ISSN: 1991-8941

Core Deformation in the Two-Neutron Halo Nucleus, ^{11}Li by Using Faddeev Equation

Waleed S. Hwash

Faculty of Education for Pure Sciences- University of Anbar

Abstract

The properties of the two-neutron halo nucleus ^{11}Li are investigated. The calculations are performed using the Faddeev equation with core excitation and Woods-Saxon potential. The core has been deformed and allowed to be excited to first 2^+ state. This model enables to include the excitation of core of the three bodies, while the other two particles remain inert. Also, it is particularly suitable for obtaining the bound states structure compositions and binding energies of light exotic nuclei considered as three-body systems, which given the three effective of two body interactions. Results were compared with experimental data and three-body model. The handling of ^{11}Li with an excited core in a microscopic cluster model was more accurate than that with an inert core in the three-body model. The dependence of three-body system energy on the quadruple of the core was investigated, and the core has an oblate shape.

Keywords: Halo-nuclei, ^{11}Li, Microscopic cluster model, Faddeev equation, Woods-Saxon equation

1. Introduction

The developments that occurred in the radioactive nuclear beams field led to a new field of nuclear structure physics, that is the study of nuclei away from the p-stable nuclei line. It was found in experimental data [l–3] in light nuclei that ^{11}Li, ^{14}Be, and ^{17}B have an exotic large root-mean-square nuclear radii. Hansen et al. [4] have reported the halo-neutron structure in the halo nucleus, which gave very new consequences to the philosophical study of "nuclear physics and reaction." These studies on structure of halo-nuclei have attracted the attention of many nuclear physicists [5].

Generally, the halo phenomenon is a threshold effect that occurs in most weakly bound systems, where nucleons are held with short-distance potential wells. Under favorable circumstances, a just restricted nucleon or nucleons (or a cluster of nucleons) may tunnel out into a forbidden area. This "leakage" populates very fragile and dilute structures, that are near particle emission threshold effect. As the binding of the halo-nuclei weakened further, the "halo-stratosphere" developed more [6]. In nuclear structure physics, the most observable two-neutron halo candidates, which have been in light nuclei, which have two neutrons surrounding a core. Among them ^{11}Li (= ^{9}Li + n + n) is a stereotypical prototype of halo systems [7], and it enjoyed the consideration of the current work. Actually, ^{11}Li has been considered as the prima donna for all halo nuclei, because of the very small separation energy of the two-neutron 378 keV [8] of valence neutrons. Various works on ^{11}Li nucleus used the approach of the three-body models with an inert core. Several calculation methods of the three-body system have been applied, mostly to ^{6}He and ^{11}Li. Researchers have included the Faddeev method [7, 9–10], the hyper-spherical harmonics approach [7, 11], the various techniques on a harmonic oscillator basis [12], the Green's function of two-body [13], and the cluster-orbital shell model [14,15]. Some studies of pairing model have also been reported [16]. Different approaches of the two-body interaction, such as Gaussian, Woods-Saxon, and Yukawa potentials have been used; each has to reproduce the properties of low-energy for a two-body subsystem.

A non-spherical shape of the core affects the binding energy of the two-neutron and radius of nucleus. The core in ^{11}Li consists of 9 particles, 3 protons and 6 neutrons; therefore, 1 proton and 4 neutrons are outside the closed shell (magic number). Thus, the core (^{9}Li) in ^{11}Li is not spherical; it has a prolate or an oblate shape. When we describe the two neutron-halo nuclei using microscopic cluster model and compared the result with experimental data and other models, we may be able to calculate this deformation.

