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Solution BBM-Burger Equation via Quartic Trigonometric B-spline Approach

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Abstract. In the present work, the new quartic trigonometric B-spline approach based on finite difference scheme is described to solve the one dimensional non-linear equation of (Benjamin – Bona-Mahony- Burger). Dirichlet boundary with the help of applying the von-Neumann stability analysis is also used in this description. While the time derivative part is discretized by using the finite difference scheme. In the space dimension, the quartic trigonometric B-spline is also used as an interpolation function. The execution of this method which is used in the present work showed that the quartic trigonometric B-spline method is a more efficient and effective tool and gives better results according to the comparisons that are made with the precise solution for a different time and some other published numerical methods.

1. Introduction

Nonlinear phenomena have important roles in physical and engineering issues in addition to that in applied mathematics they take different parameters depending on various factors. The mathematics model of propagation of small amplitude long waves in nonlinear dispersive media is described by the following (B-B-M-B) equation [3].

$$\frac{\partial \varpi}{\partial t}+\delta\frac{\partial \varpi}{\partial x}+\varpi\frac{\partial \varpi}{\partial x}-\lambda\frac{\partial \varpi}{\partial xx}-\frac{\partial \varpi}{\partial xxt}=0 \qquad x\in[\psi,\tau]\ ,t\in[0,T] \tag{1}$$

For both δ and λ are constants, and applying the flowed initial and boundary conditions

$$u(x,0)=g(x) \qquad x\in[\psi,\tau] \tag{2}$$

$$u(\psi,t)=u(\tau,t)=0 \tag{3}$$

In the case of physical applications, the dispersive effect of Equation 1 same as (B-B-M-B) equation, while the dissipative effect is similar to Burgers equation that is an alternative sample for Korteweg-de Vries Burger’s equation.

In the previous studies, many researchers are focused to solve same these equations by applying different numerical methods’. Arora et al. [1] solved the BBM-Burger by used the quartic B-spline approach, and he got good results as compared with the exact solution and the other researchers’ solutions. Omrain & Ayadi [2] are used the Crank-Nicolson-type finite difference method to prove the stability and uniqueness of the corresponding approaches by the means of the discrete energy method. Salih et al. [3] presented the cubic trigonometric B-spline approach to solve (B-B-M-B) equations. They have obtained good results with high rate of accuracy and efficiency when they have compared the fining results with those ones obtained by Zarebnia & Parvaz [4], where they have used the cubic B-spline collection method to solve the same equation. Yin & Piao [5] solved the equation by using the quadratic B-Spline finite element method. The predicted numerical solutions give that the scheme is efficient and feasible when compared with the exact solution.

Because of the effectiveness of the B-spline method in a numerical solution for different linear partial and non-linear differential equations, it has great attention on interest in its use in many previous works that are mentioned in the literature. It also has many geometric properties such as local support and the ability to deal with local phenomena, which makes it used in a solution of partial non-linear differential equations effortlessly and easily. The trigonometric B-spline (T-BS) gives more accurate results than the T-BS functions for solving the non-linear initial boundary value problems [6].

In the present work, the (QT-BS) method will represent to obtain the approximate solution of the (B-B-M-B) equation. Zin et al [7] used this approach to obtain a numerical solution to the Korteweg-de Vries equation. The detailed outline of this work deals with the following: In section2, discuss the (QT-BS) method will explain. In section 3, discussing the proposed numerical solution. In section 4, investigate the stability of the method. In section 5, the obtained results of approximate experiments will present and then compared with the obtained results for some of the previous methods. Finally, in section 6, the conclusions of the current work will write down.

QT-BS Method

In this section, we give the QT basis function based on [7, 8].

$$TB_{5,j}(x)=\frac{1}{z}\begin{cases}\eta^4(x_j), & x\in[x_j,x_{j+1})\\ \eta^3(x_j)\sigma(x_{j+2})+\eta^2(x_j)\sigma(x_{j+3}) & x\in[x_{j+1},x_{j+2})\\ +\eta(x_j)\sigma(x_{j+4})\eta^2(x_{j+1})+\sigma(x_{j+5})\eta^3(x_{j+1}), & \\ \eta^2(x_j)\sigma^2(x_{j+3})+\eta(x_j)\sigma(x_{j+4})\eta(x_{j+1})\sigma(x_{j+3}) & \\ +\eta(x_j)\sigma^2(x_{j+4})\eta(x_{j+2})+\sigma(x_{j+5})\eta^2(x_{j+1})\sigma(x_{j+3}) & x\in[x_{j+2},x_{j+3})\\ +\sigma(x_{j+5})\eta(x_{j+1})\sigma(x_{j+4})\eta(x_{j+2})+\sigma^2(x_{j+5})\eta^2(x_{j+2}), & \\ \eta(x_j)\sigma^3(x_{j+4})+\sigma(x_j+5)\eta(x_{j+1})\sigma^2(x_{j+4}) & x\in[x_{j+3},x_{j+4}]\\ +\sigma^2(x_{j+5})\eta(x_{j+2})\sigma(x_{j+4})+\sigma^3(x_{j+5})\eta(x_{j+3}), & \\ \sigma^4(x_{j+5}), & x\in[x_{j+4},x_{j+5}]\\ 0 & otherwise\end{cases} \tag{4}$$

