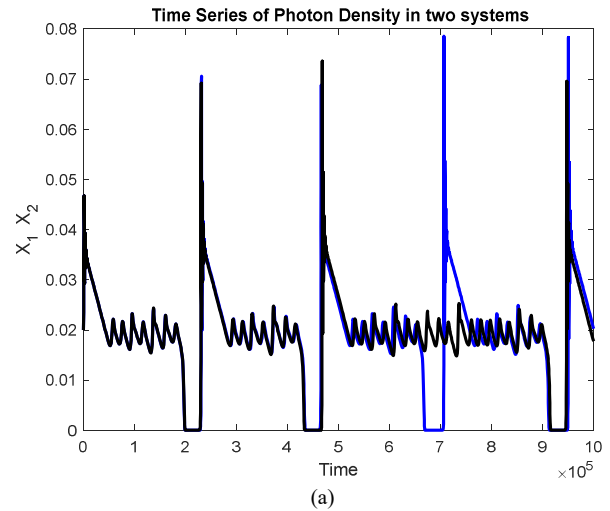
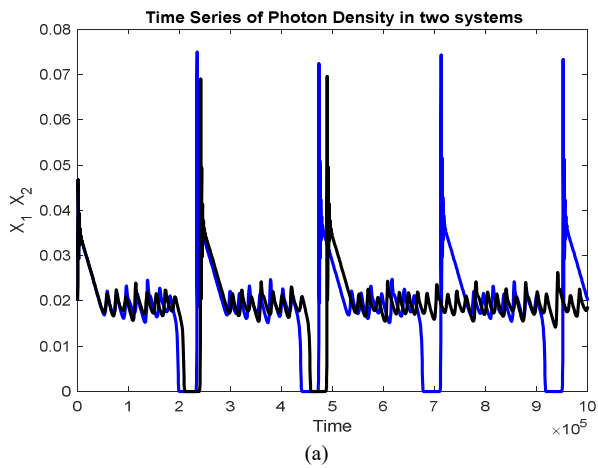


Fig. 4. (a) The time series and (b) ISI of photon number for both systems in phase-coupling with optoelectronic feedback loops.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$  and  $k = 2 \times 10^{-5}$  parameters are used.

Fig. 5. (a) The time series and (b) correlation map (c) ISI of photon number for both systems in phase-coupling with optoelectronic feedback loops.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$  and  $k = 2 \times 10^{-8}$  parameters are used.



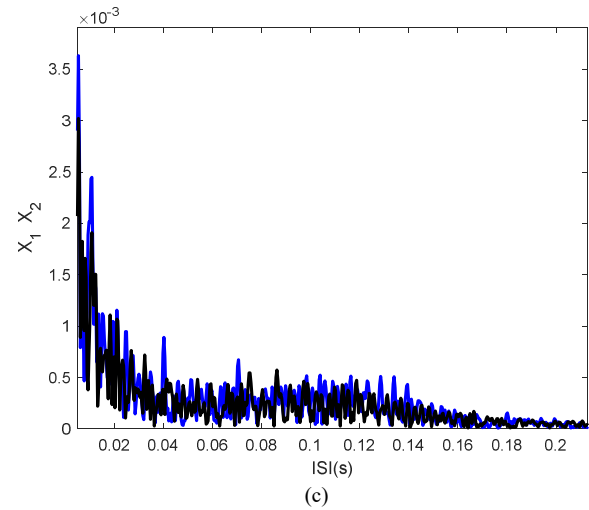
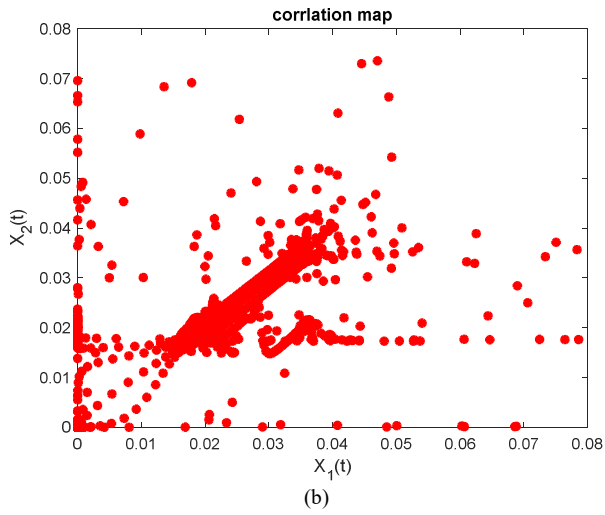


Fig. 6. (a) The time series and (b) correlation map (c) ISI of photon number for both systems in phase-coupling with optoelectronic feedback loops.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$  and  $k = 2 \times 10^{-11}$  parameters are used.