2. Theoretical Methods

Three-body halo nucleus (^{11}Li) is described using two ways.

a- Faddeev equation

$$\begin{aligned}(T_1+h+V_1-E)\psi_1^{JM} &= -V_1(\psi_2^{JM}+\psi_3^{JM}),\\ (T_2+h+V_2-E)\psi_2^{JM} &= -V_2(\psi_3^{JM}+\psi_1^{JM}),\\ (T_3+h+V_3-E)\psi_3^{JM} &= -V_3(\psi_1^{JM}+\psi_2^{JM}),\end{aligned} \qquad (1)$$

The Faddeev equation is defined as three components of the wave function ψ_i^{JM}, such that the full wave function of three-body is as follows.

$$\psi^{JM} = \psi_1^{JM}(x_1, y_1) + \psi_2^{JM}(x_2, y_2) + \psi_3^{JM}(x_1, y_1) \qquad (2)$$

It enables the inclusion of excited core, whereas the other two particles remain inert.

Here, the components ψ_i^{JM} are functions of their own "natural" Jacobi coordinate pairs *i*, as shown in Fig. 1.

The distances between both particles pair $\vec{r}_{jk}$ and the Jacopian coordinate has been used to describe the distances in a three-body system, between the center of the pair and the third particle (Fig. 1).

$(\vec{x}_i, \vec{y}_i)$ where $\vec{x}_i = \sqrt{A_{jk}}\,\vec{r}_{jk}$ and $\vec{y}_i = \sqrt{A_{(jk)i}}\,\vec{r}_{(jk)i}$:

$$\begin{aligned}\vec{r}_{jk} &= \vec{r}_j - \vec{r}_k = \vec{y}_i + A_{jk}\vec{x}_i\\ \vec{r}_{(jk)i} &= \vec{r}_i - (A_j\vec{r}_j + A_k\vec{r}_k)/(A_j + A_k)\end{aligned} \qquad (3)$$

The intrinsic Hamiltonian for the core has been define a set of eigenstates ϕ_{s_c} and eigenvalues ε_{s_c} ,

$$\hat{h}(\xi_c)\phi_{s_c}(\xi_c) = \varepsilon_{s_c}\phi_{s_c}(\xi_c) \qquad (4)$$

The wave function of the three-body nucleus has been expanded in this model in terms of these ϕsc states, and it factorizes the freedom degrees of core from the Jacobi-coordinates in every Faddeev component:
(for more details refer to [17]).

$$\psi_i^{JM}(x_i, y_i, \hat{\xi}_c) = \sum_{s_c}\phi_{s_c}(\hat{\xi}_c)\psi_{s_c}(x_i, y_i),$$

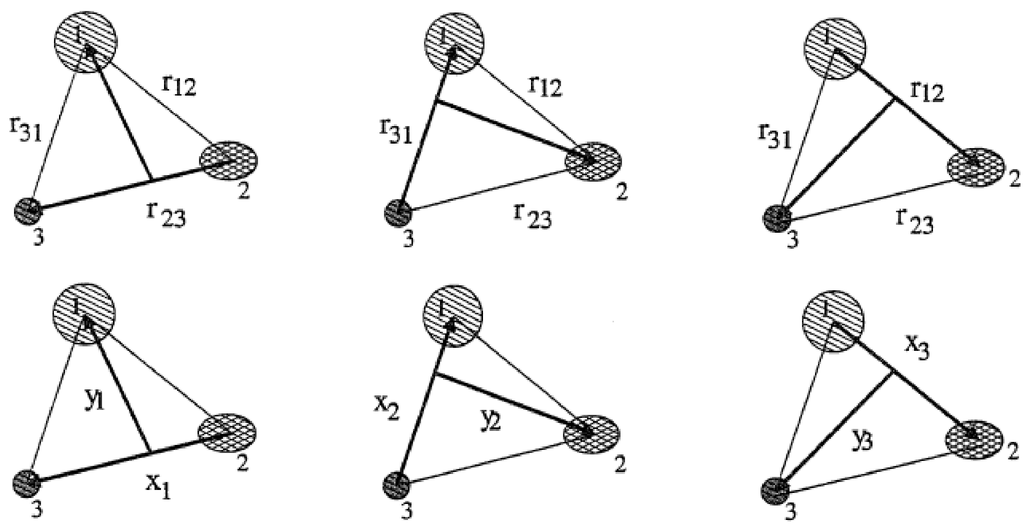

Figure 1: Three sets of Jacobi coordinates used in the Faddeev formalism[17].

