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A study of *E*-Connectedness and Strongly *E*-Connectedness in Topological spaces and Their Applications

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ABSTRACT

This paper is devoted to introduce and study new concept of separated sets called, E-separated sets by utilizing another generalized open set namely, E-open, as well with this concept we introduce a new class of connected spaces which is called strongly E-connected spaces and investigate some basic properties of E-connected spaces. Several characterizations and fundamental properties concerning of such classes of connected spaces with some E -Separation axioms and compact spaces are obtained. The behavior of E-connected spaces with respect to several types of well-known mappings is discussed. Furthermore, we construct new topological spaces on a connected graph.

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1. Introduction

The notions of connectedness, strongly connectedness and compactness are useful and fundamental notions not only in general topology but also of other advanced branches of mathematics. In topology and related branches of mathematics, connectedness plays a crucial rule in topological spaces where many problems use connectedness to distinguish topological spaces. Many other stronger types of connectedness were studied such path connected, widely connected, bi-connected, and *n*-connected spaces to study the structure of topological spaces and the geometry of the topological spaces, see {[**1**], [**2**]}.

"Over the last thirty years several concepts of connectedness have been studied and considered, many researchers [**3-6**] have investigated the basic properties of connectedness and compactness. The productivity and fruitfulness of these notions of connectedness and compactness motivated mathematicians to generalize these notions".

Connectedness and compactness are powerful tools in topology but they have many dissimilar properties. Connectedness in [**7–9**] is used to expand some topological spaces, as well in [**10**], authors studied some types of connected topological spaces.

"As well known, connectivity occupies very important place in topology, several form of connectedness were investigated in the literature such as semi-connectedness [**11**], pre-connectedness [**12**], α-connectedness [**13**] and *ß*-connectedness [**13-15**] in topological spaces are based on the notions of semi-open set , pre-open set, α-open and *ß*-open set, respectively.

The classes of semi-connectedness, pre-connectedness and *ß*-connectedness in topological spaces are subclasses of the class of connected topological spaces". Many authors have presented different kinds of connectivity in fuzzy setting ([**16**], [**17**], [**18**], [**19**], [**20**]).

The classes of (*E*-open and *δ*-*ß*-open) sets was introduced and discussed via Erdal Ekici [**21, 22**], and E. Hatir, T. Noiri [**23**], and since then these notions have used to define and investigate many topological properties. Moreover A. A. El-Atik et al. [**24**] presented a new class of connectedness in topological spaces and studied a new notion of separated sets and utilize it to study other types of connected and strongly connected sets. Recently, Noiri and Modak [**25**] introduced half b-connectedness in topological spaces. Tyagi et al. studied several forms of connectedness in topological spaces using the ideal concepts (see [**26, 27**]) and also in generalized topological spaces, see [**28, 29, 30**]. The Present paper offers another extension for the classical meaning of connectedness (of subsets in topological spaces). The aim of this paper is to introduce a new type of connected spaces which is called strongly *E* -connected spaces and study the notion of *E*-connected spaces by using *E*-open sets. We also introduce some characterizations and basic properties concerning of such types of connected spaces with some *E*-Separation axioms and compact spaces. As well the digital spaces in the context of these new concepts are examined. Finally we construct a new topological space on a connected graph.

2. PRELIMINARIES

Throughout this paper, $(\mathcal{X},\mathcal{T})$, $(\mathcal{Y},\ \mathcal{T}^*)$ and $(\mathcal{Z},\mathcal{T}^{**})$(or simply $\mathcal{X},\mathcal{Y}$ and $\mathcal{Z}$) meanTopo − logical spaces on which no separation axioms are assumed unless explicitly stated. For any subset $\mathcal{A}$ of $\mathcal{X}$, the closure and interiorof $\mathcal{A}$ are denoted by $Cl(\mathcal{A})$and $Int(\mathcal{A})$, respectively. We recall the following definitions and results of generalized open sets, which will be used often throughout this work.

Definition 2.1: Let$(\mathcal{X},\ \mathcal{T})$be a topological space. A subset $\mathcal{A}\ of\ \mathcal{X}$ is said to be:

a) Regular open (resp. regular closed) [**31**] if$\mathcal{A} = Int(Cl(\mathcal{A}))(resp.\ \mathcal{A} = Cl(Int(\mathcal{A})))$.

b) $\delta-$open [**32**] if for each $x \in \mathcal{A}$ there exists a regular open set $\mathcal{V}$ such that $x \in \mathcal{V} \subseteq \mathcal{A}$. The δ-interior of $\mathcal{A}$ is the union of all regular open sets contained in $\mathcal{A}$ and is denoted by $Int_\delta(\mathcal{A})$. The subset A is called $\delta-$open [**32**] if $\mathcal{A} = Int_\delta(\mathcal{A})$. A point $x \in \mathcal{X}$ is called a δ-cluster points of $\mathcal{A}$ [**32**] if $\mathcal{A} \cap \mathrm{Int}(\mathrm{Cl}(\mathcal{V})) \neq \emptyset$, for each open set $\mathcal{V}$ containing x. The set of all δ-cluster points of A is called the δ-closure of $\mathcal{A}$ and is denoted by $Cl_\delta(\mathcal{A})$. If $\mathcal{A} = Cl_\delta(\mathcal{A})$, then $\mathcal{A}$ is said to be $\delta-$closed [**32**]. The complement of $\delta-$ closed set is said to be $\delta-$ open set. A subset $\mathcal{A}$ of a Topological space $\mathcal{X}$ is called $\delta-$ open [**32**] if for each $x\in\mathcal{A}$ there exists an open set $\mathcal{G}$ such that, $x\in\mathcal{G}\subseteq \mathrm{Int}(\mathrm{Cl}(\mathcal{G}))\subseteq\mathcal{A}$. The family of all $\delta-$open sets in $\mathcal{X}$ is denoted by. $\delta\Sigma(\mathcal{X},\ \mathcal{T})$.

c) $\alpha-$ open[**33**] (resp. semi $-$ open [**34**], pre $-$ open [**35**], ß $-$ open [**36**] or semi $-$ pre $-$ open [**37**], b $-$ open [**38**] or $\gamma-$ open [**39**], $\boldsymbol{\delta}-$ pre $-$ open [**40**]) if $\mathcal{A}\subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(\mathcal{A})))$ (resp. $\mathcal{A}\subseteq \mathrm{Cl}(\mathrm{Int}(\mathcal{A}))$, $\mathcal{A}\subseteq \mathrm{Int}(\mathrm{Cl}(\mathcal{A}))$, $\mathcal{A}\subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(\mathcal{A})))$, $\mathcal{A}\subseteq \mathrm{Int}(\mathrm{Cl}(\mathcal{A}))\cup \mathrm{Cl}(\mathrm{Int}(\mathcal{A}))$, $\mathcal{A}\subseteq \mathrm{Int}(Cl_{\delta}(\mathcal{A}))$.

Remark 2.2: The complement of a semi $-$ open(resp. $\alpha-$ open, pre $-$ open, ß $-$ open, b $-$ open, $\delta-$ pre $-$ open) set is said to be semi $-$ closed(resp. $\alpha-$ closed, pre $-$ closed, ß $-$ closed, b $-$ closed, $\delta-$ pre $-$ closed). The intersection of all b $-$ closed (resp. semi $-$ closed, $\alpha-$ closed, pre $-$ closed, ß $-$ closed, $\delta-$ pre $-$ closed) sets of $\mathcal{X}$ Containing $\mathcal{A}$ is called the $b-$ closure (resp. s $-$ closure, $\alpha-$ closure, pre $-$ closure, ß $-$ closure, $\delta-$ pre $-$ closure)of $\mathcal{A}$ and are denoted by $bCl(\mathcal{A})$, $(resp. SCl(\mathcal{A}), \alpha Cl(\mathcal{A}), PCl(\mathcal{A}),$ ß$Cl(\mathcal{A}), \delta Cl(\mathcal{A}))$.

