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RESEARCH ARTICLE

Theoretical evidence for synchronous and multi-scroll attractors in coupled quantum dot light-emitting diode

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Abstract The current research work presents an evidence on overall synchronization of loosely bound strength in chaotic systems along with a new coupled design based on dual quantum dot light-emitting diodes (QDLEDs) in order to generate n-scroll attractors. To characterize these phenomena, the researchers used a theoretical approach on the basis of time series and phase space maps, i.e., attractors. In case of coupled QDLED attractors, the phases are generally locked during synchronous regime while the amplitudes are correlated. With the proposed construction scheme, both frequency detuning and coupling strength of two systems can be tuned independently. Further, chaotic attractors with even or else odd count of scrolls can also be easily generated. The study also demonstrated distinct attractors with n scrolls obtained using coupled design.

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Introduction

Semiconductor QDLED—as they are often called—are an integral part of modern technology. QDLED is widely used as incoherent light sources in applications such as lighting and short-distance optical fiber communications. An important performance characteristic of a QDLED is the output efficiency, i.e., the amount of light extracted from the structure at a given input current. Since higher modulation speed implies a larger information capacity, a high modulation speed is important for short-distance communication applications [1]. Some light sources exhibit intensity and phase fluctuations. These fluctuations are of great importance since they induce errors in optical measurements. Their origin lies in the quantum nature of transition process itself. In fact, every spontaneous emission event in the oscillating mode varying the phase of the electromagnetic field (quantum noise) is responsible for the carrier density variation [2, 3].

Various methods have been deployed by the researchers since it was discovered. These methods are aimed at improving the performance of lasers without external influences, for instance, the injection process of other lasers, controlling the quality parameter, pattern stabilization, addition of feedback inside or outside the laser as the latter gets delayed. It was found that this technique has a significant effect on laser kinematics in general since it affects the stability of the device and affects the width of the emission line, etc.; this technique is the most preferred one for several years now [4]; negative and positive late optoelectronic feedback [5], controllable chaotic dynamics in semiconductor lasers (SCLs), optoelectronic-delayed feedback [6] in synchronization process between lasers and filtered optical-delayed feedback [3, 4] to control nonlinear dynamics in SCLs and delayed nonlinear feedback [7]. When there is a delayed optical feedback, the semiconductor lasers exhibit a several of complex dynamic behaviors that are governed by nonlinearities among the deterministic field, deterministic interactions of both electromagnetic and semiconductor material and the noise produced by quantum mechanical processes of spontaneously emitted photons. The quality of the specific behavior heavily depends on the distance measured from reflective surface to laser and on the basis of amount of reflectivity of the surface. Thus, low reflections from optical fiber connections or CDs can significantly alter the kinetics of semiconductor laser [8].

As per the literature [9, 10], wide ranges of nonlinearities are exhibited by QDLEDs under optical injection. It is possible to detail about the QDLED dynamics through the frequency of the addition of optical injected signal to slave laser. This detuning difference maintains a control on slave system, whereas the master one is maintained as it is and provides no disturbance. Here, injection locking remains a highly beneficial tool in the stabilization of semiconductor devices. But the injection-locked SCLs exhibit a prism of dynamics as per the literature [11].

When it comes to framing the design for multi-scroll attractors, it is always a challenge to design multiple equilibrium points that may scroll a messy attractant, on the basis of changes brought by nonlinear treatment feature. In the literature [12], this change is brought through the submission of additional breakpoints in Chua circuit's nonlinear characteristic through which one can attain double n-scroll attractants.

In the study conducted earlier [13], the same modification was done using sine function. In the studies conducted earlier [14] and [15], the authors used a set of step functions and hyperbolic smooth tangent functions to conduct the modifications. Indeed, both design and realization of multi-scroll attractants rely upon nonlinear assembly with electricity circuit. The question arises whether an electrician drives a device that can naturally allow multi-pass mess design attractant.