b- Woods-Saxon potential equation

The Wood-Saxon potential equation is given by the following Equation.

$$V(r) = \frac{-V_o}{1+\exp\left(\dfrac{r-R}{a}\right)} \qquad (5)$$

In the present work, the Woods-Saxon Equation (5) was used to calculate the potential for the two neutrons in ^{11}Li with some degree of deformation. The Hamiltonian of this system is as follows:

$$\hat{H} = \hat{T} + \hat{h}_{core}(\vec{\xi}) + \hat{V}_{core-n1}(r_{core-n1}, \vec{\xi}) + \hat{V}_{core-n2}(r_{core-n2}, \vec{\xi}) + \hat{V}_{n-n}(r_{n-n}) \qquad (6)$$

The Hamiltonian covers the kinetic energy $\hat{T} = \hat{T}_x + \hat{T}_y$, the core Hamiltonian $\hat{h}_{core}(\vec{\xi})$ which relies on the internal parameters $\vec{\xi}$, an interaction of two-body V_{core-n} and V_{n-n} are for all pairs of bodies interaction. Here the potential is taken as a deformed Wood-Saxon potential as well as a spin-orbit interaction as in Eqs (7) and (8). where *T* is the kinetic energy of the two neutrons, r is the root-mean-square (RMS) matter radius of the neutron-halo nuclei. *R* is radius of the core and is given in the following equation:

$$\hat{V}_{core-n}(r_{core-n},\vec{\xi}) = \frac{-V_0}{\left[1+\exp\left(\frac{r_{core-n}-R(\theta,\phi)}{a}\right)\right]} + \frac{-\hbar^2}{m^2c^2}(2l.s)\frac{V_{s.o}}{4r_{core-n}}\frac{d}{dr_{core-n}}\left(\left[1+\exp\left(\frac{r_{core-n}-R_{so}}{a_{so}}\right)^{-1}\right]\right) \quad (7)$$

$$V_{n-n}(r_{n-n}) = -\frac{\hbar^2}{m^2c^2}(2l.s)\frac{V_{s.o}}{4r_{n-n}}\frac{d}{dr_{n-n}}\left(\left[1+\exp\left(\frac{r_{n-n}-R_{so}}{a_{so}}\right)^{-1}\right]\right) \quad (8)$$

$$R = R_0\left[1+\beta_2 Y_{20}(\theta,\phi)\right] \quad (9)$$

where θ and ϕ are spherical angles in the rest frame of the core, *Ro = 1.25A*$^{1/3}$, β_2 parameter of deformation [18].

3. Results and Discussion

The ground-state properties of ^{11}Li are calculated. The results are listed in Table 1. The E_{FD} and R_{FD} are the theoretical eigen-energies, and the RMS matter radius of the neutron-halo nucleus is calculated with the method of Faddeev equation with an excited core. The $E_{W.S}$ and $R_{W.S}$ are the energy and radius of ^{11}Li, respectively, and are calculated using Woods-Saxon equation formula. The experimental data of energies for the three-body system are cited from Ref. [19], and the theoretical-energy calculations of three-body model ($E_{thr-bod}$ and $R_{thr-bod}$) with an inert core is cited from Ref. [20].

Table 1.The results of the three-body energy (MeV) and radius(fm) of ^{11}Li.

Nuclei	J^{π}	E_{FD}	$E_{W.S}$	E_{exp}	$E_{thr-bod}$	R_{FD}	$R_{W.S}$	R_{exp}	$R_{thr-bod}$
^{11}Li	1/2$^-$	- 0.53	-0.292	-0.300(±0.019)	-0.54	3.596	3.6	3.1(±0.17)	2.95

Some data of ^{11}Li were presented here based on [6]. In this study, the value of separation energy of two valance neutrons was 295 ± 26 keV [21]. Furthermore, it was stated in another experimental data 376 ± 5 keV [22] as separation energy of two-neutron, this value was finally modified to 378 ± 5 keV [8]. Table 1 shows that the three-body energies of the ground states are calculated using Faddeev equation with freedom degree of excited core and Wood-Saxon equation. These energies are nearest to experimental values than the three-body model with an inert core.