Remark 2.3: The collection of all $b-$ open (resp. ß $-$ open, $\alpha-$ open, semi $-$ open, pre $-$ open, $\delta-$ pre $-$ open and regular open)subsets of $\mathcal{X}$ containing a point $x\in\mathcal{X}$is denoted by $B\Sigma(\mathcal{X},x)$(resp. ß$\Sigma(\mathcal{X},x), \alpha\Sigma(\mathcal{X},x), S\Sigma(\mathcal{X},x), P\Sigma(\mathcal{X},x), \delta P\Sigma(\mathcal{X},x)$and $R\Sigma(\mathcal{X},x)$), the family of all b $-$ open (resp. ß $-$ open, $\alpha-$ open, semi $-$ open, pre $-$ open, $\delta-$ pre $-$ open and regular open) sets in $\mathcal{X}$ are denoted by $B\Sigma(\mathcal{X},\mathcal{T})$ $(resp.$ ß$\Sigma(\mathcal{X},\mathcal{T}), \alpha\Sigma(\mathcal{X},\mathcal{T}), S\Sigma(\mathcal{X},\mathcal{T}), P\Sigma(\mathcal{X},\mathcal{T}), \delta P\Sigma(\mathcal{X},\mathcal{T})$and $R\Sigma(\mathcal{X},\mathcal{T}))$.

Definition 2.4: Let $(\mathcal{X},\mathcal{T})$ be a Topologicalspace. Thensubset $\mathcal{A}$ of a space $\mathcal{X}$ is called $E-$ open [21]if$\mathcal{A}\subseteq \mathrm{Cl}(\delta-Int(\mathcal{A}))\cup Int(\delta-Cl(\mathcal{A}))$.
The complement of an $E-$ open set is called $E-$ closed. The intersection of all $E-$ closed sets containing $\mathcal{A}$ is called the $E-$ closure of $\mathcal{A}$ [21]and is denoted by $E-Cl(\mathcal{A})$. The union of all $E-$ open sets of $\mathcal{X}$ contained in $\mathcal{A}$ is called the $E-$ interior [21] of $\mathcal{A}$ and is denoted by $E-\mathrm{Int}(\mathcal{A})$.

Remark 2.5: The family of all $E-$ open(resp. $E-$ closed)subsets of $\mathcal{X}$containing a point$x\in\mathcal{X}$ is denoted by $E\Sigma(\mathcal{X},\ x)$ $(resp. EC(\mathcal{X},\ x))$. The family of all $E-$ open $(resp. E-$ closed) sets in $\mathcal{X}$ are denoted by $E\Sigma(\mathcal{X},\mathcal{T})(resp. EC(\mathcal{X},\mathcal{T}))$.

Definition 2.6: Let $(\mathcal{X},\mathcal{T})$be a Topological space. Then, A subset $\mathcal{A}$ of $\mathcal{X}$ is said to be $\boldsymbol{\theta}-$ **open**[32]if for each $x\in\mathcal{A}$ $\exists$ an **open** set $\mathcal{G}$ such that, $x\in\mathcal{G}\subseteq \mathrm{Cl}(\mathcal{G})\subseteq\mathcal{A}$. (i. e)

A point $x \in \mathcal{X}$ is called a $\theta-$cluster point of $\mathcal{A}$ if $\mathrm{Cl}(\mathcal{V}) \cap \mathcal{A} \neq \emptyset$ for every open subset $\mathcal{V}$ of $\mathcal{X}$ containing x. The set of all $\theta-$cluster points of $\mathcal{A}$ is called the $\theta-$closure of $\mathcal{A}$ and is denoted by $Cl_\theta(\mathcal{A})$. If $\mathcal{A} = Cl_\theta(\mathcal{A})$, then $\mathcal{A}$ is said to be $\theta-$closed[**32**]. The complement of a $\theta-$closed set is said to be $\boldsymbol{\theta}-$**open**. The family of all $\boldsymbol{\theta}-$**open** sets in $\mathcal{X}$ is denoted by $\theta\Sigma(\mathcal{X},\mathcal{T})$.

Remark 2.7: The collection of $\theta-$open sets in a Topological space $\mathcal{X}$ forms a Topology $\mathcal{T}_\theta$ which is coarser than $\mathcal{T}$. as well, the family of $\delta-$open sets in a Topological space $\mathcal{X}$ forms a Topology $\mathcal{T}_\delta$ such that $\mathcal{T}_\delta \subseteq \mathcal{T}$.

Proposition 2.8: [**21, 23**] the following properties hold for a space $\mathcal{X}$:

a) The Arbitrary union of any family of $E-$open sets in $\mathcal{X}$, is an $E-$open set.

b) The Arbitrary intersection of any family of $E-$closed sets in $\mathcal{X}$, is an $E-$closed.

Remark 2.9: We have the following figure in which the converses of implications need not be true, see the examples in [**21**], [**22**] and [**23**].

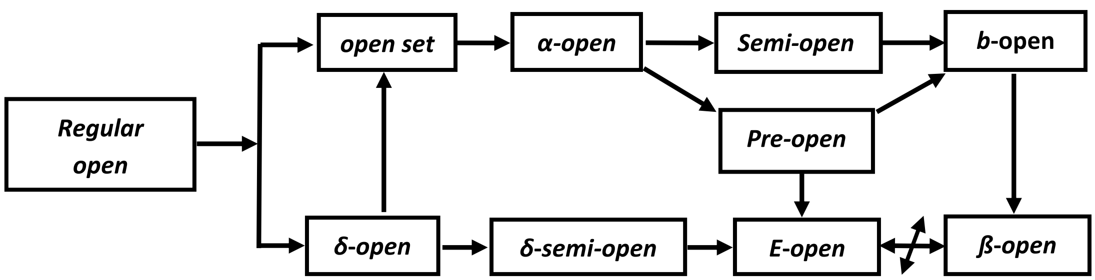

Figure (1): The relationships among some well-known generalized open sets in Topological spaces

Definition 2.10: [**42**] A subset $\mathcal{U}$ of a topological space $(\mathcal{X},\mathcal{T})$ is said to be $E-$neighborhood of a point $x \in \mathcal{X}$ if there exists an $E-$open set $\mathcal{A}$ of $\mathcal{X}$ such that $x \in \mathcal{A} \subseteq \mathcal{U}$.

3. *CHARACTERIZATIONS OF* E − *SEPARATENESS AND* E − *CONNECTED SPACES*

In this section, several characterizations and fundamental properties concerning of E − Separated sets and E − connected spaces by utilizing E − open sets are obtained.

Definition 3.1: Two nonempty subsets $\mathcal{A}$ and $\mathcal{B}$ of a topological space $(\mathcal{X},\mathcal{T})$ are called E − Separated sets iff $\mathcal{A} \cap E-Cl(\mathcal{B}) = \emptyset$ and $E-\mathrm{Cl}(\mathcal{A}) \cap \mathcal{B} = \emptyset$.