where $\eta(x_j)=\sin\left(\frac{x-x_j}{2}\right)$, $\sigma(x_j)=\sin\left(\frac{x_j-x}{2}\right)$ and $z=\sin\left(\frac{h}{2}\right)\sin\left(h\right)\sin\left(\frac{3h}{2}\right)\sin(2h)$

Due to the B-spline's domestic support characteristics, there are only four non-zero functions, $B_{5,j-4}(x_j), B_{5,j-3}(x_j), B_{5,j-2}(x_j)\, and\, B_{5,j-1}(x_j)$ over subinterval $[x_j,x_{j+1}]$.

Where $h=(\tau-\psi)/n$ and values of $TB_{5,j}(x)$ are tabulated in Table 1

Table 1: Values of $TB_i^4(x)$

x	x_{j-4}	x_{j-3}	x_{j-2}	x_{j-1}	x_j
$TB_{5,j}$	κ_1	κ_2	κ_2	κ_1	0
TB'_j	κ_3	κ_4	κ_4	κ_3	0
TB''_j	κ_5	κ_5	κ_5	κ_5	0

TB_j'''	κ_6	κ_7	κ_7	κ_6	0

Where

$$\kappa_1=\frac{\sin^3(\frac{h}{2})}{\sin(h)\sin(\frac{3h}{2})\sin(2h)}\quad ,\kappa_2=\frac{5+6\cos(h)}{8\cos^2(h)\cos(h)(1+2\cos(h))}$$

$$\kappa_3=\frac{1}{2\sin(h)\cos(h)(1+2\cos(h))}\quad ,\kappa_4=-\frac{1}{\sin(2\mathrm{h})},\kappa_5=\frac{1}{\sin(h)\sin(2h)}$$

$$\kappa_6=\frac{\cos(\frac{h}{2})(1-4\cos(h))}{\sin(h)\sin(\frac{3h}{2})\sin(2h)}\quad ,\quad \kappa_7=\frac{1+2\cos(h)}{2\sin^2(\frac{h}{2})\sin(2h)}$$

3. Numerical Approach

The QT-BS method will be discussed in this section for solving the (B-B-M-B) equation numerically. The solution domain will be divided equally in the knots into subintervals. Our aim for (B-B-M-B) equation is using QT-BS to find an approximate solution like [8, 9].

$$\varpi_j=\sum_{j=-4}^{n-1}C_jTB_{5,j}(x) \tag{5}$$

where $C_j(t)$ is a time dependent unknown to be determined where $j=0,1,2,...,n$. In order to obtain approximate estimates of the solution, the values of $B_{5,j}(x)$ and its derivatives at the node points are required and these derivatives are tabulated by using approximate functions (4) and (5). The values at the U_i^j node and its derivatives up to second order are

$$\begin{cases}(\varpi)_j^i=\kappa_1C_{j-4}^i+\kappa_2C_{j-3}^i+\kappa_2C_{j-2}^i+\kappa_1C_{j-1}^i,\\ (\varpi_x)_j^i=\kappa_3C_{j-4}^i+\kappa_4C_{j-3}^i-\kappa_4C_{j-2}^i-\kappa_3C_{j-1}^i\\ (\varpi_{xx})_j^i=\kappa_5C_{j-4}^i-\kappa_5C_{j-3}^i-\kappa_5C_{j-2}^i+\kappa_5C_{j-1}^i\\ (\varpi_{xxx})_j^i=\kappa_6C_{j-4}^i+\kappa_7C_{j-3}^i-\kappa_7C_{j-2}^i-\kappa_6C_{j-1}^i\end{cases} \tag{6}$$

The approximations for the solutions of (B-B-M-B) equation at t_{j+1} th time level can be as

$$[\frac{(\varpi-\varpi_{xx})^{n+1}-(\varpi-\varpi_{xx})^n}{\Delta t}]+\theta\kappa_j^{n+1}+(1-\theta)\kappa_j^n=0 \tag{7}$$

