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THE BETA$^+$-DECAY IN PROTON HALO NUCLEUS

LA DESINTEGRACIÓN BETA$^+$ EN NÚCLEOS CON HALO DE PROTONES

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The main point of this study is to determine when and where proton emission and β^+-decay happen. The study focused on three factors, the Coulomb effect, a core deformation, and the clustering configurations. In this work it was used the Microscopic Cluster Model to describe the system. The description of any system that will fragment by clustering forms has been considered in order to expand this methodology to all radioactive nuclei. The results confirmed this logical description and support its use for all radioactive isotopes. The $^{1}{}_{7}$Ne has been investigated within this work.

El principal objetivo del estudio ha sido determinar cuándo y dónde ocurren la emisión de protones y la descomposición β^+ en núcleos con halo de protones. El análisis se centra en tres factores: el efecto Coulomb, la deformación del núcleo y las configuraciones de agrupamiento. Este estudio utilizó el modelo de clúster microscópico para caracterizar el sistema. Se ha considerado el uso de formas de agrupamiento en la descripción de cualquier sistema que se fragmentará para expandir esta metodología a todos los núcleos radiactivos. Los resultados confirmaron esta descripción lógica y apoyan que la misma pueda ser utilizada para todos los isótopos radiactivos. En este trabajo se estudia el elemento ^{17}Ne.

PACS: Nucleon distribution and halo features (distribución nucleónica y propiedades de halo), 21.10.Gv, beta-decay (desintegración beta), 23.40.-s; decay by proton emission (desintagración por emisión protónica), 23.50,+z proton emission, nuclear cluster models (modelos nucleares de clúster), 21.60.Gx

I. INTRODUCTION

The progress achieved in the field of nuclear beam radioactivity (within high energies physics) led to a new period in the physics of nuclear structure. The halo property exists in few light proton-rich nuclei like ^{17}Ne. The study of weak binding energies for some exotic nuclei led to the discovery of a halo structure of new light nuclei [1]. This property was detected in the interaction sizes of a cross-section when exotic large radii and proton or neutron r.m.s. radii were noticed in specific light nuclei [2,3].

The proton halo system indicates a proton-rich nucleus which is placed close to the proton-drip line; consequently, this structure is unstable. A valence proton easily can be scattered in the continuum into resonant orbitals of a single-particle. Because of weakly bound energy, the valence protons penetrate the small nuclear centrifugal potential barrier, increasing the radii and forming the halo nuclei.

As a result, the resonant orbitals play an important role in achieving a description of the phenomena of halo in the coupling between continuum and bound states threshold [4,5].

The nuclear beams with low, medium, and high energies are named radioactive nuclear beams; those beams allow the discovery of many halo nuclei. The features of those nuclei are studied by the particle fragmentation method in reactions of high and intermediate energies. These properties are diverse from the nuclear-known structures near or at the β-stability line, denoting halo proton structures. So far, only 3–4 proton-halo nuclei were recognized experimentally, but several atomic nuclei are proposed as possible candidates.

The two-proton halo nucleus Neon-17 is considered as a good candidate for investigating the presence of two valence protons in the halo structure due to the Borromean property and loosely separation energy of ^{17}Ne(S_{2p} = 0,94 MeV). In the literature it is reported that the large cross-section of Neon-17 is related to the interaction cross-section for the A = 17 isobaric, which means that it has a halo structure [6]. A large asymmetry of β-decay was also noticed for this atomic nucleus [7]. Both observations showed the halo structure; however, the observations must be totally understood. Theoretically, β-decay is explained by low admixture s-wave possibility for two valence nucleons of ^{17}Ne, collected with a d-wave possibility [8]: no halo was proposed. Studies suggested no halo shape according to the Coulomb energy [9]. Some studies point to a big probability of the valence protons s-wave [10,11].

The 17Ne nucleus has been investigated by using Jacobi coordinates. The Jacobi coordinates system is good to describe such as halo structure as shown in figure 1. The ^{17}Nehas Borromean property, so this system has been described in T-configuration within Jacobi coordinates. The angle in the figure defines an angle of halo proton motion around the core.