The numbers of protons and neutrons inside the core and data in Table 1 indicate that the core of ^{11}Li (^{9}Li) has some deformations. What is the deformation value and whether the core oblate or prolate? The RMS matter radius of ^{9}Li is 2.43 ± 0.07 fm [23,24]. The observed energy states can be identified with the first five predicted p-shell levels in Table 2 .

Table 2. The observed levels of ^{9}Li.

E (MeV±keV)	J^{π}
G S	$3/2^-$
2.691±5	$1/2^-$
4.296±15	$5/2^-$
5.38±6	$7/2^-$
6.43±15	$9/2^-$

According to the above mentioned value of ^{9}Li, and using Equation (9), the radius of the core for various deformation parameters (β_2) was calculated. In Figure 2, R_c radius of the core increases with increasing β_2.

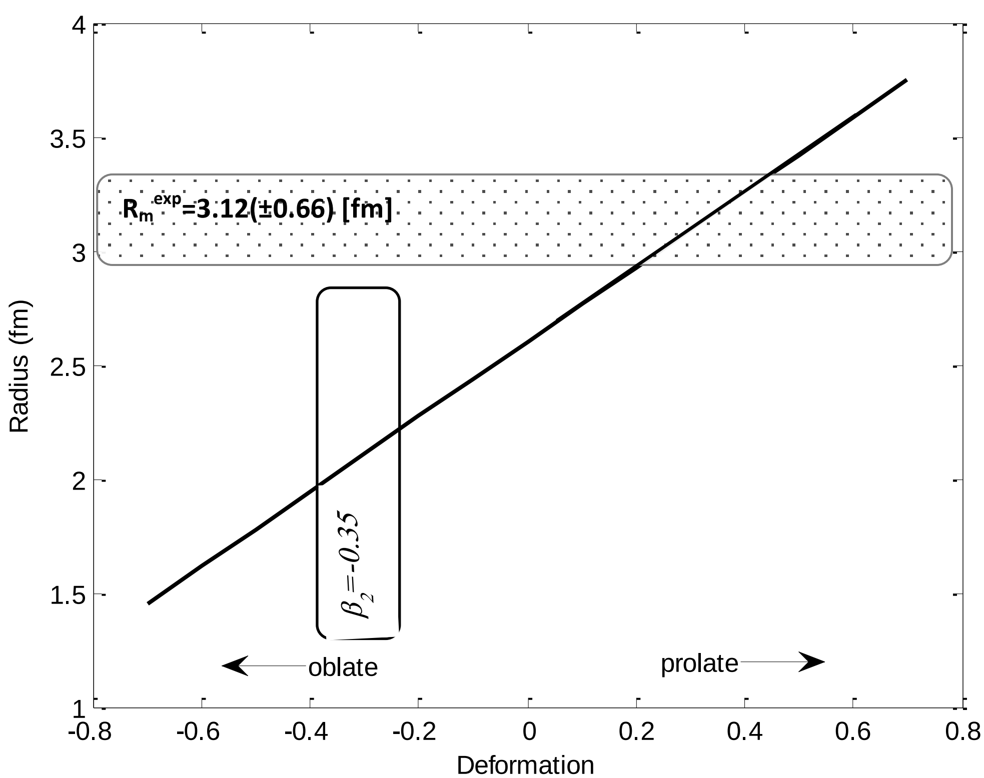

Figure 2: Radius of the core as function of deformation

From Figure 2 and according to the radius of the core from [23], the parameter of deformation is equal ($\beta_2 \approx -0.3$). Therefore, the core has an oblate shape. This deformation because the spin and parity of ^{9}Li are $1p_{3/2}^-$, regarding to shell model. Figure 3 shows the dependence of three-body energy on the deformation of ^{9}Li, and it shows a decrease in binding energy with the deformation (with oblate shape), thereby indicating the increasing size of the nucleus (e.g., radius increases with the deformation).

