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H-C and H*-C Semi compactness in bitopological- space

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Abstract: In this paper we introduce two new concepts, namely H-C-Semi compact and H*- C-Semi compact in bitopological space several propositions and examples about these concepts are introduced .

Key words : H-C , H*-C , Semi compactness , bitopological- space

Introduction

Let X be a non empty set. Let T1 and T2 be two topolgies on X then the triple (X,T1,T2) is called a bitopological space, this concept was first introduced by Kelly [1]

In this work, introduce new concepts namely H-C- Semi compact and H* -C semi compact in bitopological space.

Preliminaries

2-1 Remarks

i) If T1 is a topology on X and T2 is also a topology on X then T1 $\cup$ T2 is not necessarily a topology on X

ii) (T1 $\cup$ T2) means the topology on X generated by T1 $\cup$ T2

Definition : [2]

Let (X, T1, T2) be a bitopological- space let A $\subseteq$ X , we say that A is N-open if and only if is open in the space (X , T3) where T3 = (T1 $\cup$ T2) is the supremum topology on X containing T1 and T2 . A $\subseteq$ X is called S- open if and only if it is T1–open or T2 - open

2-3 Remarks and Example [2]

i) The complement of N- open (S- open) is called N -closed (S- closed)

ii) Each S-open in (X, T1 ,T2) is N- open but the converse is not necessarily true

iii) X= {a, b , c}

T1={Ø, X , {a}}

T2={ Ø , X , {b}}

(T1 $\cup$ T2) = {Ø , X , {a} , {b} , {a,b}}.

{a,b} is N- open but not S - open

{a} is S- open , hence it's also N- open

{ a , b}c = {c} N-closed

{ a }c = {b, c} S-closed

Definition [2] , [3]

A bitopological - space (X, T1, T2) is called N – compact (S- compact) if every N- open cover (S- open cover) of X has a finite subcover.

Definition [2] , [3]

Let (X ,T) be a toplogical - space we say that X is C- compact if : for each closed set A $\subseteq$ X

And each open cover

F = {Wα | $\alpha \in \Omega$ } of A, there exists

α_1 , α_2, …., α_n such that A $\subseteq \overline{W}\alpha_1 \cup \overline{W}\alpha_2 \cup \dots \overline{W}\alpha_n$

(that is, there exist a finite sub family whose closures covers A)

3-H-C-Semi- compact- space

In this section, we introduce the concept of H-C-Semi-compact space several properties of this concept are proved

First, we introduce the following definition

Definition

Let (X , T1 , T2) be a bitopological–space let A $\subseteq$ X , we say that A is H- semi open in X iff it is semi - open in the space (X , (T1$\cup$T2))

Remarks and examples

i) The complement of H-semi- open is called H- semi- closed.

ii) Every N- open is H- semi- open but the converse is not necessarily true.

iii) Let X= {a ,b,c}

T1= { Ø, X, {a} }

T2={ Ø, X }

(T1$\cup$ T2) = { Ø ,X ,{a}}=T1

{ a } is N – open , Hence it will be H-semi – open

{a}c= { b, c} is H- semi – closed

Consider A={ a , c } , A is

H-semi – open but A is not N-open

Definition

Let (X ,T1 , T2) be abitopological – space

Let $A \subseteq X$

$F = \{ W\alpha \mid \alpha \in \Omega \}$ is called

H-semi – open cover of A if

1- $W\alpha$ is H- semi-open in X for each $\alpha \in \Omega$

2- $A \subseteq \bigcup_{\alpha \in \Omega} W\alpha$

Definition

i) A bitopological space (X , T1 , T2) is called H-semi compact iff every

H-semi open cover of X has a finite sub cover

ii) let $A \subseteq X$, We say that A is H- semi compact iff every

H-semi open cover of A has a finite sub cover

Definition

Let (X , T) be a topological-space, X is called C-semi compact if :