**Case II.** This case is more complicated, as we deal here with the significant effect of the coupling force, which we will fix at the highest value  $k = 0.02$ , while we search for the phase inversion case by changing the value of the strength of the feedback circuit of the slave. Where “Fig. 7” and “Fig. 8” represent the beginning of phase coupling and the frequency of each of the two systems coupled with an increase in the feedback power is the same, where  $\epsilon_2 = 4.9 \times 10^{-5}$  in the “Fig. 7” and  $\epsilon_2 = 8.9 \times 10^{-5}$  in the “Fig. 8” respectively. “Fig. 9, a” shows the optimum synchronization state in this work where the phases and frequencies coincide at fixed intervals and this happens at a certain value of the feedback strength  $\epsilon_2 = 15.9 \times 10^{-5}$ . “Figs. 9.b” and “9.c” show the phase space (attractor) for both systems. Continuing to increase the strength of the feedback  $\epsilon_2 = 30.9 \times 10^{-5}$  maintains the continuation of the phase coupling, but for the same time, it leads to the emergence of new phase frequencies resulting from the excessive influence in increasing the transitions, which in turn leads to an increase in the fluctuations in the resulting behavior, as shown in “Fig. 10”.

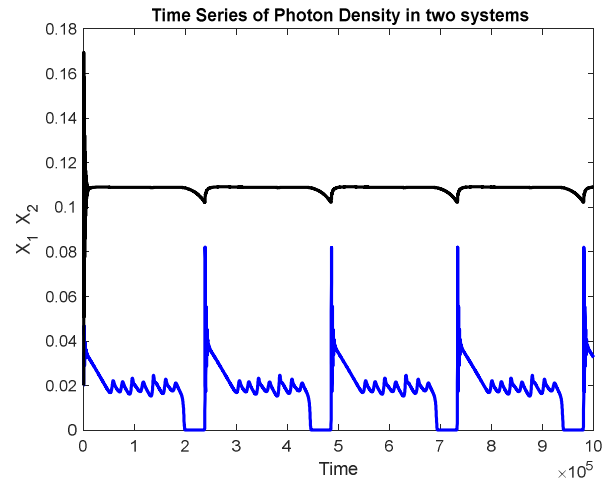


Fig. 7. The time series of photon number for both systems in phase-coupling with optoelectronic feedback loops.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$ ,  $k = 0.02$  and  $\epsilon_2 = 4.9 \times 10^{-5}$  parameters are used.

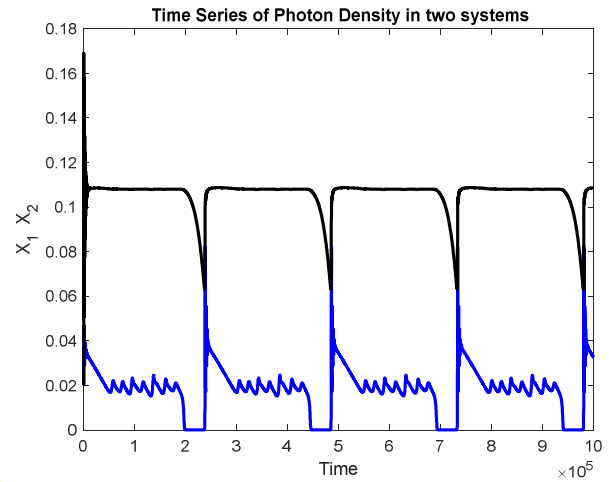
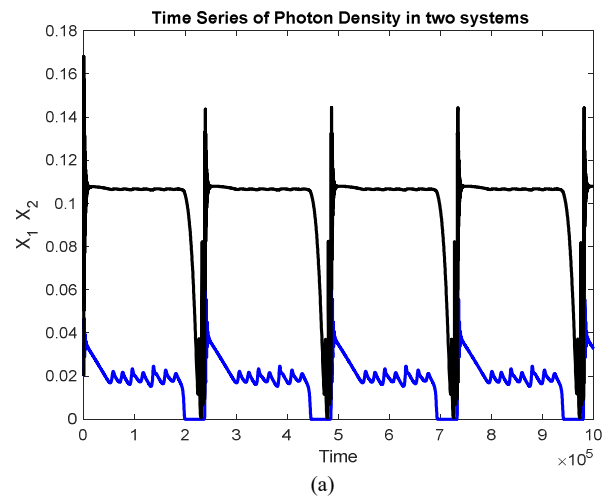


Fig. 8. The time series of photon number for both systems in phase-coupling with optoelectronic feedback loops.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$ ,  $k = 0.02$  and  $\epsilon_2 = 8.9 \times 10^{-5}$  parameters are used.