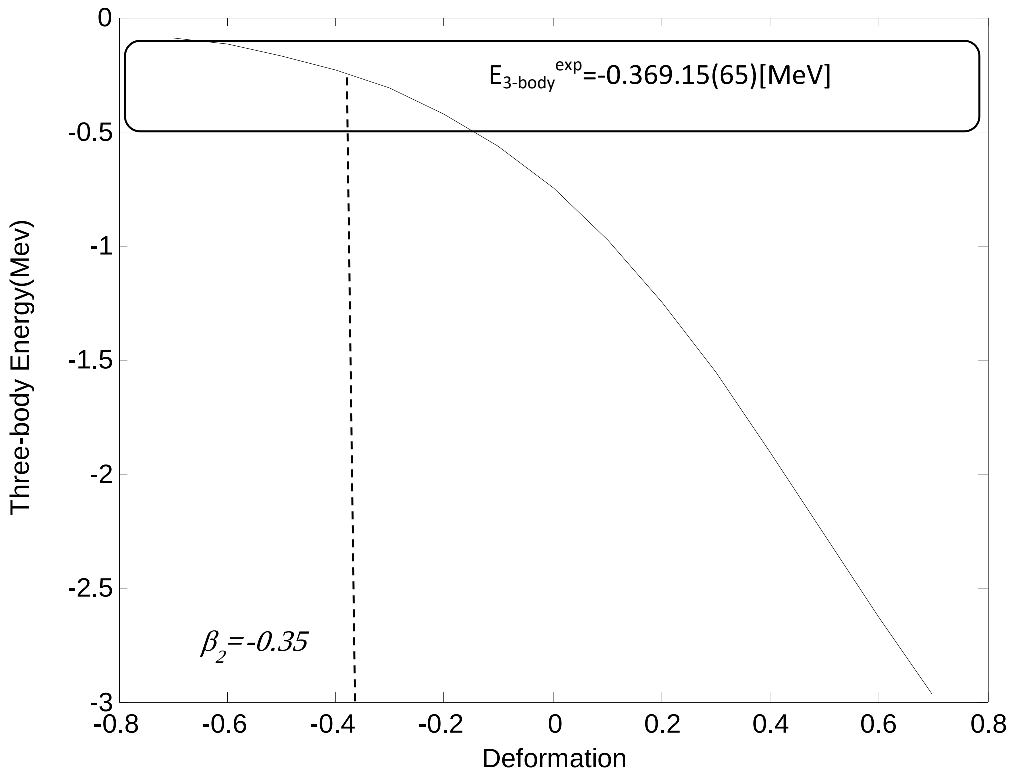

Figure 3: the relationship between three-body energy and β_2

The dependence of the three-body energy E and RMS matter radius of nucleus on β_2 is shown in the three dimensions plot of E, R, and β_2 (Figure 4). Regarding the case of two-body, the Pauli principle is involved in the calculation automatically regardless of the core excitation, if it is supposed that the neutrons of the core occupy ground states are considered as the core-neutron interactions. The energy states of core are already orthogonal to the states of valence-neutrons. In the case of three-body system, the system wave function has to be anti-symmetrized with consideration to the replacement of the two valence nucleons in their places, as well as the replacement of neutrons between halo and the core neutrons. The first point is simply satisfied in the **T**-coordinate by imposing the appropriate-selection rule. The wave function must be anti-symmetric with all consideration to the replacement of the halo neutrons and the core; so, this case will satisfy the second point. Actually, the Pauli principle is considered in an appropriate way by demanding to the two-halo system wave function has been orthogonal with the core plus two valence neutron states, which are taken as the ground states of the deformation of ^{9}Li and Hamiltonian of two-neutron. When these energy states are built, the projection operator on the forbidden-states space can be defined, and these can be projected out of the allowed two-halo space along with the standard Feshbach technique. This approach is appropriate for dealing with the unclosed shells of the nuclei's core. This approximation has been used in the current work, we assume that ^{9}Li is an unclosed *p*-shell nucleus. If ^{9}Li is assumed not to be spherical and inert, the $1s_{1/2}$ and $1p_{3/2}$ states are not fully occupied. When core excitation is considered, an alternative of these three states are 11 blocked eigenstates that are joined with excited and ground states of the core.