The current study attempts to investigate the overall synchronization of chaotic systems. With the help of Poincaré map methods (which means that periodic orbits start on the subspace flow through it and not parallel to it), the researchers arrive at the conclusion that when non-identical autonomous chaotic oscillators interact with each other, it results in optimum locking of the phases with chaotic amplitudes. In case of handling a weaker synchronization type, most of the frequencies are locked while the phase difference showcases a random-walk-type motion. One such state variable is phase difference of the junctions which produces the n-scroll attractor. Here we made use of a closed chain of coupled identical n-scroll attractors to analyze the n-scroll hypercube attractors. In this work used QDLED leaving beside the QD lasers, thanks to the broad emission of QDLED. This broad mission poses serious challenges to academicians in terms of controlling the multi-mode output and fulfillment of injection locking requirements.

QDLED model

In order to model the proposed QDLED rate equations under external optical feedback, we are using multiple equations given below. With complex electric field E, (real) number of carriers denoted by QDs n_{QD} and WL n_{wl}, the following equations are applicable [3, 9].

$$\dot{E} = -\frac{1}{2}(1+i\alpha)\, W n_{\mathrm{QD}} E - \gamma_{\mathrm{s}}\frac{E}{2} + \frac{k}{2}E_{\tau}\, e^{-i\Theta} + E_{\mathrm{sp}}(t) \tag{1a}$$

$$\dot{n}_{\mathrm{QD}} = \gamma_c n_{\mathrm{wl}}\left(1 - \frac{n_{\mathrm{QD}}}{2N_d}\right) - \gamma_{r_{\mathrm{QD}}} n_{\mathrm{QD}} - \left(E_{\mathrm{sp}}(t) - W n_{\mathrm{QD}}|E|^2\right) \tag{1b}$$

$$\dot{n}_{\mathrm{wl}} = \frac{I}{e} - \gamma_{r_{\mathrm{wl}}} n_{\mathrm{wl}} - \gamma_c n_{\mathrm{wl}}\left(1 - \frac{n_{\mathrm{QD}}}{2N_d}\right) \tag{1c}$$

Here, $E(t) = \sqrt{S}e^{-i\psi(t)}$ denotes the generalized mildly differentiable complicated amplitude of the electrical field, S denotes the number of photons, whereas ψ denotes the phase. Here, α parameter denotes the linewidth enhancement factor, ω_o denotes single optical mode frequency while the symbol k is the parameter which can measure the injection feedback strength. At the time of one-round trip in external cavity ($\tau = 2l/c$) where l is length of external cavity, the phase shift value is derived using $\Theta = \omega_o\tau$. Here, c denotes the light speed. Since the field contains two subscripts such as τ and E_{τ}, so, the symbol ψ_{τ} denotes the electric field amplitude followed by optical phase considered for the delayed time (t-τ). The $R_{ind} = W n_{\mathrm{QD}} S$ models the induced processes of absorption in which W denotes the Einstein coefficient. $\gamma_{r_{\mathrm{QD}}}$ and $\gamma_{r_{\mathrm{wl}}}$ denote the nonradiative decay rates for the count of carriers in QD and WL, respectively. Here, N_d denotes the overall count of QDs, I corresponds to injection current, e corresponds to elementary charge, γ_c is pointed at the capture rate from WL into dot, and finally, γ_{s} corresponds to the output coupling rate of photons in optical mode. In both WL and QD, the population distributions is explicitly considered so as to correlate between absorption and spontaneous emission spectra. $E_{\mathrm{sp}}(t)$ corresponds to the stochastic function which in turn point toward the zero-mean random field for spontaneous emissions. As per the literature [8], the field relation is shown herewith empirically $\langle E_{sp}(t)E^{*}(t)\rangle = R_{sp}/2$ [8]. According to the equation, (*) symbol corresponds to complex conjugate. Here, the notation R_{sp} is utilized for the impact created by spontaneous emission in photon number equation and is given in equation [9].

$$R_{sp} = Wn_{QD}^{2}$$

Furthermore, when the fundamental dynamics of instability and chaos in nonlinear systems are investigated, the deterministic terms can be treated after considering the statistical noises.

Phase controlling is one of the direct methods to operate a QD in single mode. Figure 1 shows that the light can travel the controlled frequencies through the cavity.