Definition 3.2: Two non − empty subsets $\mathcal{A}$ and $\mathcal{B}$ of a space $\mathcal{X}$ are said to be separated (resp. semi − separated[**11**], pre − separated[12], α − separated [**13**] , ß − separated [13]), if $\mathcal{A} \cap \mathrm{Cl}(\mathcal{B}) = \emptyset = \mathrm{Cl}(\mathcal{A}) \cap \mathcal{B}$ (resp. $\mathcal{A} \cap \mathrm{SCl}(\mathcal{B}) = \emptyset = \mathrm{SCl}(\mathcal{A}) \cap \mathcal{B}$, $\mathcal{A} \cap PCl(\mathcal{B}) = \emptyset = P\mathrm{Cl}(\mathcal{A}) \cap \mathcal{B}$, $\mathcal{A} \cap \alpha Cl(\mathcal{B}) = \emptyset = \alpha\mathrm{Cl}(\mathcal{A}) \cap \mathcal{B}$, $\mathcal{A} \cap ßCl(\mathcal{B}) = \emptyset = ß\mathrm{Cl}(\mathcal{A}) \cap \mathcal{B}$).

Remark 3.3: From the above definitions **{(3.1)** and **(3.2)}**, we have the following implications. However, the converse of these are not always true as shown in the examples of **(3.4), (3.5)** and **(3.6)** in [**43**] and example **(3.3)** above.

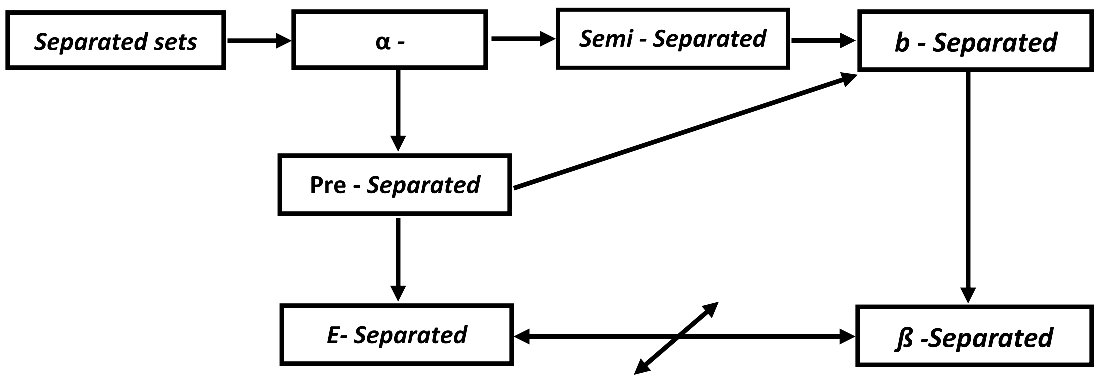

Figure (2): The relationships among some generalized separated sets in topological spaces

Remark 3.4: For each subset $\mathcal{A}$ of a topological space $(\mathcal{X}, \mathcal{T})$, and from the following fact, $E - Cl(\mathcal{A}) \subseteq \mathrm{Cl}(\mathcal{A})$, we obtain that every separated set is E − Separated. However, the converse may not be true as shown in the following example:

Example 3.5: Let $\mathcal{X} = \{x, y, w, z\}$, define a topology $\mathcal{T}$ on $\mathcal{X}$ as follows:
$\mathcal{T} = \{\emptyset, \mathcal{X}, \{x\}, \{y\}, \{x, y\}, \{x, y, w\}\}$ and $E\Sigma(\mathcal{X}, \mathcal{T}) = \{\emptyset, \mathcal{X}, \{x\}, \{y\}, \{x, y\}, \{x, w\}, \{x, z\}, \{y, w\}, \{y, z\}, \{x, y, w\}, \{y, w, z\}, \{x, y, z\}, \{x, w, z\}\}$

Then: The subsets $\mathcal{A} = \{x, z\}$ and $\mathcal{B} = \{y, w\}$ are E − Separated sets, but not separated.

Remark 3.6: Since $\mathcal{A} \cap \mathcal{B} \subseteq \mathcal{A} \cap E - Cl(\mathcal{B}) = \emptyset$. So it is clear that every two E − Separated sets $\mathcal{A}$ and $\mathcal{B}$ of a topological space $(\mathcal{X}, \mathcal{T})$ are disjoint, but the converse may be not true in general as show in the following example:

Example 3.7: Let $\mathcal{X} = \{1, 2, 3, 4\}$, define a topology $\mathcal{T}$ on $\mathcal{X}$ as follows:
$\mathcal{T} = \{\emptyset, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}, \{1, 3, 4\}, \mathcal{X}\}$, we have
$E\Sigma(\mathcal{X}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\mathcal{X}\}$
it is obvious that the subsets $\{1, 3\}$ and $\{2, 4\}$ are disjoint sets, but not E − Separated.

Theorem 3.8:
The following properties are hold for each subsets $\mathcal{A}$ and $\mathcal{B}$ of a space $\mathcal{X}$:

a) If $\mathcal{A}$ and $\mathcal{B}$ are E − Separated sets and $\mathcal{C} \subseteq \mathcal{A}$ and $\mathcal{D} \subseteq \mathcal{B}$, then $\mathcal{C}$ and $\mathcal{D}$ are

$\mathrm{E}-$ Separated.

b) If $\mathcal{A}$ and $\mathcal{B}$ are both $\mathrm{E}-$ closed($\mathrm{E}-$ open) and $\mathcal{A} \cap \mathcal{B} = \emptyset$, then $\mathcal{A}$ and $\mathcal{B}$ are $\mathrm{E}-$ Separated sets.

c) If $\mathcal{A}$ and $\mathcal{B}$ are both $\mathrm{E}-$ closed($\mathrm{E}-$ open) and $\mathcal{M} = \mathcal{A} \cap (\mathcal{X} / \mathcal{B})$ and $\mathcal{K} = \mathcal{B} \cap (\mathcal{X} / \mathcal{A})$, then $\mathcal{M}$ and $\mathcal{K}$ are $\mathrm{E}-$ Separated sets.

Proof: (**a**) – Since $\mathcal{C} \subseteq \mathcal{A}$, then wehave
$E - Cl(\mathcal{C}) \subseteq E - Cl(\mathcal{A})$, and since $\mathcal{A}$ and $\mathcal{B}$ are$\mathrm{E}-$ Separated , then
$\mathcal{B} \cap E - Cl(\mathcal{A}) = \emptyset \implies \mathcal{D} \cap E - Cl(\mathcal{A}) = \emptyset$ and $\mathcal{D} \cap E - Cl(\mathcal{C}) = \emptyset$, and by same method we get, $\mathcal{C} \cap E - Cl(\mathcal{D}) = \emptyset$. Thus, $\mathcal{C}$ and $\mathcal{D}$ are $\mathrm{E}-$ Separated sets.

Proof: (**b**) – Suppose that, $\mathcal{A}$ and $\mathcal{B}$ are both $\mathrm{E}-$ closed sets, thus we have,
$\mathcal{A} = E - Cl(\mathcal{A})$ and $\mathcal{B} = E - Cl(\mathcal{B})$and since $\mathcal{A} \cap \mathcal{B} = \emptyset$, then we obtain,
$E - Cl(\mathcal{A}) \cap \mathcal{B} = \emptyset = E - Cl(\mathcal{B}) \cap \mathcal{A}$. Hence, $\mathcal{A}$ and $\mathcal{B}$ are $\mathrm{E}-$ Separated sets.
Now, if $\mathcal{A}$ and $\mathcal{B}$ are both $\mathrm{E}-$ open sets, so their complements $\mathcal{X} / \mathcal{A}$ and $\mathcal{X} / \mathcal{B}$ are
$\mathrm{E}-$ closed sets, and similarly we have, $\mathcal{A}$ and $\mathcal{B}$ are$\mathrm{E}-$ Separated sets.

