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Nuclear Structure of ^{14}Be Nucleus

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We investigated in this work the two-neutron halo nuclei of ^{14}Be. The cluster model was used in the present work to study the properties of a three-body system. The three-body system described depended on the Jacobi coordinates using two configurations: the T-configuration and the Y-configuration. In this work, the binding energy, the root-mean-square (r.m.s.) radius and effect of deformation of the core (^{12}Be) on the properties of two-neutron halo were studied. The calculations confirmed that the core had a prolate-shaped deformation, which affected the structure of the three-body system.

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I. INTRODUCTION

The development of radioactive nuclear-beam facilities has brought to light many interesting phenomena in nuclear physics, the study of nuclei far from the P-stability line. It has been found in the experiments [1–3] on light nuclei that ^{11}Li, ^{14}Be, and ^{17}B have an abnormally large root-mean-square r.m.s radii. Reference 4 pointed out that this was due to the neutron halo in the nucleus, which gave new implications to the theory of nuclear structure and nuclear reaction. Studies on halo nuclei have attracted many nuclear physicists and have led to a series of experiments to study ^{14}Be. The r.m.s matter radius of the ^{14}Be nucleus has been deduced from the interaction cross-sections of light radioactive nuclei close to the neutron drip line (^{14}Be). Numerous matter radii of ^{14}Be have been found: 3.36 ± 0.19 fm [5], 3.11 ± 0.38 fm [1], and [3.10 ± 0.15]fm [6]. The separation energy of the two-neutron halo nucleus of ^{14}Be S_{2n}, has been determined by using two experiments: a pion double-charge-exchange measurement [7] that gave a value of 1.12 ± 0.16 MeV and a time-of-flight experiment [8] that gave a value of 1.48 ± 0.14 MeV. The weighted average of S_{2n} = 1.34 ± 0.11 MeV [9] is the usual cited value.

The properties of ^{14}Be have been studied using a particle-particle random phase approximation (RPA) model. Two-body correlations within the RPA model have provided good descriptions of the energies and the amplitudes in comparison to models that assume a neutron closed shell [10]. The dissociation process of light exotic ^{14}Be nuclei has been treated as two neutrons and a core (two neutrons with an inert core) [11]. Similarly, a three-body model has also been applied to describe two-neutron halo nuclei of ^{14}Be [12], with an antisymmetrized molecular dynamics method being used to investigate its excited states. The theoretical results of the study indicated rotational bands with new cluster structures in ^{14}Be [13].

The two-neutron pairing model has also been used to calculate the two-neutron dissociation energy in ^{14}Be and the $d_{5/2}$ resonance in ^{13}Be at a measured energy of 2 MeV [14]. A three-body system of ^{14}Be with the Efimov effect, employing a separable potential for binary systems, was found to be in agreement with the experimental data [15]. A Skyrme-Hartree-Fock approach to study the ground-state properties of the ^{14}Be nucleus (with new force parameters SKI4 of Reinhard and Flocard) has succeeded in reproducing neutron halos in the ^{14}Be nucleus and provides good descriptions of the binding energy and the radii for Be isotopes [16]. The ^{14}Be nucleus described in a three-cluster generator coordinate method [17] has provided the energy spectrum of ^{14}Be up to a 5-MeV excitation energy and matter densities that support a halo structure of the ground state.

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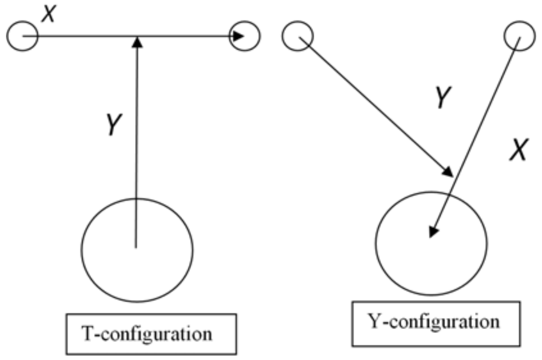

Fig. 1. Jacobi coordinates for the three-body system.