In Eq. (1), a delay term $\frac{k}{2}E(t-\tau)e^{-iw_o\tau}$ models the feedback from external flat mirror. In order to analyze the phase controlling dynamics using the optically injected QD, Eq. (1) is once again inducted into the photon number for master and slave lasers, i.e., $S_m(t)$ and $S_s(t)$, while the equations for phase $\psi(t)$ and carrier number (t) are as follows.

$$S_s^{\cdot}(t) = -Wn_{QD}(t)S_s(t) - \gamma_s S_s(t) + Wn_{QD}^2(t) + \zeta\sqrt{S_m(t)}\cos\psi(t) \tag{2a}$$

$$\psi^{\cdot}(t) = \frac{1}{2}\alpha Wn_{QD}(t) - \zeta\sqrt{\frac{S_m(t)}{S_s(t)}}\sin\psi(t) - \Delta\omega \tag{2b}$$

$$n_{QD}^{\cdot}(t) = \gamma_c n_{wl}(t)\left(1 - \frac{n_{QD}(t)}{2N_d}\right) - \gamma_{r_{QD}} n_{QD}(t) - (Wn_{QD}^2(t) - Wn_{QD}(t)S_s(t)) \tag{2c}$$

$$n_{wl}^{\cdot}(t) = \frac{I}{e} - \gamma_{r_{wl}} n_{wl}(t) - \gamma_c n_{wl}(t)\left(1 - \frac{n_{QD}(t)}{2N_d}\right) \tag{2d}$$

$$\psi(t) = \phi_s(t) - \phi_m(t) - \Delta\omega$$

Here $\zeta = \frac{K_{in}}{2\tau_{in}}$ is the coupling strength rate. A phase $\psi(t) = \varphi_m(t) - \varphi_s(t) - \Delta\omega$ is introduced in which $\Delta\omega = 2\pi\Delta\upsilon = \omega_m - \omega_s$ denotes the detuning process between angular frequencies while ω_m and ω_s correspond to master and slave systems, respectively. However, the phase control plays a rule of injection. Then, it controls the selection of frequency and phase shift of the electric field E in optical mode.

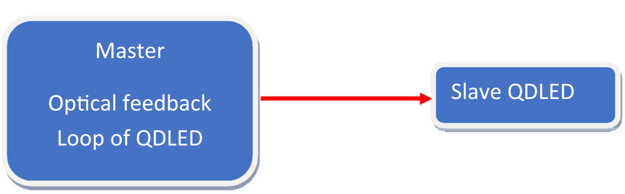

Fig. 1 Configuration of optical feedback system for both master and silver QDLEDs

Equation (2) is rewritten for numerical application in dimensionless form. The new variables are thus introduced as given herewith.

$x = S,\ \psi \equiv \psi,\ y = \frac{W}{\gamma_s}n_{QD},\ z = \frac{n_{wl}\gamma_c}{W},$
$\gamma = \frac{\gamma_s}{\gamma_{r_{wl}}},\ \gamma_2 = \frac{W}{\gamma_{r_{wl}}},\ \gamma_1 = \frac{W}{\gamma_s},\ \gamma_3 = \frac{\gamma_{r_{QD}}}{\gamma_{r_{wl}}},\ \gamma_4 = \frac{\gamma_c}{\gamma_{r_{wl}}},$
$N_d \equiv a,\ \delta_o = \frac{I}{We},\ t' = \gamma_{r_{wl}}t$. The rate equation become

$$\gamma_1 x_s^{\cdot} = (\gamma(y^2 - \gamma_1 x_s(y+1)) + \gamma_1 C_{12}\sqrt{x_m(t)}\cos(\psi)) \tag{3a}$$

$$\psi^{\cdot} = \frac{\alpha\gamma}{2}y - C_{12}\sqrt{\frac{x_m}{x_s}}\sin(\psi) - \delta w \tag{3b}$$

$$y^{\cdot} = \gamma_2 z(\gamma_1 - y/a) - y(\gamma_3 + \gamma y) + \gamma_2 x_s y \tag{3c}$$

$$z^{\cdot} = \gamma_4(\delta_o - z + yz/\gamma_1 a) - z \tag{3d}$$

Here, $\delta w = \Delta\omega/\gamma_{r_{wl}}$ and the injection ratio are also normalized in the form of $C_{12} = \zeta/\gamma_{r_{wl}}$. It is an established assumption that the delay time τ is higher compared to the round trip time taken inside the active region.