Proof: (**c**) – If $\mathcal{A}$ and $\mathcal{B}$ are$\mathrm{E}-$ open sets, so $\mathcal{X} / \mathcal{A}$ and $\mathcal{X} / \mathcal{B}$ are
$\mathrm{E}-$ closed sets, Since $\mathcal{M} \subseteq (\mathcal{X} / \mathcal{B}) \implies$
$E - Cl(\mathcal{M}) \subseteq E - Cl(\mathcal{X} / \mathcal{B}) = \mathcal{X} / \mathcal{B}$
therefore, $E - Cl(\mathcal{M}) \cap \mathcal{B} = \emptyset$.
and Hence, $E - Cl(\mathcal{M}) \cap \mathcal{K} = \emptyset$, … … … … … … (1)
and by same method we get, $\mathcal{K} \subseteq (\mathcal{X} / \mathcal{A}) \implies$
$E - Cl(\mathcal{K}) \subseteq E - Cl(\mathcal{X} / \mathcal{A}) = \mathcal{X}/\mathcal{A}$, therefore, $E - Cl(\mathcal{K}) \cap \mathcal{A} = \emptyset$.
and Hence, $E - Cl(\mathcal{K}) \cap \mathcal{M} = \emptyset$, … … … … … … (2)
Thus, from (1) and (2), we have $\mathcal{M}$ and $\mathcal{K}$ are $\mathrm{E}-$ Separated sets.

Theorem 3.9: A subsets $\mathcal{A}$ and $\mathcal{B}$ ofa topological space $(\mathcal{X}, \mathcal{T})$ are $\mathrm{E}-$ Separated sets
iff there exist two $\mathrm{E}-$ open sets $\mathcal{U}$ and $\mathcal{V}$ of $\mathcal{X}$ such that $\mathcal{A} \subseteq \mathcal{U}$ & $\mathcal{B} \subseteq \mathcal{V}$ and $\mathcal{A} \cap \mathcal{V} = \emptyset = \mathcal{B} \cap \mathcal{U}$.

Proof: Assume that, $\mathcal{A}$ and $\mathcal{B}$ are $\mathrm{E}-$ Separated sets.
Put $\mathcal{U} = \mathcal{X} / E - Cl(\mathcal{B})$and $\mathcal{V} = \mathcal{X}/E - Cl(\mathcal{A})$. Therefore,
$\mathcal{U}$ and $\mathcal{V} \in E\Sigma(\mathcal{X}, \mathcal{T})$, such that $\mathcal{A} \subseteq \mathcal{U}$ & $\mathcal{B} \subseteq \mathcal{V}$ and $\mathcal{A} \cap \mathcal{V} = \emptyset = \mathcal{B} \cap \mathcal{U}$.
(**Conversely**): Suppose that, $\mathcal{U}$ and $\mathcal{V} \in E\Sigma(\mathcal{X},\mathcal{T})$ such that
$\mathcal{A} \subseteq \mathcal{U}$ and $\mathcal{B} \subseteq \mathcal{V}$ and $\mathcal{A} \cap \mathcal{V} = \emptyset = \mathcal{B} \cap \mathcal{U}$.
Now, since $\mathcal{X}/\mathcal{U}$ and $\mathcal{X}/\mathcal{V}$ are $\mathrm{E}-$ open sets, so $E - Cl(\mathcal{B}) \subseteq \mathcal{X}/\mathcal{U} \subseteq \mathcal{X}/\mathcal{A}$ and
$E - Cl(\mathcal{A}) \subseteq \mathcal{X}/\mathcal{V} \subseteq \mathcal{X}/\mathcal{B}$. Therefore,
$E - Cl(\mathcal{B}) \cap \mathcal{A} = \emptyset = E - Cl(\mathcal{A}) \cap \mathcal{B}$. So, $\mathcal{A}$ and $\mathcal{B}$ o are $\mathrm{E}-$ Separated sets.

Definition 3.10: A point $x \in \mathcal{X}$ is called an$E -$ Limit point of a set $\mathcal{A} \subseteq \mathcal{X}$ if every $\mathcal{U} \in E\Sigma(\mathcal{X}, x)$ contains a point of $\mathcal{A}$ different than$x$.

Theorem 3.11: If$\mathcal{A}$ and $\mathcal{B}$ are non $-$ empty disjoint subsets of a space $\mathcal{X}$ and $\mathcal{G} = \mathcal{A} \cup \mathcal{B}$. Then$\mathcal{A}$ and $\mathcal{B}$ are$\mathrm{E}-$ Separated iff every of $\mathcal{A}$ and $\mathcal{B}$ are $\mathrm{E}-$ closed ($\mathrm{E}-$ open)of $\mathcal{G}$.

Proof: Assume that, $\mathcal{A}$ and $\mathcal{B}$ are E $-$ Separated sets. Via definition of separated sets

(**3**. **1**), we have $\mathcal{A}$ contains no $E-$ Limit points of $\mathcal{B}$.
Then, $\mathcal{B}$ contains all $E-$ Limit points of $\mathcal{B}$ which are in $\mathcal{A}\cup\mathcal{B}$ and $\mathcal{B}$ is E $-$ closed in $\mathcal{A}\cup\mathcal{B}$. Therefore $\mathcal{B}$ is E $-$ closed in $\mathcal{G}$. Similarly $\mathcal{A}$ is E $-$ closed in $\mathcal{G}$.
(Conversely) of this theorem is clear it's consequently from part **(b)** of **Theorem(3.8).**

Definition 3.12: A subset $\mathcal{R}$ of a topological space $(\mathcal{X},\mathcal{T})$ is called $E-$ connected relativeto $\mathcal{X}$ if $\nexists$ two E $-$ Separated subsets $\mathcal{A}$ and $\mathcal{B}$ relative to $\mathcal{X}$ and $\mathcal{R}=\mathcal{A}\cup\mathcal{B}$. Otherwise, $\mathcal{R}$ is called $E-$ disconnected.

Definition 3.13: A subset $\mathcal{S}$ of a space $\mathcal{X}$ is called connected (*resp.* semi $-$ connected [**11**], pre $-$ connected[12], α $-$ connected [13], ß $-$ connected[13]) if $\nexists$ two separated subsets $\mathcal{A}$ and $\mathcal{B}$ (*resp.* semi $-$ separated, pre $-$ separated, α $-$ separated, ß $-$ separated) such that, $\mathcal{S}=\mathcal{A}\cup\mathcal{B}$.

Definition 3.14: A topologicalspace $(\mathcal{X},\ \mathcal{T})$ is said to be:

a) $E-$ Connected [**44**] if $\mathcal{X}$ cannot be written as the unionof two disjoint nonempty $E-$ open sets.

b) $Pre-$ connected [**12**] (*resp.* semi $-$ connected [**11**]) if $\mathcal{X}$can not be expressed as the union of two nonempty disjoint pre $-$ open (*resp.* semi $-$ open) sets of $\mathcal{X}$.

c) $b-$ connected [**45**](resp. ß $-$ connected [**15**]) if $\mathcal{X}$ cannot be expressed as the union of two disjoint nonempty $b-$ open (resp. ß $-$ open) sets of $\mathcal{X}$.

Remark 3.15: "From the above definitions, we have the following implications. However, converse of these are not always true as shown via the **examples (2.1 and 2.2)** in [**13**], (**6 and 7)** in [**45**], **(3.6)** in [**24**] and the implications in [**43**], and the below examples. In other words, every disconnected space is $E-$ disconnected .