In treating the ^{13}Be and the ^{14}B nuclei with a deformed ^{12}Be core as a rigid rotor in a one-neutron halo nucleus and a two-neutron halo nucleus, Tarutina *et al.* [18], found that the deformation of the ^{12}Be core has a large positive quadrupole of about ($\beta_2 > 0.8$) when a Wood-Saxon potential between the core and neutrons was used. Thompson and Zhukov [19] used the Faddeev three-body approach with the ^{14}Be treated as an inert ^{12}Be core interacting with valence neutrons.

In the present work, the ^{14}Be nucleus was investigated using cluster model, and the binding energy of the two-neutron halo ^{14}Be nucleus, the r.m.s. matter radius of the ^{14}Be and the effect of deformation of the (^{12}Be) core on the binding energy and the r.m.s. radius of ^{14}Be were calculated. The ^{14}Be nucleus was treated as having a core(^{12}Be)+n+n in a two-neutron halo nucleus. The core is not inert, but has some deformation dependence as it is not a closed shell for neutrons and protons while the extra neutrons and protons outside the closed shell contribute to some deformation in the core and has an effect on the three-body energy in two-neutron halo nuclei. The ^{14}Be nucleus was described using the Jacobi coordinates for the three-body problem.

II. THEORETICAL FRAMEWORK

The three-body system or two-neutron halo nuclei should be defined in terms of core and valence neutrons. The distances between each pair of particles $\vec{r}_{jk}$ and the distance between the center of mass of the pair and the corresponding third particle (represented in Fig. 1) can be expressed in terms of the Jacobi coordinates ($\vec{x}$, $\vec{y}$):

$$x = \sqrt{A_{jk}}\vec{r}_{jk} = \sqrt{\frac{A_j A_k}{A_j + A_k}}\vec{r}_{jk}$$

and

$$y_i = \sqrt{A_{(jk)i}}\vec{r}_{(jk)i} = \sqrt{\frac{(A_j A_k)A_i}{A_i + A_j + A_k}}\vec{r}_{(jk)i}.$$

The intrinsic Hamiltonian of the core determines a set of eigenstates ϕ_{core} and eigenvalues ε_{core} with

$$\hat{h}_{core}(\xi_{core})\phi_{core}(\xi_{core}) = \varepsilon_{core}\phi_{core}(\xi_{core}). \quad (1)$$

The total wavefunction of the system from Jacobi coordinates is

$$\Psi^{JM}(x, y, \vec{\xi}) = \phi_{core}(\xi_{core})\psi(x, y), \quad (2)$$

where $\psi(x, y)$ contains the radial, angular and spin of the remaining two particles relative to the core. The hyperspherical method was used to convert the two-dimensional partial differential equation into a set of coupled one-dimensional equations. The Jacobi coordinates (x, y) are transformed into the hyperspherical coordinates (hyper-radius ρ and hyper-angle θ) defined as $\rho^2 = x^2 + y^2$ and $\theta = \arctan\left(\frac{x}{y}\right)$.

The hyperspherical expansion of the three-body radial and angular wave functions is

$$R_n(\rho) = \frac{\rho^{5/2}}{\rho_o^2}\sqrt{\frac{n!}{(n+5)!}}L^5_{nlag}(z)\exp\left(\frac{-z}{2}\right), \quad (3)$$

where$z = \rho/\rho_o$ and

$$\psi_k^{l_x l_y}(\theta) = N_k^{l_x l_y}(\sin\theta)^{l_x}(\cos\theta)^{l_y}P_n^{l_x+\frac{1}{2}, l_y+\frac{1}{2}}(\cos 2\theta). \quad (4)$$

The wave function of the valence neutrons is

$$\psi_{n,k}^{l_x l_y}(\rho, \theta) = R_n(\rho)\psi_k^{l_x l_y}(\theta). \quad (5)$$

Using $\psi(x, y)$ in Eq. (2) as

$$\psi(x, y) = \psi_{n,k}^{l_x l_y}(\rho, \theta),$$

where L^5_{nlag} are associated Laguerre polynomials of order $nlag = 0, 1, 2, \ldots$, $P_n^{l_x+\frac{1}{2}, l_y+\frac{1}{2}}(\cos 2\theta)$ is the Jacobi Polynomial. Equation (3) is a function of (ρ) because of the z dependence on (ρ), with $z = \rho/\rho_o$. The values of ρ and ρ_o are explained in Sec. III. We assume that $nlag = n$, where $n = l_x + 1$. $N_k^{l_x l_y}$ is a normalization coefficient, and k is the hyperangular momentum quantum number with for $n = 0, 1, 2, \ldots$.