QDLED dynamic results

A set of four coupled equations are necessary for the functioning of QDLED with optical injection. These equations exhibit dynamic oscillations and chaotic nature in their output powers as shown in Eq. (3) coupled equations of Lorenz systems. In particular, the term of the external optical feedback was added in Eq. (3a) which is providing the degree of freedom needed to generate chaos. The chaos dynamics, found in QDLED systems, are discussed in detail with external effect. Followed by, different routes are shown for chaos within the threshold of parameter differences. All the three system rate equations are numerically solved with the help of 4th order Runge–Kutta method and implemented in MATLAB/Simulink system. The study used the following parameters for simulation process such as $\gamma = 0.158$, $\gamma_1 = 0.049$, $\gamma_2 = 0.026$, $\gamma_3 = 0.03$, $\gamma_4 = 0.078$ and $a = 0.891$. The initial values were $x_o = 0.066$, $Ø_o = 0.066$, $y_o = 0.99$, $z_o = 0.0049$. These values were attained by resolving system (3) at steady-state conditions.

The dynamics of two QDLED masters under optical feedback, the dynamics of other slaves under optical injection and the realization of phase coupling enable the researchers to make use of a behavioral diagram drawn at master dynamics through several processes, for instance, frequency of phase. By keeping the master output under control, the outputs of the master as well as slave are incorporated. Master dynamic output is highly

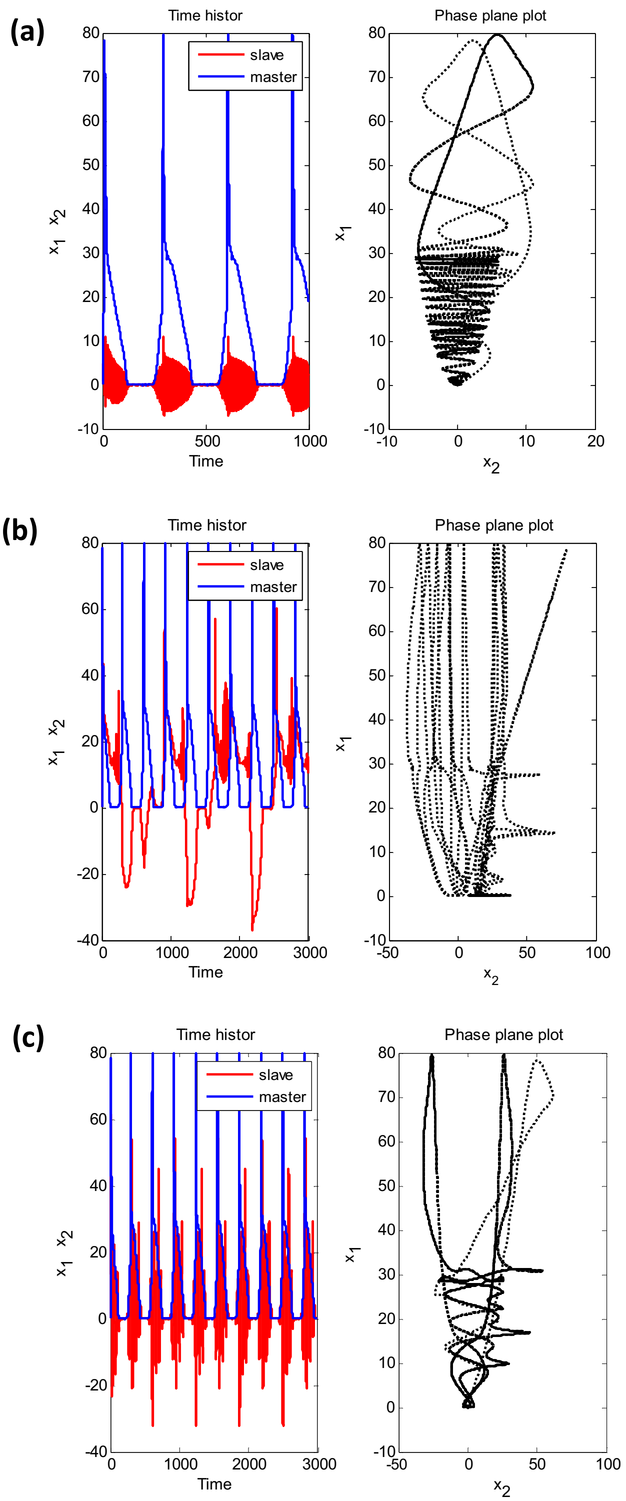

Fig. 2 Time series and phase plane of two coupled QD systems (Eq. 1) for nonsynchronous α=2 and $C_{12} = 1$ with phase differences such as **a** $\delta\omega = 0.9$, **b** $\delta\omega = 0.27$ and **c** $\delta\omega = 0.\ 15$