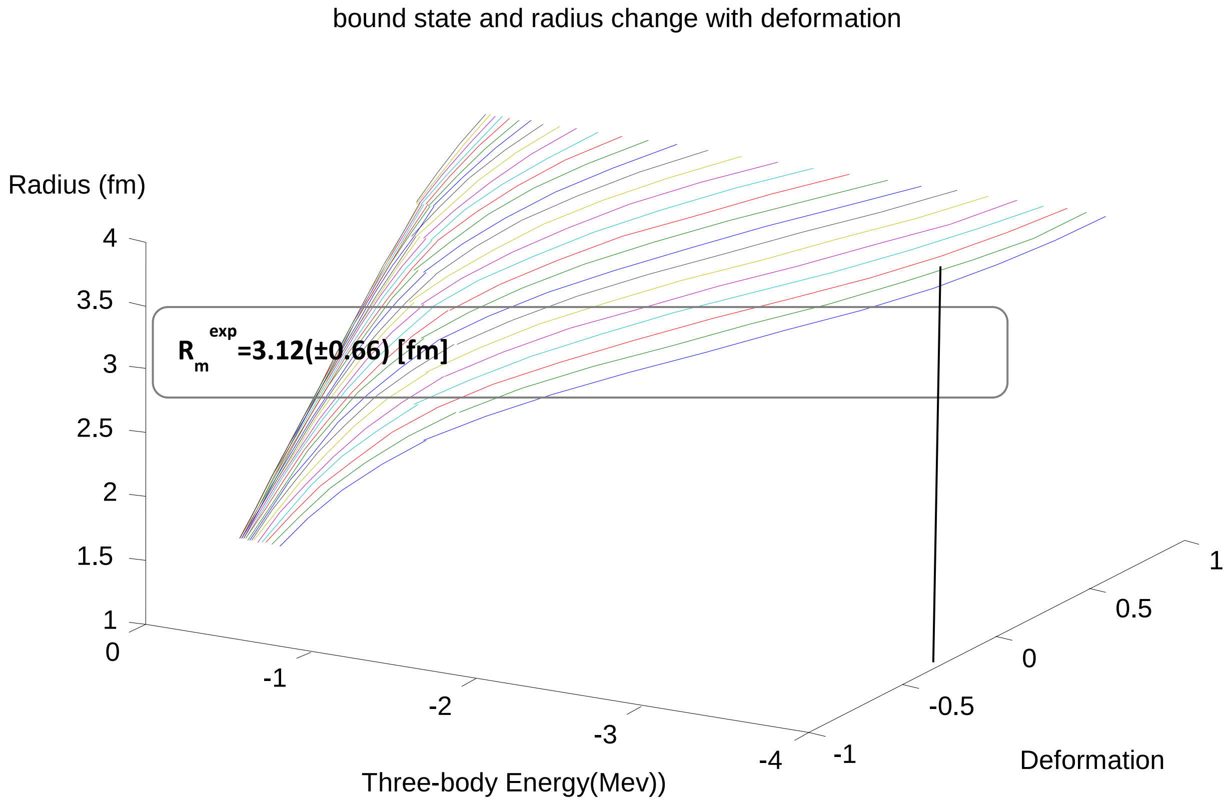

Figure 4: Bound state and radius change with deformation

Figure 4 illuminates the configuration of the lowest of $(5/2)^{+}$ resonances, stated at around 2MeV, and displays the percentage of components of the core excited state in the resonance of wave function. For slight $|V_{ls}|$ the $(5/2)^{+}$ resonance has been built on the excitation of the ^{9}Li. With increasing (in figure 4) spin–orbit potential depth, from −5 to −6 MeV, the resonance modifications its situation and come to be built on the core ground state. For large positive (prolate) β the $(5/2)^{+}$ resonance has large amounts of together excited core and inert core components therefore cannot be ascribed to either the excited core resonance or inert core. For high core deformations β, the model guesses ^{9}Li is unbound for an important variety of spin–orbit potentials. I observe that in the area of slight deformations, spin–orbit strengths are between −4 and −3MeV the resonance modifications nature, as the avoided requires a $(5/2)^+$ energy state of different essential structure to be placed at +2.0 MeV

4. Conclusions

The Faddeev equation with Woods-Saxon potential formula is used in this work to calculate the two-neutron halo (^{11}Li) properties. Three-body system calculations for the low-lying states of ^{11}Li, including excitation and deformation of ^{9}Li, have shown that effects of core deformation were clearly on the binding energy of two valence neutrons. The core is treated (i.e., a deformed rotor in a three-body model); thus, it is possible to reproduce the binding energy, the RMS matter radius, and the structure of ^{11}Li, as consistent with the experiment.