The wavefunction of the system comes from the internal wavefunction of every body in that system. The internal wavefunction of each neutron was obtained by solving the Schrödinger Equation in spherical coordinates. More details about the formalism of the hyperspherical harmonics method are presented in Refs. 20 and 21. The total Hamiltonian $\hat{H}$, of the system is

$$H = T + h_{core}(\vec{\xi}) + V_{core-n1}(r_{core-n1}, \vec{\xi}) + V_{core-n2}(r_{core-n2}, \vec{\xi}) + V_{n-n}(r_{n-n}). \quad (6)$$

The Hamiltonian contains the kinetic energy $T = T_x + T_y$, the intrinsic Hamiltonian of the core $\hat{h}_{core}(\vec{\xi})$, which depends on the internal variables $\vec{\xi}$, and two-body interactions V_{core-n} and V_{n-n} for all pairs of interacting bodies. Here, the potential is taken as a deformed Wood-Saxon potential, as well as a spin-orbit interaction.

The rotational model is assumed for the structure of the core; hence, the core is an axially deformed symmetric rotor where, in the body-fixed frame, the radius of this deformed core is expanded in spherical harmonic while for simplicity, we retain only the quadrupole term, as in Eq. (9):

$$\hat{V}_{core-n}(r_{core-n}, \vec{\xi}) = \frac{-V_0}{\left[1+\exp\left(\frac{r_{core-n}-R(\theta,\phi)}{a}\right)\right]} + \frac{-\hbar^2}{m^2c^2}(2l.s)\frac{V_{s.o}}{4r_{core-n}}\frac{d}{dr_{core-n}}\left[1+\exp\left(\frac{r_{core-n}-R_{so}}{a_{so}}\right)\right]^{-1}. \tag{7}$$

With the spin-orbit interaction being

$$V_{n-n}(r_{n-n}) = -\frac{-\hbar^2}{m^2c^2}(2l.s)\frac{V_{s.o}}{4r_{n-n}}\frac{d}{dr_{n-n}}\left[1+\exp\left(\frac{r_{n-n}-R_{so}}{a_{so}}\right)\right]^{-1} \tag{8}$$

with

$$R = R_0[1+\beta_2 Y_{20}(\theta,\phi)] \tag{9}$$

and $R_0 = 1.25A_{core}^{1/3}$. We assumed $R_{so} = R$. $\vec{l}$ is the operator of orbital momentum between core and a neutron, $\vec{s}$ is the operator of a neutron's spin and $m = m_\pi$ is the mass of a pion.

For practical calculations $\left(\frac{\hbar}{m_\pi}\right)^2 = 2.0$ fm^2. β_2 is the core's quadrupole deformation, and A_{core} is the mass number of the core. The operator for the average squared distance of nucleons for an A-body system from the position of the total center of mass is

$$\vec{r}_{CM} = \frac{1}{A}\sum_{i=1}^{A}\vec{r_i}, \tag{10}$$

$$r_m^2 = \frac{1}{A}\sum_{i=1}^{A}(\vec{r_i}-\vec{r}_{CM})^2, \tag{11}$$

where r_i is the position of the *ith* nucleon, r_{CM} is the centre of mass and the r.m.s. matter radius $\langle r_m^2\rangle^{1/2}$ of the nucleus is

$$\langle r_m^2\rangle^{1/2} = \frac{1}{A}\left[A_{core}\langle r_m^2(core)\rangle + \langle\rho^2\rangle\right]. \tag{12}$$

The total quadrupole moment for a halo nucleus can be written as $Q = Q_j + Q_c$, where Q is the total quadrupole moment composed of Q_j due to a loose nucleon and Q_c is due to the core. Generally, $Q_c \gg Q_j$ [22]:

$$Q_c = Q'\left[\left(3\Omega^2/2J^2\right)-\frac{1}{2}\right]. \tag{13}$$

Equation (13) can be written as

$$Q_c = Q'\frac{J}{2J+3}\left[\frac{3\Omega^2}{J(J+1)}-1\right], \tag{14}$$

where J is the total nuclear angular momentum, Ω is the projection of j (nuclear angular momentum) and Ω' can be taken equal to

$$Q' = \frac{4}{5}\delta Z R^2, \tag{15}$$

with Z being the atomic number, R radius of the nucleus calculated before and δ being related with the deformation parameter β_2 ($\beta_2 = 2/3(4\pi/5)^{1/2}\delta$) [22].