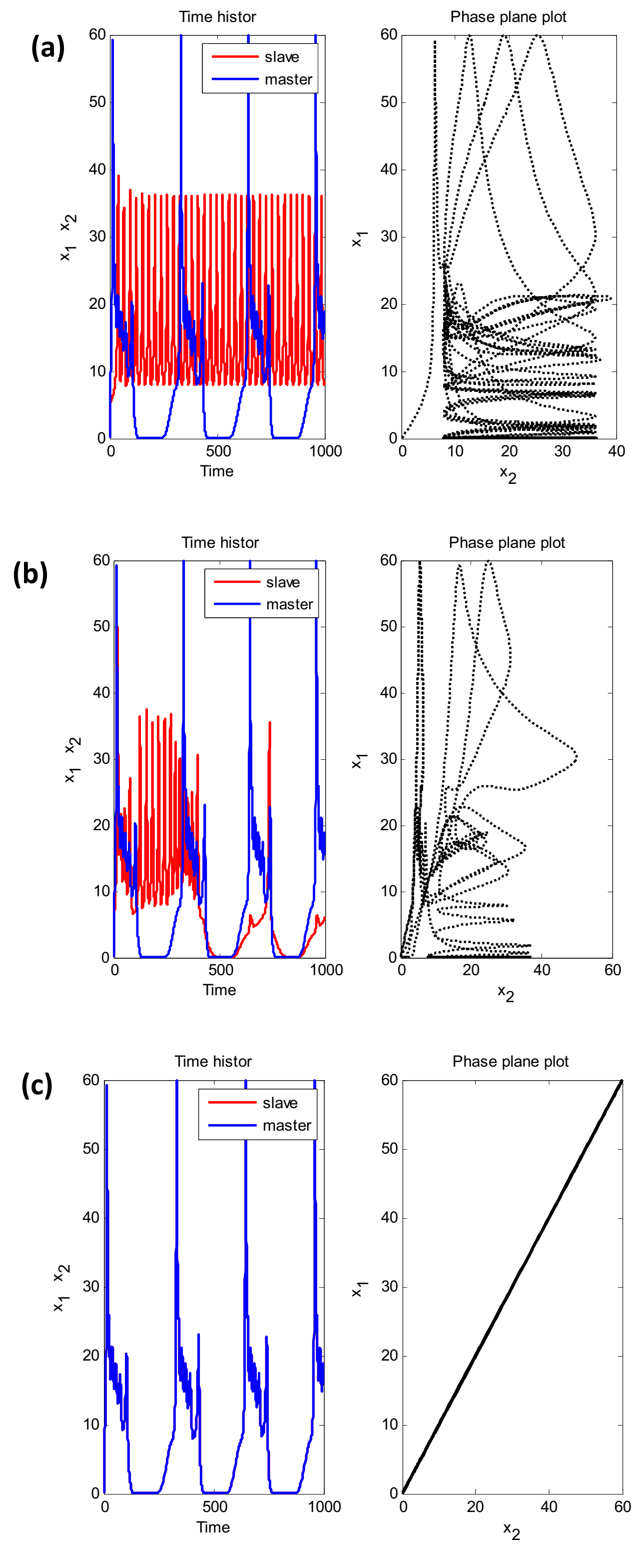

Fig. 3 Time series and phase plane of two coupled QD systems for nonsynchronous α=2 and $\delta\omega = 0$ with coupled strength, **a** $C_{12} = 0.01$ and **b** $C_{12} = 0.\ 4$; and synchronous states, **c** C_{12} = 0.5, respectively

predictable due to which the slave hand is also predicted in advance. Due to the corresponding phase locking process, the predictable output of the master can be utilized for the slave as well. The coupling scheme's performances were evaluated. Some appropriate behaviors of systems were