The all probabilities of $(0d_{5/2})^2$, $(1s_{1/2})^2$ *and* $(0d_{3/2})^2$ show slightly probabilities to be occupied by valence neutrons. The $(0P_{1/2})^2$ has been excluded in this calculation. Solving the coupled-Fadeev equations and Woods-Saxon equation, the three-body energy and the RMS matter radius of the system are obtained.

Therefore, calculations for ^{11}Li nucleus through the solution of Faddeev equation with core excitation and Woods-Saxon equation gave more accurate results than those obtained through the Faddeev equation with an inert core. The deformation of ^{9}Li plays the main role in the effect on binding energy of the three-body system.

5. References

[1] Tanihata, I.; Kobayashi, T.; Yamakawa, O.; Shimoura, S.; Ekuni, K.; Sugimoto, K Takahashi, N.; Shimoda, T.; Sato, H. *Phys. Lett. B.* **1988**, 206, 592-596

[2] Mittig, W.; Chouvel, J. M.; Wen-Long Zhan; Bianchi, L.; Cunsolo, A.; Fernandez, B.; Foti, A.; Gastebois, J.; Gillibert, A.; Gregoire, C.; et al. Phys. Rev. Lett. 1987, 59, 1889-1897.

[3] Saint-Laurent, M. G.; Anne, R.; Bazin, D.; Guillemaud-Mueller, D.; Jahnke, U.; Jin Gen Ming; Mueller, A. C.; Bruandet, J. F.; Glasser, F.; Kox, S. et al. Z. Phys. A 1989, 332, 457-465.

[4] Hansen, P.; Jonson, B.; Europhys. Lett. 1987, 4, 409-413.

[5] Zhongzhou Ren; Gongou Xu, Baoqui Chen; Zhongyu Ma; Mittig, W. phys. Lett. B. 1995, 351, 11-17.

[6] Brida, I. Ph.D. Thesis, Department of Physics, Michigan State University, Michigan State, USA, (2009).

[7] Zhukov, M. V.; Danilin, B. V.; Fedorov, D. V.; Bang, J. M.; Thompson, I. J.; Vaagen, J. S. Phys. Rep. 1993, 231, 151-199.

[8] Bachelet, C.; Audi, G.; Gaulard, C.; Guenaut, C.; Herfurth, F.; Lunney, D.; de Saint Simon, M.; Thibault, C. Phys. Rev. Lett. 2008, 100, 182501-182504.

[9] Bang, J. M.; Thompson, I. J. Phys. Lett. B 1992, 279, 201-206.

[10] Garrido, E.; Fedorov, D. V.; Jensen, A. S. Nucl. Phys. A 2002, 700, 117-141.