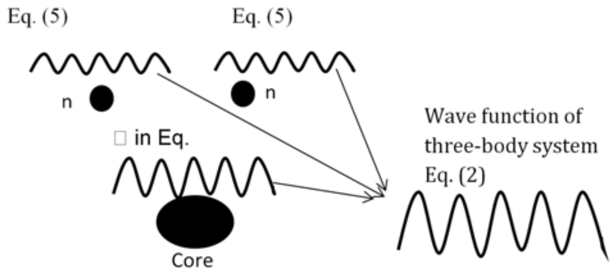

Fig. 2. Description of the wave functions of a three-body nucleus.

III. CALCULATION METHOD AND RESULTS

Figure 2 shows the three-body core and two neutrons with the wave function of each particle, as described in the theoretical section. Equation (5) describes the wave function of the valence neutrons while ϕ in Eq. (2) describes the wave function of the core calculated using the shell model, and Eq. (2) itself describe the total wave function of the three-body system.

The three-body Hamiltonian in Eq. (6) has been applied to calculate the energy of a two-neutron halo nucleus. The relationship between the three bodies depends on the central Wood-Saxon potential and the spin-orbit interaction as in Eq. (7). Two configurations (T-configuration and Y-configuration) were applied in the calculation using Jacobi coordinates. The core, assumed

Table 1. Parameters used in the calculations.

l_x	l_y	n	K	ρ	ρ_o	r_o (fm)	a (fm)	a_{so} (fm)	V_o (MeV)	V_{so} (MeV)
0	0	1	2	0.866	0.866	1.25	0.65	0.65	–74	–7.5
1	1	2	6	0.866	1.936	1.25	0.65	0.65	–74	–7.5
–	–	–	–	1.936	–	1.25	0.65	–	–	–
2	2	3	10	0.866	2.958	1.25	0.65	–	–	-
2	2	3	10	1.936	–	1.25	–	–	–	–
2	–	–	-	2.958	–	1.25	–	–	–	–

Table 2. Experimental data for the binding energy, matter radius of ^{14}Be and deformation parameter of ^{12}Be.

Nucleus	E^{exp} (keV)	R_m^{exp} (fm)	Deformation parameter (β_2)
^{14}Be	-1.12 ± 0.16 [7]	3.36 ± 0.19 [5]	
–	-1.48 ± 0.14 [8]	3.11 ± 0.38 [1]	
–	-1.34 ± 0.11 [9]		
^{12}Be			0.614(47) – 0.725(54) [23]

to be deformed, is connected with the two neutrons while the bounded-state energies of ^{14}Be, with the matter r.m.s. radius and a deformation of ^{12}Be, were calculated. The values of ρ and ρ_o in Eq. (3) were approximated using the formulae

$$\rho_0 = \sqrt{j(j+1)} \quad \text{where} \quad j = l_x + \frac{1}{2},$$
$$\rho = \sqrt{m_j(m_j+1)} \quad \text{where} \quad m_j = -j, -j+1, \ldots, j,$$

where the total angular momentum (j) of the valence neutron depends on the core-n radius. With these approximations, ρ and ρ_o were calculated.

In Eq. (9), $Y_{20}(\theta,\phi)$ is taken as

$$Y_{20}(\theta,\phi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2(\theta) - 1). \tag{16}$$

The central Wood-Saxon potential depends on the core's quadrupole deformation parameter β_2 through the radius R in Eq. (9). Throughout the present work, the spin-orbit term was left deformed (with radius $r_o = 1.25$ fm, where $R_o = r_o A^{1/3}$). The radius R_{SO} was made equal to R at any deformation. The diffuseness was fixed to the standard value $a_{ws} = a_{so} = 0.65$ fm, and the calculations were performed for the full range of the deformation parameter $\beta_2 = [-0.7, 0.7]$. The properties of ^{14}Be were calculated for various core deformations β_2 with a fixed spin-orbit depth, V_{so}, at 7.5 MeV and a central Wood-Saxon depth, V_o, at 74 MeV. Table 1 contains all the parameters used in the calculations.