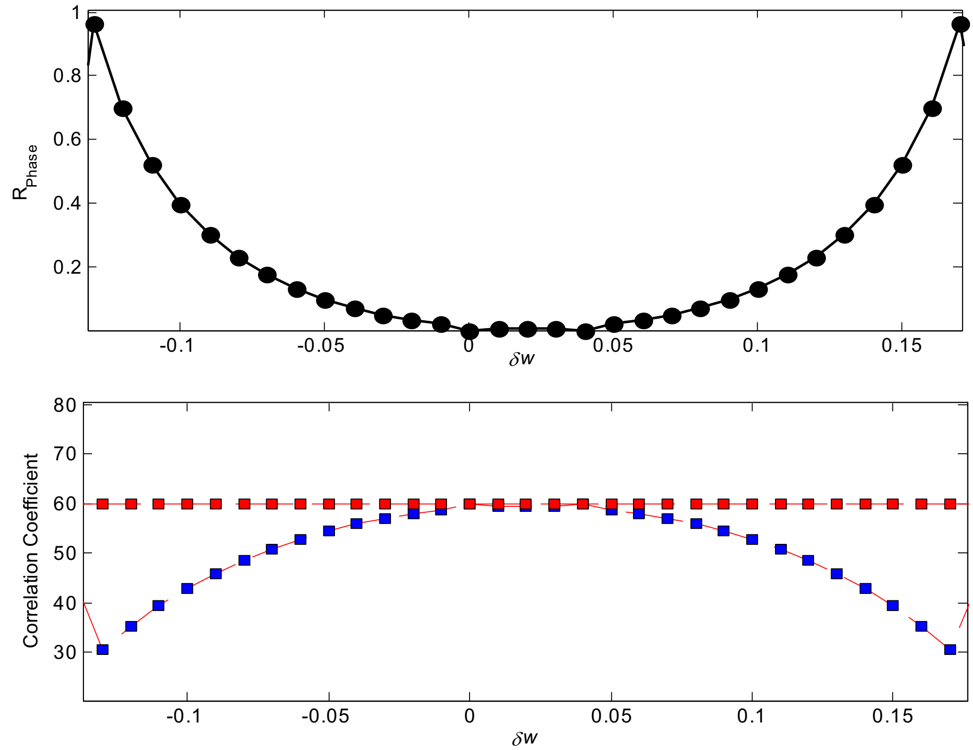

Fig. 4 Typical residual of synchronous and correlation record of the phase difference relative to two coupled QD systems under fixed strength coupling

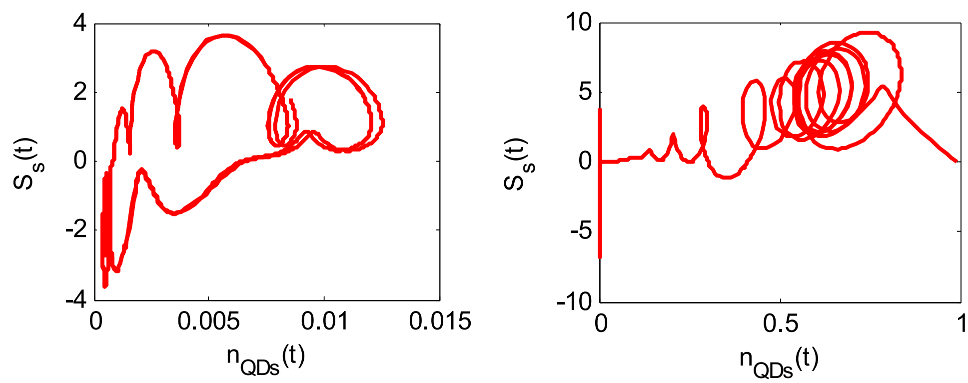

Fig. 5 n-scroll attractor from the model [two coupled QD systems] (1) with $C_{12} = 0.5$, $\delta w = 0.18$ and $\delta w = 0.6$