The Borromean property and weak separation energy S_{2p} = 0,94 MeV of the ^{17}Nenucleus, is a significant candidate for investigating a probable protons halo. Several experimental works (the calculation of cross-section) proposed the halo property for A = 17 isobars [6]. The β-decay noticed between Neon-17 (^{17}Ne) and Nitrogen ($^{1}{}_{7}$N) is indicative of an abnormal orbital for the halo protons in ^{17}Ne [7]. The measurements of a cross-section at intermediate energies have suggested that a halo is not present in ^{17}Ne [12]. Some theoretical investigations displayed contradiction on protons occupying dominance of the 1d5/2 orbital, taking Coulomb energy into consideration [9]. The study of the β-decay reached a similar conclusion for a d-wave. The probability of a proton halo structure in (Neon-17) is taken into consideration from Glauber with Hartree–Fock in Ref. [13]. Also, a calculation has suggested the two valence protons halo β^{+}-decay and proton emission for ^{17}Ne, based on the Faddeev model [10, 14].

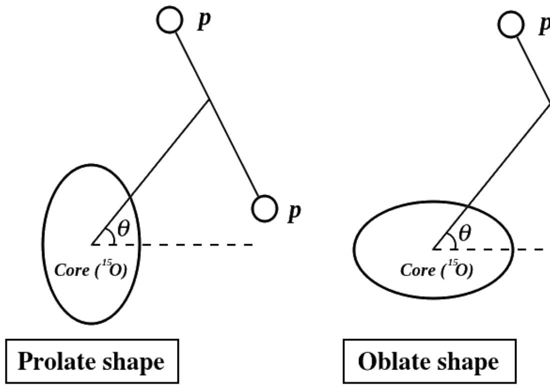

Figure 1. Jacobi coordinates for three-body system with two shapes of the core and angle of two halo protons.

The Hamiltonian of $^{1}{}_{5}$O is,

$$\hat{h}_{core}(\xi_{core})\phi_{core}(\xi_{core}) = \varepsilon_{core}\phi_{core}(\xi_{core}) \tag{1}$$

, and total wavefunction is,

$$\Psi^{JM}(x, y, \xi) = \phi_{core}(\xi_{core})\psi(x, y) \tag{2}$$

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The wavefunction of the valence protons is,

$$\psi^{l_x,l_y}_{n,k}(\rho, \theta) = R_n(\rho)\psi^{l_x,l_y}_{k}(\theta). \tag{3}$$

The total Hamiltonian $\hat{H}$ is,

$$\hat{H} = \hat{T} + \hat{h}_{core}(\vec{\xi}) + \hat{V}_{core-n1}(r_{core-n1}, \vec{\xi}) + \hat{V}_{core-n2}(r_{core-n2}, \vec{\xi}) + \hat{V}_{n-n}(r_{n-n}) + V_c, \tag{4}$$

where

$$\hat{V}_{core-n}(r_{core} - n, \vec{\xi}) = \frac{-V_0}{\left[1 + \exp\left(\frac{r_{core-n} - R(\theta,\phi)}{a}\right)\right]} - \frac{\hbar^2}{m^2c^2} \times (2l.s)\frac{V_{s,o}}{4r_{core-n}}\frac{d}{d_{core-n}}\left(\left[1 + \exp\left(\frac{r_{core-n} - R_{so}}{a_{so}}\right)\right]^{-1}\right), \tag{5}$$

and

$$\hat{V}_{n-n}(r_{n-n}) = -\frac{\hbar^2}{m^2c^2}(2l.s)\frac{V_{s.o}}{4r_{n-n}}\frac{d}{dr_{n-n}}\left(\left[1 + \exp\left(\frac{r_{n-n} - R_{so}}{a_{so}}\right)\right]^{-1}\right), \tag{6}$$

with

$$R = R_0[1+\beta_2 Y_{20}(\theta, \phi)], \qquad Y_{20}(\theta, \phi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1). \tag{7}$$

More details about this formalism are in [15–19].

II. RESULTS

The β^{+}-decay and proton emission are important in nuclear radioactivity and its applications. The knowledge of when and where β^{+}-decay and proton emission take place in nuclei is very significant. The present study has focused on this matter. The ^{17}Neis a radio nucleus and has a half-life of around 109.2(5) ms. The decay probabilities of ^{17}Neare β^{+}, p (96.0 %), β^{+}, α (2.7 %) and β^{+} (1.3 %). The dominant probability is β^{+}, p (96.0 %), which we have considered in the present work. The ^{17}Nehas two protons away from the other nucleons that are the reason for proton emission and the decay. In the ^{17}Ne, the two valence protons surround the $^{1}{}_{5}$O and move around the probably deformed core ($^{1}{}_{5}$O). The Hamiltonian of the ^{17}Nestructure relied on the microscopic core and the valence protons clusterization. The configuration and the clusterization of the system depended on several factors; the (θ) of valence proton position is one of them. The energy of the clustered valence protons is calculated based on Eq. 6, taking into consideration the Coulomb effect.