Given a semi–closed subset $A \subseteq X$ and given a semi – open cover

$F = \{W\alpha \mid \alpha \in \Omega\}$ of A

Then there exist

$\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ Such that $A \subseteq \overline{W}\alpha_1 \cup \overline{W}\alpha_2 \cup \ldots\ldots \overline{W}\alpha_n$

Definition

let (X , T1 , T2) be abitopological space we say that X is H-C-semi compac if given H – semi closed set $A \subseteq X$ and given

$F = \{ W\alpha \mid \alpha \in \Omega \}$ where F is an H – semi open cover of A

then there exit $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that

$A \subseteq$ (H – scl $W\alpha_1$) $\cup$ (H – scl $W\alpha_2$) $\cup \ldots \cup$ (H – scl $W\alpha_n$) (where H- scl $W\alpha$ = the smallest H-semi closed Set containing $W\alpha_1$)

Proposition

Every H- semi closed sub set of H-semi compact space is H-semi compact

Proof

Let (X ,T1 ,T2) be H-semi compact and let $A \subseteq X$ be H-semi closed subset of X let $F = \{W\alpha \mid \alpha \in \Omega\}$ be an H-semi open cover of A.

Now A is H-semi closed, so AC =X-A is H - semi open

Now $F^* = F \cup \{Ac\}$ is an H-semi open cover of X , but X is H-semi compact so $\exists\ \alpha_1, \alpha_2, \ldots, \alpha_n$

such that

$X = W\alpha_1 \cup W\alpha_2 \cup \ldots \cup W\alpha_n \cup Ac$

Hence $A \subseteq W\alpha_1 \cup W\alpha_2 \cup \ldots \cup W\alpha_n$, which means that A is H- semi compact.

The proof of the following proposition is clear .

3.8 Proposition :

i) Every compact space is C-compact

ii) Every semi compact space is C-semi compact

iii) Every H-semi compact space is H-C- semi compact .

iv) the converse of (iii) is not necessarily true . as show by the following example.

Let (N ,T1 , T2) be a bitopological space where N= the set of natural numbers

T1= the indiscrete topology on N T2= $F \cup \{ N, Ø \}$

where $F = \{ Wn \mid Wn = \{1, 2, \ldots, n\}, n \in N \}$

Now (N , T1 , T2) is H- C- semi compact but not H- semi compact

3.9 Proposition

Let (X , T1 , T2) be H-C-semi compact then (X , T1) and (X ,T2) are C- semi compact space.

Proof

Let $A \subseteq$ (X , T1) be semi closed and let $F = \{ W\alpha \mid \alpha \in \Omega \}$ be a T1 semi open cover of A .Now A is H-semi closed subset of (X , T1 , T2) and F is an H-semi open cover of A

But (X, T1, T2) is H – C- semi compact so $\exists\ \alpha_1, \alpha_2, \ldots, \alpha_n$ such that

$A \subseteq$ (H – scl $W\alpha_1$) $\cup$ (H – scl $W\alpha_2$) $\cup \ldots \cup$ (H – scl $W\alpha_n$) . Now H – scl $W\alpha_1 \subseteq$ T1- scl $W\alpha_1$

H – scl $W\alpha_1 \subseteq$ T1- scl $W\alpha_n$

So $A \subseteq$ (T1 – scl $W\alpha_1$) $\cup \ldots \cup$ (T1 – scl $W\alpha_n$)

So (X ,T1) is C-semi compact

Similarly we prove that (X , T2) is C- semi compact .