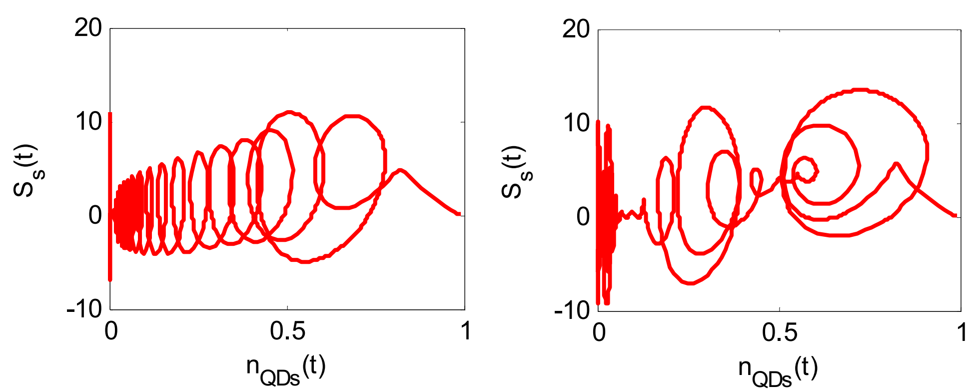

Fig. 6 n-scroll attractor from the model [two coupled QD systems] (1) with $C_{12} = 1$, $\delta w = 0.9$, and $\delta w = 0.8$

found for a wide range of factors in communication applications.

Figures 2 and 3 show the results of periodic pulsing regime analysis for both normalized time series and phase space, respectively.

In case of the simple attractor, it is straightforward to introduce the phase, thanks to its symmetrical nature. In Figs. 2 and 3, the researchers introduce the parameters $\delta\omega$ and C_{12} which govern frequency detuning and the strength of coupling, respectively. Figure 2 shows the decline in the fixed coupling strength for mismatch, $\delta\omega = 0.9, 0.27, 0.15$. From this result, one can understand a regime's transition where the phases tend to revolve around under different velocities. This occurs in a phase synchronous state during when the phase difference evolves in the due course of time. With locked phases in this synchronous regime, the amplitudes differ in a chaotic manner and there exists an impractical correlation between them. In every figure, it is shown that the time periodic interval gets changed due to the master that developed to a state of extreme synchronization related to two coupled QD systems, i.e., $\alpha = 2$ and strength coupled $C_{12} = 1$. It is stressed that, in comparison with other synchronization kinds found in chaotic systems [16], there is no coincidence available between the instant fields such as x_1 and x_2. Further, even if the correlation is present between x_1 and x_2, it remains pretty small (Fig. 2). In spite of the fact that the phases are completely locked, the motions remain highly coherent.

Figure 3 shows one more synchronization kind in which the frequency detuning is entrained while the phase difference is unbounded, i.e., $\delta\omega = 0$. Further, the coupling strength remains changing i.e., $C_{12} = 0.01$, 0. 4 and 0.5. In the figures given above, it can be noticed that the slave's behavior got developed due to the increased strength of the coupling. This is because the dynamics of the slave is forced to meet the dynamics of the master completely. This occurred in spite of the fact that the initial behavior of the slave does not significantly agree with the elementary behavior of master. Here the impulsive behavior or mixed-mode behavior interrupted and changing this notion. When coupling strength value reached half of the total value, the total synchronization occurred, a result of commonly found type and the most concerned application of secure communications.

Figure 4 shows the relation between the correlation amplitude and the properties of synchronization residue. According to Fig. 4, the synchronization was achieved in coupled QDLED systems, thanks to the appropriate choice of detuning frequency in both systems that coincided together.

Figure 4a shows the synchronization region in the plane of parameters, whereas "coupling–frequency mismatch" was obtained using the residue map. It is to be noted that there is no threshold when the frequency mismatch is small, i.e., $\delta\omega \approx 0$. During this scenario, the synchronization has already appeared for coupling. This is a particular feature of QDLED systems, where the motion is highly coherent. On the other side, it is possible to predict the synchronization of systems with frequency mismatch from correlation coefficient plotting (see Fig. 4b).

A new approach was introduced and shown in Fig. 5 on the basis of nonlinearity and dynamics of QDLED so as to produce n-scroll attractors. The phase difference ($\delta\omega = 0.18, 0.6$) of the coupled QDLEDs ($C_{12} = 0.5$) was one of the state variables for the resultant n-scroll attractor. The production of the n-scroll attractors is shown in a closed chain of coupled mismatched n-scroll attractors (see Fig. 5). This chain of coupled mismatched n-scroll attractors is introduced from two coupling systems with the help of a new coupling way. When the phase difference was increased ($\delta\omega = 0.8,\ 0.9$), by increasing the coupling strength ($C_{12} = 1$), it in turn increased the chances of obtaining even or odd scroll numbers as well as obtaining some symmetry (see Fig. 6).