In the present work, the binding energy, matter radius of the three-body system (^{14}Be) and deformation of the core (^{12}Be) were calculated. Normalization was used to determine the above properties by using experimental data as given in Table 2.

The ^{12}Be nucleus was shown to be a stable particle with a ground-state spin and parity of about $J^\pi = 3/2^-$.

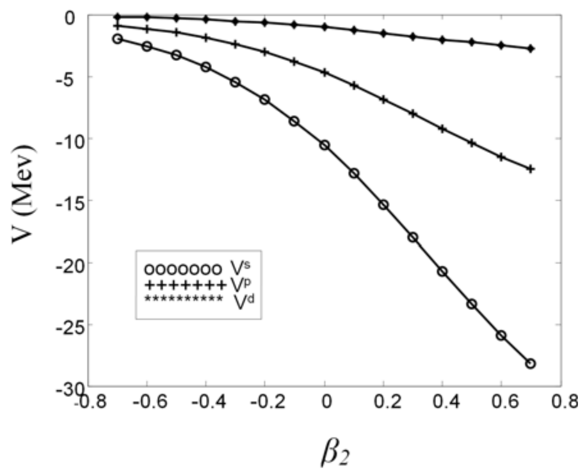

Fig. 3. ^{12}Be-n potential as a function of the deformation.

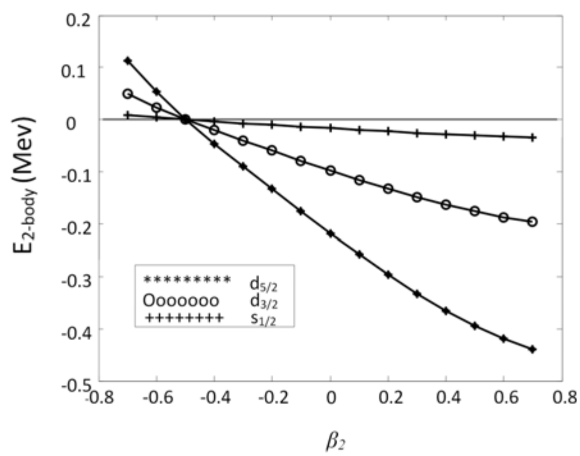

Fig. 4. Energy of the bound state in ^{13}Be as a function of the deformation for the ground state of the $p_{3/2}$- ^{12}Be core.

Table 3. Experimental value of the deformation parameter of ^{12}Be with the theoretical values (present work) of the binding energy and the matter radius of ^{14}Be.

Nucleus	Deformation parameter (β_2) exp	Binding energy (MeV) present work	Matter radius (fm) present work
^{12}Be	0.614 [23]		
^{14}Be		−1.531	5.44

Table 4. Experimental value of the matter radius of ^{14}Be with the theoretical values (this work) of the binding energy of ^{14}Be and the deformation parameter of ^{12}Be.

Nucleus	Matter radius (fm) exp	Deformation parameter (β_2) present work	Binding energy (MeV) present work
^{14}Be	3.36 ± 0.19 [5]		−1.02
^{12}Be		0.1	

Table 5. Experimental value of the binding energy of ^{14}Be with the theoretical values (this work) of the matter radius of ^{14}Be and the deformation parameter of ^{12}Be.

Nucleus	Binding energy (MeV) exp	Deformation parameter (β_2) present work	Matter radius (fm) present work
^{14}Be	−1.34 ± 0.11 [9]		4.5
^{12}Be		0.4	

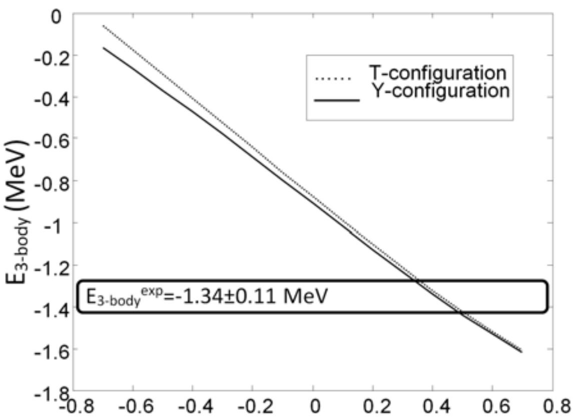

Fig. 5. Energy of the bound state in the ^{14}Be T- configuration and Y-configuration as functions of the deformation for the ground state of the $p_{1/2}$ ^{12}Be core.