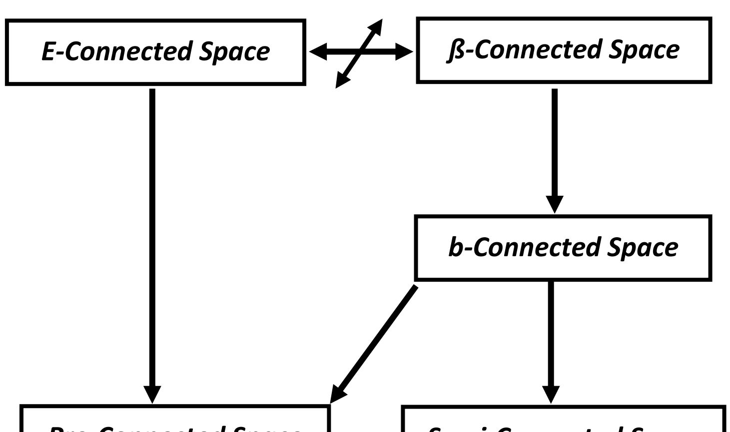

Figure (2): The relationships among of $\mathrm{E}-$ connected spaces and different other weaker and stronger types of generalized connected spaces

Example 3.16: **1** $-$ Let $\mathcal{X} = \{a, b, c\}$ with a topology $\mathcal{T} = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \mathcal{X}\}$.
Then, $(\mathcal{X}, \mathcal{T})$ is Pre $-$ connected space, but it is neither $E -$ connected nor Semi $-$ connected. Furthermore, $(\mathcal{X}, \mathcal{T})$ is neither $b -$ connected nor ß $-$ connected.

2 $-$ Let $\mathcal{X} = \{a, b, c\}$ with a topology $\mathcal{T} = \{\emptyset, \{b, c\}, \mathcal{X}\}$.
Then, $(\mathcal{X}, \mathcal{T})$ is Semi $-$ connected space, but it is neither $E -$ connected nor Pre $-$ connected. Furthermore, $(\mathcal{X}, \mathcal{T})$ is neither $b -$ connected nor ß $-$ connected.

3 $-$ Let $\mathcal{X} = \{a, b, c, d\}$ with a topology $\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \mathcal{X}\}$.
Then, $(\mathcal{X}, \mathcal{T})$ is $b -$ connected and ß $-$ connected, but it is not $E -$ connected space.

Theorem 3.17: Let $\mathcal{R}$ *be a* $E -$ connected subset of a space $\mathcal{X}$. *If* $\mathcal{S}$ *is a* subset of $\mathcal{X}$ such that $\mathcal{R} \subseteq \mathcal{S} \subseteq E - Cl(\mathcal{R})$, then $\mathcal{S}$ is $E -$ connected.

Proof: If $\mathcal{S}$ is not $E -$ connected $\{\mathcal{S}$ is $E -$ disconnected$\}$, then there exist two $\mathrm{E} -$ Separated subsets $\mathcal{A}$ and $\mathcal{B}$ relative to $\mathcal{X}$ and $\mathcal{S} = \mathcal{A} \cup \mathcal{B}$.
Since $\mathcal{R}$ is $E -$ connected, then either $\mathcal{R} \subseteq \mathcal{A}$ *or* $\mathcal{R} \subseteq \mathcal{B}$.
Without loss of generality, let $\mathcal{R} \subseteq \mathcal{A}$.
$As, \mathcal{R} \subseteq \mathcal{A} \subseteq \mathcal{S}$, now we take closure of $\mathcal{R}$ and $\mathcal{A}$ in $\mathcal{S}$,
$E - Cl_{\mathcal{S}}(\mathcal{R}) \subseteq E - Cl_{\mathcal{S}}(\mathcal{A}) \subseteq E - Cl(\mathcal{A})$. Furthermore,
$E - Cl_{\mathcal{S}}(\mathcal{R})\big(resp. \delta - ß - Cl_{\mathcal{S}}(\mathcal{R})\big) = \mathcal{S} \cap E - Cl(\mathcal{R}) = \mathcal{S} \supseteq E - Cl(\mathcal{A})$.
This implise that , $\mathcal{S} = E - Cl(\mathcal{A})$. Consequently $\mathcal{A}$ and $\mathcal{B}$ are not $\mathrm{E} -$ Separated and $\mathcal{S}$ *is* $E -$ connected.

"The concept of locally connectedness is often mentioned when talking about connectedness. A locally connected space is defined in term of neighborhood".

Definition 3.18: A topologicalspace $\mathcal{X}$ is said to be locally $E -$ connected at a point

$q \in \mathcal{X}$ iff every $E-$neighborhood of q contains $E-$connected $E-$neighborhood of q.

A space $\mathcal{X}$ is said to be locally $E-$connected if it is locally $E-$connected at eachof its points.

Lemma 3.19: [41] Let $(\mathcal{X}, \mathcal{T})$ a topological space and $\mathcal{A}, \mathcal{B} \subseteq \mathcal{X}$. if $\mathcal{A} \in \delta\Sigma(\mathcal{X})$ and $\mathcal{B} \in E\Sigma(\mathcal{X})$then, $\mathcal{A} \cap \mathcal{B} \in E\Sigma(\mathcal{X})$.

"The following two results are very useful when a locally connected space is involved".

Theorem 3.20: Every$\delta-$open subspace $\mathcal{B}$ of a locally$E-$connected spaces is locally $E-$connected in $\mathcal{B}$.

Proof: "This is a direct consequence of the respective **definitions (3. 18** and **2.10)** and **Lemma (3.19)**"

Theorem 3.21: The following properties are equivalent for any topological space $\mathcal{X}$:

- **i)** $\mathcal{X}$ *is* locally $E-$connected.
- **ii)** The components of each $\delta-$open subspace $\mathcal{U}$ of $\mathcal{X}$ are E $-$ open in $\mathcal{U}$.
- **iii)** The $E-$connected open sets of $\mathcal{X}$ form basis of the topology of $\mathcal{X}$.

Proof: (**i**) $\Longrightarrow$ (**ii**)

Assume that $\mathcal{X}$ is locally $E-$connected space and let $\mathcal{U}$ be an $\delta-$open subspace of $\mathcal{X}$.

Via **Theorem**(**3.20**), we have $\mathcal{U}$ is a locally $E-$connected space in $\mathcal{U}$. Moreover, the components of $\mathcal{U}$ are $\delta-$open sets of $\mathcal{U}$. Since $\mathcal{U}$ is $\delta-$open subspace, so via **Lemma** (**3.19**) these components are also E $-$ open in $\mathcal{U}$.

(**ii**) $\Longrightarrow$ (**iii**) Suppose that the components of each$\delta-$open subspace $\mathcal{U}$ of $\mathcal{X}$ are E $-$ open sets in $\mathcal{U}$ and let $\mathcal{U}$ be any $\delta-$open set of $\mathcal{X}$.

Since the components of $\mathcal{U}$ are E $-$ connected, then $\mathcal{U}$ is the union of a family *of* E $-$ connected open sets of $\mathcal{X}$. This complete proof of part (**iii**).

(**iii**) $\Longrightarrow$ (**i**) suppose that the part (**iii**) holds, and let $\mathcal{U}$ be any open $E-$neighborhood of an arbitrary point $q \in \mathcal{X}$. Via part (**iii**) $\mathcal{U}$ is the union of a family *of*E $-$ connected open sets. Thus there exists an E $-$ connected open set $\mathcal{V}$ such that, $q \in \mathcal{V} \subseteq \mathcal{U}$. Hence $\mathcal{X}$ *is* locally $E-$connected at a point q. Since q arbitrary , thus $\mathcal{X}$ is locally $E-$connected.

Theorem 3.22: If a subset$\mathcal{A}$ of a space$(\mathcal{X}, \mathcal{T})$ is $E-$connected, then $E-Cl(\mathcal{A})$ is $E-$connected.

Proof: Assume that$E-Cl(\mathcal{A})$ is$E-$disconnected.