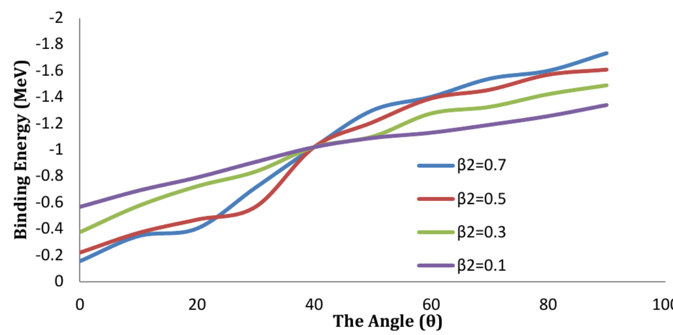

Figure 2. The Binding energy of the valence neutrons as function of the angle with prolate shape.

Eq.3 describes the valence protons wavefunction, whereas ϕ_{core} in eq. 2, defines the wavefunction of the core described by the shell model. Therefore the valence protons wave function depends on the angle as seen in eq. 3.

Table 1. The binding energy and angle of valence electrons, and prolate deformation parameters of $^{1}{}_{5}$O.

θ	Binding Energy (MeV)			
	β_2=0.7	β_2=0.5	β_2=0.3	β_2=0.1
0	-0.1547	-0.22	-0.375	-0.567
10	-0.345	-0.369	-0.574	-0.69
20	-0.4023	-0.47	-0.723	-0.79
30	-0.713	-0.567	-0.8345	-0.908
40	-1.02	-1.02	-1.02	-1.02
50	-1.3	-1.209	-1.1	-1.09
60	-1.402	-1.39	-1.276	-1.13
70	-1.54	-1.456	-1.326	-1.19
80	-1.6	-1.57	-1.422	-1.256
90	-1.734	-1.609	-1.49	-1.34

Movement of the valence protons around the 15O makes the energy of these protons to vary regarding the deformation shape (oblate or prolate) and also the angle (θ). In fig. 2, it is shown how the energy varies from -0.1547 MeV to -1.734 MeV for $\beta_2 = 0{,}7$, from -0.22 MeV to -1.609 MeV for $\beta_2 = 0{,}5$, from -0.375 MeV to -1.422 MeV) for $\beta_2 = 0{,}3$ and from -0.567 MeV to -1.256 MeV for $\beta_2 = 0{,}1$ with angle values from 0° to 90° as they are listed in the table 1.

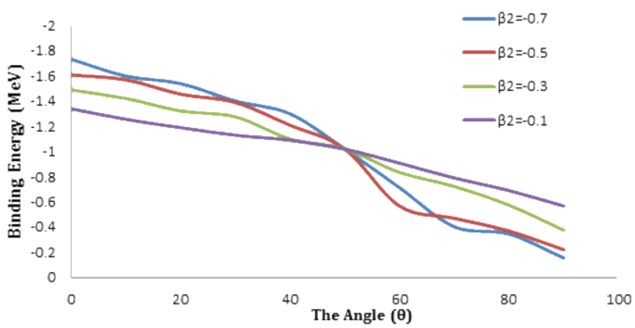

Figure 3. The Binding energy of the valence neutrons as function of the angle with oblate shape.

In fig. 3, it is shown how the energy varies from -1.734 MeV to -0.1547 MeV for $\beta_2 = -0{,}7$, from -1.609 MeV to -0.22 MeV for $\beta_2 = -0{,}5$, from -1.422 MeV to -0.375 MeV for $\beta_2 = -0{,}3$ and from -1.256 MeV to -0.567 MeV for $\beta_2 = -0{,}1$ with angle values from 0^o to 90^o as they are listed in the table 2.

Table 2. The binding energy and angle of valence electrons, and prolate deformation parameters of $^{1}_{5}$O.

θ	Binding Energy (MeV)			
	β₂=-0.7	*β₂=-0.5*	*β₂=-0.3*	*β₂=-0.1*
0	-1.734	-1.609	-1.49	-1.34
10	-1.6	-1.57	-1.422	-1.256
20	-1.54	-1.456	-1.3226	-1.19
30	-1.402	-1.39	-1.276	-1.13
40	-1.3	-1.209	-1.1	-1.09
50	-1.02	-1.02	-1.02	-1.02
60	-0.713	-0.567	-0.8345	-0.908
70	-0.4023	-0.47	-0.723	-0.79
80	-0.345	-0.369	-0.574	-0.69
90	-0.1547	-0.22	-0.375	-0.567

In figure 2, the core has been considered prolate with parameter of deformation β_2 $(0{,}7, 0{,}5, 0{,}3, and 0{,}1)$. According to the shell model structure, the core of $^{1}_{5}$O has five neutrons outside the first shell and closed shell for proton with magic number 8. The majority effect of quadrupole comes from atomic number (charged particles) and minority role from neutron number because the neutrons have a negative moment.