3-10 Remark

The converse of proposition (3-9) is not necessarily true as shown in the following

Example

Let (N, T1,T2) be a bitoplogical space, let $T1 = P(O+) \cup \{ N \}$

and $T2 = P(E+) \cup \{N\}$

where P (O +) is the power set of O+ = set of all odd natural numbers and P(E+) is the power set of

E+ = set of all even natural numbers then (N , T1) and (N , T2) are C- semi compact but (N , T1 , T2) is not H-semi compact space

3-11 Definition

Let $f: (X, T1,T2) \to (Y, T'1,T'2)$

Be a function , we say that if is H- semi continuous if the inverse image of H- semi open set in Y is H- semi open in X

3-12 Proposition:

The H- semi continuous image of an H-C-semi compact space is also H-C- semi compact

Proof :
Let (X, T1, T2) be H- C- semi compact we have to prove that (Y, T'1, T'2) is also H-C-semi compact
Let A $\subseteq$ Y be H- semi closed Now B= f-1(A) is H-semi closed in X
Let F = { Wα | $\alpha \in \Omega$ } be an H- semi open cover of A
Hence F*={f-1 (Wα | $\alpha \in \Omega$ } is an H-semi open cover of
B= f-1 (A)
But X is H – C- semi of compact
So $\exists$ α 1 , α 2, ……., α n $\ni$
B $\subseteq$ (H – scl f-1 Wα 1)
$\cup$ …. $\cup$ (H – scl f-1 Wα n)
Hence
A= f (B) $\subseteq$ (H – scl Wα 1)
$\cup$ … $\cup$ (H – scl Wα n)
Hance Y is H-C-semi compact

4- H*- C- compact space

In this section we introduce the concept of H*-C- compact space

Definition

A sub set A of bitopological space (X, T1 , T2) is said to be H*- sime open if it is T1 – semi open or T2 semi open
The complement of H*- sime open set is called H*- semi closed

Definition

Let (X , T1 , T2) be abitopological space , let A $\subseteq$ X. A sub collection of the family T1 $\cup$ T2 is called H* - semi open cover of A if the union of members of this sub collection contains A.

Definition

A bitopological space(X, T1, T2) is said to be H*-semi compact if every H*- semi open cover of X has finite sub cover .

4-4 Definition

A bitopological space (X , T1,T2) is said to be H*-C-semi compact if give H*- semi closed set A $\subseteq$ X and given F = { Wα | $\alpha \in \Omega$ } Which is H*- semi open cover of A then $\exists$ α 1 , α 2, …, α n such that
A $\subseteq$ (H*-sclWα 1)$\cup \cdots \cup$ (H*-scl Wα n)
The proof of the following propositions is similar to the previous one.

Proposition

Every H*- semi closed subset of H*-semi compact space is H*-semi compact.

Proposition

Every H*- semi compact space is H*- C - semi compact.

Proposition

Let (X,T1,T2) be an H*- C- semi compact space, then (X, T1) and (X, T2) are both C- semi Compact

Example

Let(X,T1,T2) be a bitopological space where X= [0,1] and
T1= {X, Ǿ,{0}} $\cup$ {[0, $\frac{1}{n}$)|n$\in N$ } T2={X, Ǿ, (0,1]} $\cup$ { ($\frac{1}{n}$, 1]| n$\in N$ } Then (X,T1) and (X,T2) are C-semi compact but (X ,T1,T2) is not H*- C- semi compact

Remark

Let (X, T1,T2) be a bitopological space and let A $\subseteq$ X then
i) H-scl A $\subseteq$ H*-scl A
ii) H- Scl A $\subseteq$ T1-scl A
iii) H- scl A $\subseteq$ T2- scl A
The proof of the following proposition is clear

4-10 proposition

Every H-C-semi compact space is H*-C-semi compact

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الفضاءات شبه المرصوصة H-C والفضاءت شبه المرصوصة H*-C

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الخلاصة:في هذا البحث قمنا بتعريف أنواع جديدة من الفضاءات شبه المرصوصة على الفضاءات الثنائية وقد أسميناها ((الفضاءات شبه المرصوصة H -C)والفضاءات شبه المرصوصة H*-C))وقمنا بدراسة بعض خواص هذه الفضاءات ودراسة العلاقه بينهما