Then, there are two non $-$ empty E $-$ Separated subsets $\mathcal{B}$ and $\mathcal{C}$ in $\mathcal{X}$ such that

$E - Cl(\mathcal{A}) = \mathcal{B} \cup \mathcal{C}$. *Since* $\mathcal{A} = (\mathcal{B} \cap \mathcal{A}) \cup (\mathcal{C} \cap \mathcal{A})$ and
$E - Cl(\mathcal{B} \cap \mathcal{A}) \subseteq E - Cl(\mathcal{B})$ and $E - Cl(\mathcal{C} \cap \mathcal{A}) \subseteq E - Cl(\mathcal{C})$ and $\mathcal{B} \cap \mathcal{C} = \emptyset$,
then $\{E - Cl(\mathcal{B} \cap \mathcal{A})\} \cap \mathcal{C} = \emptyset$. Hence, $\{E - Cl(\mathcal{B} \cap \mathcal{A})\} \cap (\mathcal{C} \cap \mathcal{A}) = \emptyset$.
By same method $\{E - Cl(\mathcal{C} \cap \mathcal{A})\} \cap (\mathcal{B} \cap \mathcal{A}) = \emptyset$. Therefore,
$\mathcal{A}$ is $E -$ disconnected, this contradiction, since by hypothesis $\mathcal{A}$ is E $-$ connected.

Theorem 3.23: Let $\mathcal{A}, \mathcal{B}$ *and* $\mathcal{C}$ are subsets *of a* space $\mathcal{X}$ and $\mathcal{A} \subseteq \mathcal{B} \cup \mathcal{C}$ such that $\mathcal{A}$ be a non $-$ empty $E -$ connected and $\mathcal{B}, \mathcal{C}$ *are* E $-$ Separated. Then, only one of the following statements holds:

i) $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{A} \cap \mathcal{C} = \emptyset$.

ii) $\mathcal{A} \subseteq \mathcal{C}$ and $\mathcal{A} \cap \mathcal{B} = \emptyset$

Proof: Let $\mathcal{A} \subseteq \mathcal{B} \cup \mathcal{C}$, since $\mathcal{A} \cap \mathcal{C} = \emptyset$, then $\mathcal{A} \subseteq \mathcal{B}$. Also, if $\mathcal{A} \cap \mathcal{B} = \emptyset$, then $\mathcal{A} \subseteq \mathcal{C}$.
Since $\mathcal{A} \subseteq \mathcal{B} \cap \mathcal{C}$, then both $\mathcal{A} \cap \mathcal{B} = \emptyset$ and $\mathcal{A} \cap \mathcal{C} = \emptyset$, cannot hold concurrently.
Similarly, Assume that $\mathcal{A} \cap \mathcal{B} \neq \emptyset$ and $\mathcal{A} \cap \mathcal{C} \neq \emptyset$, then via **Theorem(3.8 − part − a)**,
we get $\mathcal{A} \cap \mathcal{B}$ and $\mathcal{A} \cap \mathcal{C}$ are E $-$ Separated, such that $\mathcal{A} = (\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{A} \cap \mathcal{C})$ which contradicts with the fact of $E -$ connectedness of $\mathcal{A}$. Thus, one of the statements (**i**) and (**ii**) should be hold.

4. E − CONNECTEDNESS AND MAPPINGS

In this part, the behavior of E $-$ connected spaces with respect to several forms of well $-$ known generalized mappings is discussed.

Definition 4.1: A mapping $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is said to be a:

a) $E - open$ [**46**], *if* $f(\mathcal{U}) \in E\Sigma(\mathcal{Y}, \mathcal{T}^*)$ *for every open set* $\mathcal{U}$ *in* $(\mathcal{X}, \mathcal{T})$.

b) $E -$ continuous [**21**], if $f^{-1}(\mathcal{U})$ is $E -$ open in $\mathcal{X}$ for every open subset $\mathcal{U}$ of $\mathcal{Y}$.

Lemma 4.2: Let $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ *be an* $E -$ continuous mapping. Then, $E - Cl(f^{-1}(\mathcal{A})) \subseteq f^{-1}(Cl(\mathcal{A}))$, for every $\mathcal{A} \subseteq \mathcal{Y}$.

Theorem 4.3: Let $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ *be an* $E -$ continuous mapping, if $\mathcal{N}$ is $E -$ connected in $\mathcal{X}$, then $f(\mathcal{N})$ is connected in $\mathcal{Y}$.

Proof: Assume that $f(\mathcal{N})$ is disconnected in $\mathcal{Y}$. There exist two separated sets $\mathcal{R}$ and $\mathcal{S}$ of $\mathcal{Y}$ such that $f(\mathcal{N}) = \mathcal{R} \cup \mathcal{S}$. *Put* $\mathcal{A} = \mathcal{N} \cap f^{-1}(\mathcal{R})$ and $\mathcal{B} = \mathcal{N} \cap f^{-1}(\mathcal{S})$. Since $f(\mathcal{N}) \cap \mathcal{R} \neq \emptyset$, then $\mathcal{N} \cap f^{-1}(\mathcal{R}) \neq \emptyset$, and therefore $\mathcal{A} \neq \emptyset$. By same method $\mathcal{B} \neq \emptyset$.
Since $\mathcal{R} \cap \mathcal{S} = \emptyset$, then $\mathcal{A} \cap \mathcal{B} = \mathcal{N} \cap f^{-1}(\mathcal{R} \cap \mathcal{S}) = \emptyset$, and so $\mathcal{A} \cap \mathcal{B} = \emptyset$.
Since f $E -$ continuous, then via **Lemma(4.2)**
$E - Cl(f^{-1}(\mathcal{S})) \subseteq f^{-1}(Cl(\mathcal{S}))$, and $\mathcal{B} \subseteq f^{-1}(\mathcal{S})$, then
$E - Cl(\mathcal{B}) \subseteq f^{-1}(Cl(\mathcal{S}))$. *Since* $\mathcal{R} \cap Cl(\mathcal{S}) = \emptyset$, then
$\mathcal{A} \cap f^{-1}(Cl(\mathcal{S})) \subseteq f^{-1}(\mathcal{R}) \cap f^{-1}(Cl(\mathcal{S})) = \emptyset$, and so
$\mathcal{A} \cap E - Cl(\mathcal{B}) = \emptyset$. Hence, $\mathcal{A}$ *and* $\mathcal{B}$ are $E -$ separated sets.

Corollary 4.4: Let $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ *be an* $E-$ continuous mapping, if $\mathcal{N}$ is disconnected in $\mathcal{X}$, *then* $f(\mathcal{N})$ is $E-$ disconnected in $\mathcal{Y}$.

Proof: The proof is clear.

Theorem 4.5: Let $f: (\mathcal{X},, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ *be a bijective and* $E-$ closed mappping, if $\mathcal{N}$ is $E-$ connected in $\mathcal{Y}$, *then* $f^{-1}(\mathcal{N})$ is connected in $\mathcal{X}$.

Proof: This **proof** is similar to that of **Theorem** (**4.3**) hence omitted.

Definition 4.6: A mapping $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is said to be:

a) $E-$ irresolute [**44**] if $f^{-1}(\mathcal{V})$ is $E-$ open in $\mathcal{X}$ for each $E-$ open set $\mathcal{V}$ of $\mathcal{Y}$.

b) *Strongly* $E-$ irresolute if $f^{-1}(\mathcal{V})$ is $E-$ open in $\mathcal{X}$ for each open set $\mathcal{V}$ of $\mathcal{Y}$.

Lemma 4.7: A mapping $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is an $E-$ irresolute if and only if $E-Cl(f^{-1}(\mathcal{A})) \subseteq f^{-1}(E-Cl(\mathcal{A})) \subseteq f^{-1}(Cl(\mathcal{A}))$. for every $\mathcal{A} \subseteq \mathcal{Y}$.

Proof: Follows from the **Definition** (**4.6**).