Conclusion

The current research work proposed a new coupling method for two QDLEDs since it possesses distinct characteristics that made them a special object of interest among researchers. Initially, synchronization was investigated by coupling strength and phase difference effects. Two types of synchronizations were conducted such as phase synchronization in case of a difference in the frequency of two coupled systems and complete synchronization of amplitude and phase in case of same frequency of two systems. Subsequently, the generation of n scrolls in slave output was investigated, due to complex nonlinear behavioral properties of QDLED. Finally, the proposed design yielded excellent and wide range of results.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. T.R. Kim, *Quantum-Dot Light-Emitting Diode (LED)* (Evident Technologies, Inc., 2008) [Online]. http://www.evidenttech.com/Quantum-Dot-(LED).html

2. X. Yang, Y. Ma, X. Wei Sun, Stable, efficient, and all-solution-processed quantum dot light emitting diodes with double-sided metal oxide nanoparticle charge transport layers. ACS Appl. Mater. Interfaces **6**, 495–499 (2014)
3. H. Al Husseini, A. Al Khursan, K. Al Naimee, S.F. Abdalah, A.H. Khedir, R. Meucci, F.T. Arecchi, Modulation response, mixed mode oscillations and chaotic spiking in quantum dot light emitting diodes. Chaos Solitons Fractals **78**, 229–237 (2015)
4. M. Yousefi, D. Lenstra, G. Vemuri, A. Fischer, Control of non-linear dynamics of a semiconductor laser with filtered optical feedback. IEEE Proc. Opt. Elect. **148**, 233–237 (2001)
5. L. Thevenaz, S.L. Floch, D. Alasia, J. Troger, Novel schemes for optical signal generation using laser injection locking with application to Brillouin sensing. Meas. Sci. Technol. **15**, 1519–1524 (2004)
6. C. Guignard, P. Besnard, A. Mihaescu, N.I. Zheludev, Harmonic passive mode-locking of a single-frequency semiconductor laser submitted to nonlinear optical feedback. IEEE J. Quantum Electron. **42**, 1185–1195 (2006)
7. M. Vainio, Diode lasers with optical feedback and optical injection, PhD thesis, Helsinki University of Technology (2006)
8. E.K. Lau, H.K. Sung, M.C. Wu, Frequency response enhancement of optical injection-locked lasers. IEEE Quantum Electron. **44**, 90–99 (2008)
9. K. Al Naimee, H. Al Husseini, A. Al Khursan, S.F. Abdalah, A.H. Khedir, R. Meucci, F.T. Arecchi, Complex dynamics in quantum dot light emitting diodes. Eur. Phys. J. D **69**, 1–5 (2015)
10. H.B. Al Husseini, K.A. Al Naimee, A.H. Al-Khursan, A.H. Khedir, External modes in quantum dot light emitting diode with filtered optical feedback. J. Appl. Phys. **119**, 224–301 (2016). https://doi.org/10.1063/1.4953651
11. H.B. Al Husseini, H.A. Al Rekabie, Optical injection locking in quantum dot light emitting diode. J. Appl. Phys. Nanotechnol. **1**, 1–5 (2018)
12. J.A.K. Suykens, J. Vandewalle, Generation of n-double scrolls (n= 1,2,3,4,…). IEEE Trans. Circuits Syst. **I**(40), 861–867 (1993)
13. M.E. Yalçin, S. Özoğuz, J.A.K. Suykens, J. Vandewalle, Families of scroll grid attractors. Int. J. Bifurcation Chaos **12**, 23–41 (2002)
14. K.S. Tang, G.Q. Zhong, G. Chen, K.F. Man, Generation of nscroll attractors via sine function. IEEE Trans. Circuits Syst. **I**(48), 1369–1372 (2001)
15. S. Özoğuz, A.S. Elwakil, K.N. Salama, n-scroll chaos generator using nonlinear transconductor. Elecron. Lett. **38**(14), 685–686 (2002)
16. H.B. Al Husseini, Evaluation of quantum dot light-emitting diodes synchronization under optically feedback. J. Opt. Commun. (2020). https://doi.org/10.1515/joc-2020-0110