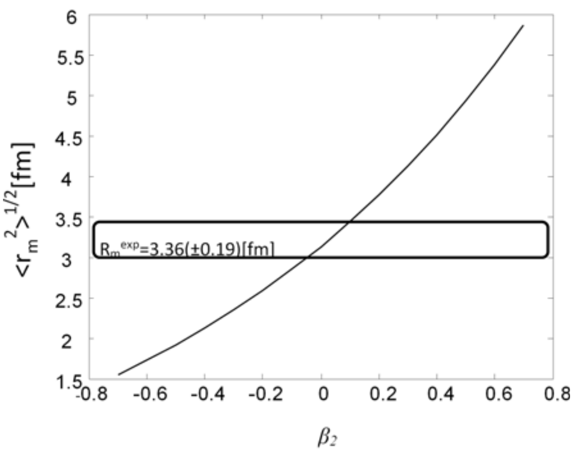

Fig. 6. Rms matter radius of ^{14}Be as a function of the deformation.

Figure 3 shows the relationship between the deformation of the core and the potential between the core and a neutron in ^{13}Be, where the potential increases with the increasing deformation, suggesting that the prolate shape of the core has a high interaction with the neutron halo. Figure 4 indicates that the two-body energy is unbounded especially when the core is oblate.

Through normalization, the experimental value of the deformation of ^{12}Be in Fig. 5 and Fig. 6 was used to determine the values of the binding energy and the matter radius of ^{14}Be, respectively, and the results are given in Table 3. The binding energy and the matter radius were found to be −1.531 MeV and 5.44 fm, respectively. The binding energy was close to the experimental value of −1.48 + 0.14 MeV [8], but the matter radius was higher than the experimental value, confirming that ^{14}Be is a halo nucleus.

Using experimental data for the matter radius of ^{14}Be, we calculated the deformation of the core (^{12}Be) and the binding energy of the ^{14}Be nucleus. The deformation parameter of ^{12}Be was determined, as shown in Fig. 6, while the binding energy from Fig. 5 is given in Table 4. The value of the matter radius of ^{14}Be indicates a small deformation of ^{12}Be at about $\beta_2 = 0.1$. In this respect, the result was more accurate than the experimental data becuase the core consists of 4 protons and 8 neutrons, meaning a closed shell for neutrons and 2 protons outside first closed shell for protons. Thus, the two protons outside the closed shell do not create a strong quadrapole deformation.

Similarly to the above, using the experimental value of the binding energy of ^{14}Be from Table 2 as in Fig. 5, we obtained the value of deformation parameter of ^{12}Be, and we used that value (value of deformation) in Fig. 6 to obtain the matter radius of ^{14}Be, as shown in Table 5. The deformation parameter β_2 was found to be 0.4, and matter radius was at 4.5 fm. This slightly differs with the experimental data, but indicates that ^{14}Be is a halo nucleus.

IV. CONCLUSION

In the present work, a cluster model was used to calculate the binding energy, the root-mean-square (r.m.s.) radius and the effect of deformation of the core on the two-neutron energy of the two-neutron halo nucleus ^{14}Be. ^{14}Be was considered in this work to be a core+n+n, a three-body system that depended on the Jacobi coordinates. The core was considered to be deformed, and we calculated the effect of the deformation on the two-body and the three-body energies.

By using experimental data and through normalization, we calculated the binding energy, the matter radius of the ^{14}Be nucleus and the deformation of ^{12}Be. This calculation showed that the core was deformed (prolate shape), which had a clear effect on the bound states of the two-body and the three-body systems. The weakly bound state of the last two neutrons and the abnormal matter radius of the ^{14}Be nucleus were confirmed in this work, as shown in the results. This further indicates that the cluster model can be successfully used to determine the properties of halo nuclei, especially the ^{14}Be nucleus.

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