Theorem 4.8: Let $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ *be an* $E-$ irresolute mapping, if $\mathcal{N}$ is $E-$ connected in $\mathcal{X}$, *then* $f(\mathcal{N})$ is $E-$ connected in $\mathcal{Y}$.

Proof: By utilizing **Definition** (**4.6**) and **Lemma** (**4.7**), the proof is immediate consequence of **Theorem**(**4.3**).

5. FUNDAMENTAL PRPERTIES OF STRONGLY *E* -CONNECTEDNESS IN COMPCT SPACES

In this section, we present new types of connected spaces which are called strongly E-connected spaces and introduce some fundamental characterizations concerning of such types of connected spaces with some E-Separation axioms and compact spaces.

Definition 5.1: A topological space$(\mathcal{X}, \mathcal{T})$ is called strongly E $-$ connected if and only if it is not a disjoint union of countably many but more than one E $-$ closed sets (i.e.) if $\mathcal{L}_i$ are non $-$ empty disjoint closed sets of $\mathcal{X}$, then $\mathcal{X} \neq \mathcal{L}_1 \cup \mathcal{L}_2 \cup \dots \dots$. Otherwise $\mathcal{X}$ is said to be strongly E $-$ disconnected.

Remark 5.2: We cannote the similarity between definition of strongly E $-$ connected (**Definition** $-$ **5.1**) and that of E $-$ connectedness. If $\mathcal{X}$ is E $-$ connected, and $\mathcal{L}_1$ and $\mathcal{L}_2$ are any two non $-$ empty disjoint closed sets of $\mathcal{X}$, then $\mathcal{X} \neq \mathcal{L}_1 \cup \mathcal{L}_2$.

Lemma 5.3: For each serjective $E-$ irresolute mapping $f: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$. The image of $f(\mathcal{X})$ is strongly E $-$ connected if $\mathcal{X}$ is strongly E $-$ connected.

Proof: Assume that, $f(\mathcal{X})$ is strongly E $-$ disconnected, so via **Definition**(**5.1**) it is a disjoint union of countably many but more than one E $-$ closed sets. Since f is $E-$ irresolute, then the inverse image of $E-$ closed sets is still E $-$ closed, also $\mathcal{X}$ is a disjoint union of $E-$ closed sets. Thus, $f(\mathcal{X})$ is strongly E − connected.

Theorem 5.4: A topologicalspace $(\mathcal{X},\mathcal{T})$ is strongly E − connected if there exisits a constant serjective $E-$ irresolute mapping $f:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{X},\mathcal{T}_D^*)$, where $\mathcal{T}_D^*$ denote to a discrete space of $\mathcal{X}$.

Proof: Assume that, $\mathcal{X}$ is stronglyE − connected and $f:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{X},\mathcal{T}_D^*)$ is a serjective $E-$ irresolute mapping, so via **Lemma(5.3)**, we get $f(\mathcal{X})$ is strongly E − connected. Now, the only strongly E − connected subset of $\mathcal{T}_D^*$ are the one − point space. Thus, f is constant.

Conversely, Assume that $\mathcal{X}$ is a disjoint union of countably many but more than one $E-$ closed sets, $\mathcal{X} = \cup_i \mathcal{L}_i$. Then, define $f:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{X},\mathcal{T}_D^*)$ by taking $f(x) = \mathrm{i}$, whenever $x \in \mathcal{L}_i$. This f is a serjective $E-$ irresolute and not constant. Hence, $\mathcal{X}$ is strongly E − connected.

Remark 5.5: Strongly E − connectedness is a stronger concept of E − connectedness. In other words, given a E − connected space, we can make it strongly E − connected via adding some conditions. But what conditions should be added is the difficulty. Our starting point is E − connected spaces, so a E − continuum may be beneficial. The notion of a E − continuum is defined on a E − connected set as follows:

Definition 5.6: A compact E − connected set in a topological space $(\mathcal{X},\mathcal{T})$ is called a E − continuum.

Definition 5.7: Let $\mathcal{A}$ be subset of a topological space $(\mathcal{X},\mathcal{T})$. The $E-$ boundary of $\mathcal{A}$ defined by $E-bd(\mathcal{A}) = E-Cl(\mathcal{A}) \cap E-Cl(\mathcal{X}-\mathcal{A})$.

Definition 5.8: [44] A space$(\mathcal{X},\ \mathcal{T})$ is said to be:

a) $E-\mathrm{T}_1$, if for each pair of distinct points x and y of $\mathcal{X}$, there exist $E-$ open
1. sets $\mathcal{A}$ and $\mathcal{B}$ containing x and y, respectively, such that, $x \notin \mathcal{B}$ and $y \notin \mathcal{A}$.

b) $E-\mathrm{T}_2$, if foreach pair of distinct points x and y of $\mathcal{X}$there exist disjoint $E-$ open sets $\mathcal{A}$ and $\mathcal{B}$ in $\mathcal{X}$such that $x \in \mathcal{A}$ *and* $y \in \mathcal{B}$.

c) E − Normal, if for each pair of disjoint E − closed sets $\mathcal{F}_1$ and $\mathcal{F}_2$there exist
2. two disjoint E − open sets $\mathcal{U}$ *and* $\mathcal{V}$ *such that*, $\mathcal{F}_1 \subseteq \mathcal{U}$, and $\mathcal{F}_2 \subseteq \mathcal{V}$.

Lemma 5.9: Let $\mathcal{A}$ be anyE − continuum in an $E-\mathrm{T}_2$ space $\mathcal{X}$ and $\mathcal{B}$ is any E − open set such that $\mathcal{A} \cap \mathcal{B} \neq \emptyset \neq \mathcal{A} \cap (\mathcal{X}-\mathcal{B})$, then every component of $\{\mathcal{A} \cap E-Cl(\mathcal{B})\} \cap E-bd(\mathcal{B}) \neq \emptyset$.

Proof: "The proof is clear by utilizing **Definitions (5.6, 5.7 and 5.8)**".

Theorem 5.10: If $(\mathcal{X},\ \mathcal{T})$ is a compact $E-\mathcal{T}_2-$ space. Then $\mathcal{X}$ is E − connected if and only if $\mathcal{X}$is strongly E − connected space.