(a)

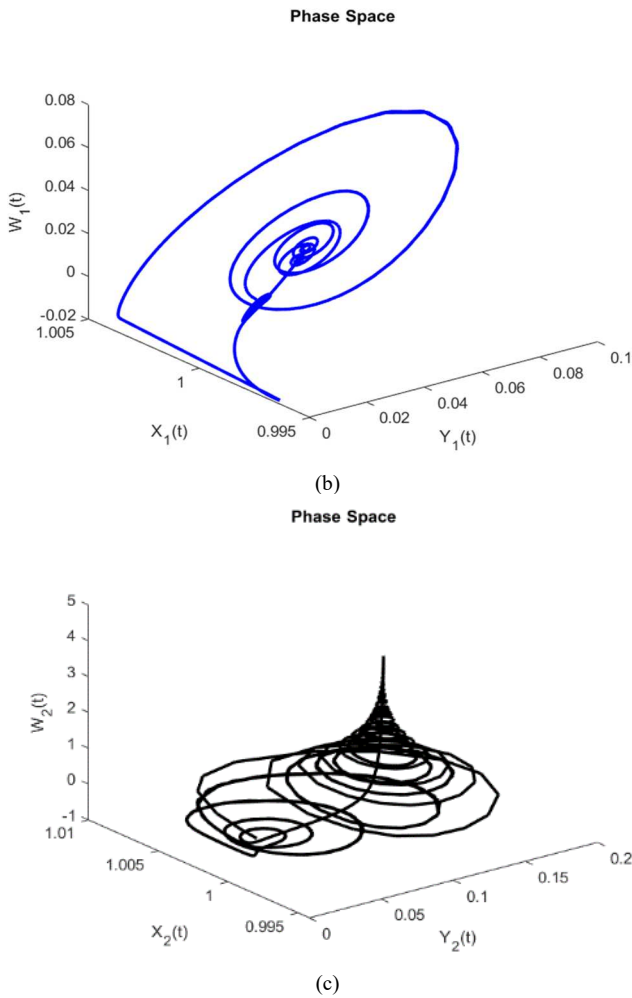


Fig. 9. (a) The time series and (b) attractor of photon number for master (c) for slave in phase-coupling with optoelectronic feedback loops.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$ ,  $k = 0.02$  and  $\epsilon_2 = 15.9 \times 10^{-5}$  parameters are used.

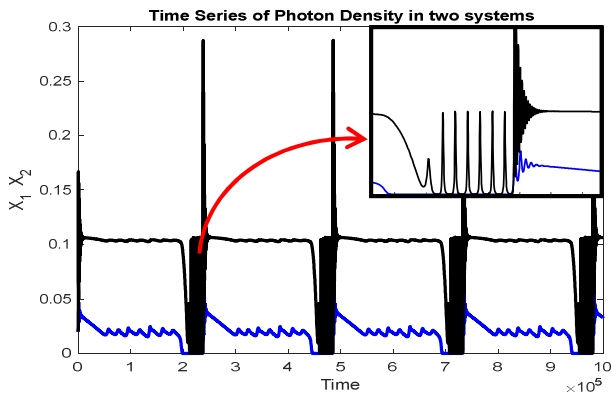


Fig. 10. The time series of photon number for both systems in phase-coupling with optoelectronic feedback loops.  $\gamma = 0.001$ ,  $\delta_o = 1.019$ ,  $s = 11$ ,  $\epsilon_1 = 0.9 \times 10^{-5}$ ,  $k = 0.02$  and  $\epsilon_2 = 30.9 \times 10^{-5}$  parameters are used.

#### IV. CONCLUSION

In conclusion, when the interaction of chaotic oscillators is very coherent, the phases are completely closed; then, the phase difference is infinite while the frequencies are restricted. The phase synchronization effecting is also possible when the accepted frequencies are in a balanced

relationship (this is closely related to a significant physical difficult related to the interaction between the cardiac and breathing systems). It is mentioned here that all phase coupling cases occur with a clear difference in capacitance in favor of the slave and the reason is due to the increase in the electric flux and thus a significant increase in the levels statistics for the slave.

#### ACKNOWLEDGEMENTS.

This work is supported by the Nassiriya Nanotechnology Research Laboratory (NNRL), Science College, University of Thi Qar, Iraq.

#### Compliance with ethical standards

**Conflicts of interest.** The authors declare that they have no conflict of interest.

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