[11] Zhukov, M. V.; Danilin, B. V.; Fedorov, D. V.; Vaagen, J. S.; Gareev, F. A.; Bang, J. Phys. Lett. B 1991, 265, 19-22.

[12] Johannsen, L.; Jensen, A. S.; Hansen, P. G. Phys. Lett. B 1990, 244, 357-362.

[13] Bertsch, G. F.; Esbensen, H. Ann. Phys. 1991, 209,327-336.

[14] Tosaka, Y.; Suzuki, Y. Nucl. Phys. A 1990, 512, 46-60.

[15] Ikeda, K. Nucl. Phys. A 1992, 538,355-365.

[16] Vinh Mau, N.; Pacheco, J. C. Nucl. Phys. A 1996, 607, 163-177.

[17] Thompson, I. J.; Nunes, F. M.; Danilin, B. V. computer physics communications, 2004, 161, 87-107.

[18] Hwash, W. S.; Yahaya, R.; Ramadan, S. Physics of Atomic Nuclei, 2014,77, 275–281.

[19] Audi, G.; Wapstra, A. H.; Thibault, C. Nucl. Phys. A 2003, 729, 337-676.

[20] Yan-Yun, C.; Shuang, C.; Zhong-Zhou, R. Chinese Physics C 2008, 32, 972-975.

[21] Young, B. M.; Benenson, W.; Fauerbach, M.; Kelley, J. H.; Pfaff, R.; Sherrill, B. M.;Steiner, M.; Winfield, J. S.; Kubo, T.;

Hellström, M. et al. Phys. Rev. Lett. 1993, 71, 4124- 4127.
[22] Bachelet, C.; Audi, G.; Gaulard, C.; Guénaut, C.; Herfurth, F.; Lunney, D.; Simon, M. S.; Thibault, C. Eur. Phys. J. A 2005, 31-32.
[23] Egelhof, P.; Alkhazov, G. D.; Andronenko, M. N.; Bauchet, A.; Dobrovolsky, A. V.; Fritz, S.; Gavrilov, G. E.; Geissel, H.; Gross, C.; Khanzadeev, A. V.; et al. Eur. Phys. J. A 2002, 15, 27-33.
[24] Young, P. G.; Stokes, R. H. Phys. Rev. C 1971, 4, 1597-1600.

تشوه القلب في نواة الهلو ذات النيوترونين ^{11}Li باستخدام معادلة فاديف

د. وليد صبحي حويش

Email: waleed973@yahoo.com

الخلاصة

تم في هذه الدراسة حسا الصفات النووية لنواة الهلو ذات النيترونين ^{11}Li. حسبت هذه الصفات باستخدام كعادلة فاديف مستخدما جهد وود-سكسون بأخذ بنظر الاعتبار تهيجات القلب. وكذلك اخذ بنظر الاعتبار تشوه قلب النواة واحتمالية ان يكون متهيج في المستوي الاول 2^{+}. النموذج المستخدم في الدراسة يسمح بإمكانية تضمين تهيجات القلب لمنظومة ثلاثية-الجسيمات، بينما تبقى الجسيمين الاخرين يبقيان خاملين. ايضا هذا النموذج يستطيع حساب مستويات الطاقة المحددة وكذلك طاقة ترابط الانوية الغريبة الخفيفة باعتبارها منظومة ثلاثية-الاجسام. قورنت النتائج النظرية المحسوبة مع النتائج العملية وكذلك مع نتائج نظرية اخرى محسوبة باستخدام نماذج اخرى. حساب صفات نواة ^{11}Li بوجود قلب متهيج باستخدام النموذج العنقودي المجهري يعطي دقة اكثر في حساب الصفات من اعتبار القلب خامل كما في نموذج الجسيمات-الثلاثة. اعتمادية طاقة نظام ثلاثي-الاجسام على عزم رباعي القطب الكهربائي اعتبر في هذه الدراسة وتم حسابة على انه ذات شكل مفلطح.