Proof: $\Rightarrow$ Suppose that, $\mathcal{X}$ is strongly $\mathrm{E}-$ connected space, then it is obvious that $\mathcal{X}$ is $\mathrm{E}-$ connected space.
$\Leftarrow$ Now, assume that $\mathcal{X}$ is a compact $E-\mathcal{T}_2$ and $\mathrm{E}-$ connected space **and** it is strongly $\mathrm{E}-$ **disconnected**, then $\mathcal{X}$ is a union of a countably many but more than one disjoint $E-$ closed sets. $\mathcal{X}=\cup\mathcal{N}_i$, where $\mathcal{N}_i$ are $E-$ disjoint sets.
Since the compact $E-\mathcal{T}_2-$ space is $E-$ Normal space, then, by **Definition (5.8)**, $\mathcal{X}$ is an $E-$ Normal space. Consequently there exist an $E-$ open set $\mathcal{U}$ such that $\mathcal{N}_2\subseteq\mathcal{U}$ and $E-Cl(\mathcal{U})\cap\mathcal{N}_1=\emptyset$. Let $\mathcal{X}_1$ be a component of $E-Cl(\mathcal{U})$ which intersects $\mathcal{N}_2$. Then $\mathcal{X}_1$ is compact and $E-$ connected. Now by utilizing **Lemma (5.9)**, we get $\mathcal{X}_1\cap E-bd(\mathcal{U})\neq\emptyset$ (i. e) $\mathcal{X}_1$ contains a point $q\in E-bd(\mathcal{U})$ such that $q\notin\mathcal{U}$ & $q\notin\mathcal{N}_1$. Thus $\mathcal{X}_1\cap\mathcal{N}_i\neq\emptyset$ for some $i>2$. Suppose that $\mathcal{N}_{\mathrm{n}_2}$ is the first $\mathcal{N}_i$ $for\ i>2$ which intersects $\mathcal{X}_1$, and let $\mathcal{V}$ be an $E-$ open set satisfying $\mathcal{N}_{\mathrm{n}_2}\subseteq\mathcal{V}$, and $E-Cl(\mathcal{V})\cap\mathcal{N}_2=\emptyset$. Then, let $\mathcal{X}_2$ be a component of $\mathcal{X}_1\cap E-Cl(\mathcal{V})$ which contains a point of $\mathcal{N}_{\mathrm{n}_2}$. Again we have $\mathcal{X}_2\cap E-bd(\mathcal{V})\neq\emptyset$, and $\mathcal{X}_2$ contains some point $q\in E-bd(\mathcal{V})$ such that $q\notin\mathcal{V}$ and $q\notin\mathcal{N}_1\cup\mathcal{N}_2$. Hence, $\mathcal{X}_2\cap\mathcal{N}_i\neq\emptyset$ for some $i>\mathrm{n}_2$, and $\mathcal{X}_2\cap\mathcal{N}_i=\emptyset$ for $i<\mathrm{n}_2$. Let $\mathcal{N}_{\mathrm{n}_3}$ be the first $\mathcal{N}_i$ $for\ i>\mathrm{n}_2$, which intersects $\mathcal{X}_2$, so by using methods similar to the above we can find a compact $E-$ connected $\mathcal{X}_3$ such that $\mathcal{X}_3\subseteq\mathcal{X}_2\subseteq\mathcal{X}_1$, and $\mathcal{X}_3$ intersectssome $\mathcal{N}_i$ with $i>\mathrm{n}_3$ but $\mathcal{X}_3\cap\mathcal{N}_i=\emptyset$ for $i<\mathrm{n}_3$. In this method, we gat a sequence of sub continua of $\mathcal{X}$: $\mathcal{X}_1\ \mathcal{X}_2\ \mathcal{X}_3\ \dots\dots$, such that for each j, $\mathcal{X}_j\cap\mathcal{N}_i=\emptyset$ for $i<\mathrm{n}_j$ and $\mathrm{n}_j\longrightarrow\infty$ when $j\longrightarrow\infty$. We know that $\cap_i\mathcal{X}_i\neq\emptyset$. As well, $(\cap_i\mathcal{X}_i)\cap\ \mathcal{N}_j=\emptyset$ for all j, sothat $(\cap_i\mathcal{X}_i)\cap(\cup_i\mathcal{N}_j)=\emptyset$ OR $(\cap_i\mathcal{X}_i)\cap\mathcal{X}=\emptyset$. But $(\cap_i\mathcal{X}_i)\subseteq\mathcal{X}$, which contradicts the truth that $\cap_i\mathcal{X}_i\neq\emptyset$. Therefore $\mathcal{X}$ is strongly $\mathrm{E}-$ connected space.

Theorem 5.11: Let $\mathcal{X}$ be a locally compact $E-\ \mathcal{T}_2-$ space. If $\mathcal{X}$ is locally $\mathrm{E}-$ connected, then $\mathcal{X}$ is locally strongly $\mathrm{E}-$ connected space.

Proof: Suppose that, $\mathcal{D}$ *is* $\mathrm{E}-$ open $\mathrm{E}-$ neighborhood of a point $x\in\mathcal{X}$. So there exists a compact $\mathrm{E}-$ neighborhood $\mathcal{U}$ of x inside $\mathcal{D}$. Assume

that $\mathcal{C}$ is a $\mathrm{E}-$connected component of $\mathcal{U}$ containing x. Since $\mathcal{U}$ *is an* $\mathrm{E}-$neighborhood of x and $\mathcal{X}$ *is* locally $\mathrm{E}-$connected, so $\mathcal{C}$ is an $\mathrm{E}-$neighborhood of x. Since $\mathcal{C}$ is $\mathrm{E}-$closed in $\mathcal{U}$ and $\mathcal{U}$ *is* compact, then $\mathcal{C}$ is compact. So $\mathcal{C}$ is a compact $\mathrm{E}-$connected $\mathrm{E}-$neighborhood of x lying inside $\mathcal{D}$.
Therefore by using **Theorem(5.10)**, we get $\mathcal{C}$ is strongly $\mathrm{E}-$connected.

Theorem 5.12: Let $\mathcal{X}$ be a locally compact $E-\mathcal{T}_2-$space. If $\mathcal{X}$ is locally $\mathrm{E}-$connected and $\mathrm{E}-$connected, then $\mathcal{X}$ is strongly $\mathrm{E}-$connected space.

Proof: The proof is consequence immediately of **Theorems (5.10) and (5.11).**

Theorem 5.13: Let $\mathcal{X}$ be a topological space. Then the following properties equivalent:

a) $\mathcal{X}$ is an $E-\mathcal{T}_1-Space$.

b) For each point $x \in \mathcal{X}$, the singleton set $\{x\}$ is $\mathrm{E}-$closed set.

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$Let X be $E-\mathcal{T}_1-Space$. For each $x, y\ (x \neq y) \in \mathcal{X}$, there exists $\mathrm{E}-$ open set $\mathcal{U}$ such that $y \in \mathcal{U}$ but $x \notin \mathcal{U}$. Consequently, $y \in \mathcal{U} \subseteq \mathcal{X} \setminus \{x\}$.
Thus $\mathcal{X} \setminus \{x\} = \cup\{\mathcal{U}: y \in \mathcal{X} \setminus \{x\}\}$ which is the union of an $\mathrm{E}-$ open sets. Then, $\mathcal{X} \setminus \{x\}$ is an $\mathrm{E}\ (resp.\ \delta-ß)-$ open set. Thus $\{x\}$ *is* $\mathrm{E}-$closed set.
$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$ Suppose that $\{\mathcal{P}\}$ is $\mathrm{E}-$ closed for each $\mathcal{P} \in \mathcal{X}$. So via supposition for each $x, y\ (x \neq y) \in \mathcal{X}, \{x\}, \{y\}$are $\mathrm{E}-$closed sets. Hence, $\mathcal{X} \setminus \{x\}, \mathcal{X} \setminus \{y\}$ are $\mathrm{E}-$open sets such that, $x \in \mathcal{X} \setminus \{y\}, y \notin \mathcal{X} \setminus \{y\}$ and $y \in \mathcal{X} \setminus \{x\}$, $x \notin \mathcal{X} \setminus \{x\}$.
Therefore, $\mathcal{X}$ is an $E-\mathcal{T}_1-$Space.

Corollary 5.14: A strongly $\mathrm{E}-$connected $E-\mathcal{T}_1-$space having more than one point is uncountable space.

Proof: by utilizing **Theorem(5.13)** we have, a singleton set in a $E-\mathcal{T}_1-$Space is $\mathrm{E}-$closed set. Hence, by Definition (5.1) a $E-\mathcal{T}_1-$ Space cannot have countably many but more than one point.

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CONCLUSION
It is well known that the effects of the investigation of properties of closed bounded sets, spaces of continuous function are the possible motivation for the notion of

connectedness. Connectedness is now one of the most important, useful, and fundamental notion of not only general topology but also other advanced branches of mathematics. "Therefore, we study the notion of E -separated sets and with this concept we introduce and investigate a new type of connected spaces which is called, strongly E -connected spaces, and study some essential properties of E -connected spaces. Several characterizations and fundamental properties concerning of such class of connected spaces with some E -Separation axioms and compact spaces are discussed. Moreover the fuzzy topological version of the concepts and results introduced in this paper are very important".

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