So the quadrupole and deformation of 15O is expected to be very little. The total quadrupole moment of the 17Ne nucleus can write as $Q = Q_j + Q_c$, where the Q_j is the contribution coming from the two protons and Q_c is the quadrupole of the core. Normally $Q_c \gg Q_j$ [20] where,

$$Q = Q' \frac{J}{2J+3}\left[\frac{3\Omega}{J(J+1)-1}\right] \quad \text{and} \quad Q' = \frac{4}{5}\delta Z R^2.$$

The δ is associated with the deformation parameter β_2 ($\beta_2 = 2/3(4\pi/5)^{1/2}\delta$) [20].

From the atomic number ($Z = 8$) and concept of the magic number, we assume the deformation parameter about 0,0 to $-0{,}1$ if it is oblate and about 0,0 to $-0{,}1$ if it is prolate.

The method used in this work depends on the Coulomb effect, the atomic number, the total angular moment and the cluster angle of the two protons with the parent. All probable deformations and clusterization angle channels were investigated. The deformation of the $^{1}_{5}$O is the starting point. The various energies of the protons regarding relative movement were calculated by using a virtual description, as shown in figures. Using a deformed ground state of the parent nucleus makes all configurations probable, which is acceptable theoretically. The acceptable deformation parameters that have been considered as an alternative of experimental data, are modified by the degree of spatial freedom of this investigation: if only experimental data are used, some probable channels cannot be opened due to lack of deformation parameter values for one or two of the clusters. An emission of radioactivity (or proton emission) can be assumed from the figures, depending on a pure cluster configuration. Always remember that "forbidden states" in the theoretical study analysis must mean bound states. We should assume that the low-lying cluster configuration of prolate nucleus doesn't refer to the pole-to-pole.

The description of the cluster configuration is very interesting as a good starting point of decay and proton emission as in the channel of a binary proton emission. Actually, the shapes of pole-pole have been chosen by penetrability calculation referred to ban the exclusion Pauli principle. Therefore, they can be small compounds only in the fundamental state of this nucleus. The calculation has addressed the allowed clusterization shapes, which are related to the valence protons. The deformation of the ground state has been considered and has an effect on the clusters of the core and protons, even when it is very small, as it is shown in the results.

An interesting question is, does the Halo clusterization atomic nuclei happen in other radioactivity isotopes just before the decay starting? The clusterization is expected for radio nuclei at excited states near the relative proton emission and β^+-decays. Essentially, the important point of this work is to determine where and when the proton emission and the decay starting take place. The location of the valence protons to emission or decay is the goal. The results appointed the starting of emission and decay according to energies. The nucleon distributions and the sizes of nuclei give us significant evidence related to a weak or a strong interaction and decay. Also, the key idea of the current work is to drive the Halo structure and clusterization of protons and neutrons to other radioactive isotopes. Those nuclei, which have radioactivity, can be dealt with in a clusterization structure. From the results and according to the core structure has been driven by the Shell model, the deformation parameter $\beta_2 = 0{,}7, 0{,}5, 0{,}3, -0{,}7, -0{,}5,$ and $-0{,}3$ have been excluded. The available evidence value of the deformation parameter is referring to the oblate shape for the $^{1}_{5}$O [21]. By normalization, the theoretical value of deformation shape is used to get the energy value, which is (-0.567 MeV).

III. CONCLUSIONS

In the present work, the two proton halo ^{17}Nenucleus has been investigated to calculate the position of the valence protons in emission or decay. The study depended on the Microscopic Cluster Model. The advantage of this model is to drive the core with more degrees of freedom. The Coulomb effect has been taken into consideration. The clusterization configurations and the deformation of the core have played an important role in the processes of proton emission and β^{+}-decay. All three factors: the Coulomb interaction, the core deformation and the clusterization have a large impact on the fragmentation of the parent nucleus. But the main responsible of a starting point for emission or decay is the clusterization configuration, because it appointed the decay and emission positions of the valence protons. The results are correlated to two factors: one built on the cluster angle and the second on the core degrees of freedom, even when its deformation is small. The motion of the valence protons around the $^{1}{}_{5}$O showed different energies depending on the angles. So, we can appoint the position of the valence proton to decay or to emission. We strongly believe that the present method can be applied to all radioactive nuclei in order to determine the energies and the proton position of decay.

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