Where $\kappa_j^n=[-\lambda(\varpi_{xx})_j^{\ n}+(\varpi\varpi_x)_j^{\ n}+\delta(\varpi_x)_j^n]$ and the subscripts n and $n+1$ are successive time levels, $n=0,1,2,..$ and Δt is the time step. By using the following formula:

$$(\varpi\varpi_x)^{n+1}=\varpi^{n+1}\varpi_x^n+\varpi^n\varpi_x^{n+1}-\varpi^{n+1}\varpi_x^n \tag{8}$$

The scheme equation (7) with the placement of the nodal w and derivatives using (6) becomes the following difference equation with variable C_j, $j=-4,...,\mathrm{n}-1$ and noted the crank-Nicolson scheme when $\theta=0.5$

$$\dagger_1C_{j-4}^{n+1}+\dagger_2C_{j-3}^{n+1}+\dagger_3C_{j-2}^{n+1}+\dagger_4C_{j-1}^{n+1}=\text{ƛ}_1C_{j-4}^{n+1}+\text{ƛ}_2C_{j-3}^{n+1}+\text{ƛ}_3C_{j-2}^{n+1}+\text{ƛ}_4C_{j-1}^{n+1} \tag{9}$$

Where

$$
\begin{aligned}
&\dagger_1 = (1+\frac{\Delta t}{2}\varpi_x^n)\kappa_1 + \frac{\Delta t}{2}(\varpi^n+\delta)\kappa_3 - (1+\frac{\alpha\,\Delta t}{2})\kappa_5 \\
&\dagger_2 = (1+\frac{\Delta t}{2}\varpi_x^n)\kappa_2 + \frac{\Delta t}{2}(\varpi^n+\delta)\kappa_4 + (1+\frac{\lambda\,\Delta t}{2})\kappa_5 \\
&\dagger_3 = (1+\frac{\Delta t}{2}\varpi_x^n)\kappa_2 - \frac{\Delta t}{2}(\varpi^n+\delta)\kappa_4 + (1+\frac{\lambda\,\Delta t}{2})\kappa_5 \\
&\dagger_4 = (1+\frac{\Delta t}{2}\varpi_x^n)\kappa_1 - \frac{\Delta t}{2}(\varpi^n+\delta)\kappa_3 - (1-\frac{\lambda\,\Delta t}{2})\kappa_5 \\
&\bar{\lambda}_1 = \kappa_1 - \frac{\Delta t\delta}{2}\kappa_3 + (\frac{\Delta t}{2}\lambda - 1)\kappa_5 \\
&\bar{\lambda}_2 = \kappa_2 - \frac{\Delta t\delta}{2}\kappa_4 + (1-\frac{\Delta t}{2}\lambda)\kappa_5 \\
&\bar{\lambda}_3 = \kappa_2 - \frac{\Delta t\delta}{2}\kappa_4 + (1-\frac{\Delta t}{2}\lambda)\kappa_5 \\
&\bar{\lambda}_4 = \kappa_1 + \frac{\Delta t\delta}{2}\kappa_3 + (\frac{\Delta t}{2}\lambda - 1)\kappa_5
\end{aligned}
\tag{10}
$$

On simplification (9) the system consists of a linear equation $(N+1)$ in $(N+4)$ unknown $C^n = [\mathrm{C}_{j-4}^n, \dots, \mathrm{C}_{N-1}^n]$ at the time level $t = t_{i+1}$. In order to obtain the unique solution to the system, adds three equations obtained from the boundary conditions. The system consists $(N+4)\times(N+4)$ as follows:

$\vartheta_{(N+4)\times(N+4)}\, C_{1xN+4}^{n+1} = \zeta_{(N+4)\times(N+4)}\, C_{1xN+4}^{n}$

From the initial conditions and its derivatives, we will compute initial vector by use it get approximate solution

$$
\begin{cases}
(\varpi_j^0)_x = g^{'}(x_j) & j=0 \\
(\varpi_j^0)_{xx} = g^{''}(x_j) & j=0 \\
\varpi_j^0 = g(x_j) & j=0,1,..N \\
(\varpi_j^0)_x = g^{'}(x_j) & j=N
\end{cases}
\tag{11}
$$

From equation (11), we obtain the system consist $(N+4)\times(N+4)$ which can be solved by Gauss-Jordan elimination method [8].

4. Stability Analysis

In this section, the investigation was performed to stability analysis of the proposed scheme using the von Neumann method. The nonlinear term $\varpi\varpi_x$ is linearized as considering a constant as α in equation (1). Therefore, the equation that is got is the same as of [1].

$$\varpi_t + \delta\varpi_x + \alpha\varpi_x - \lambda\varpi_{xx} - \varpi_{xxt} = 0$$

The linearized form of proposed scheme as following:

$$\hbar_1 C_{j-4}^{n+1} + \hbar_2\, \mathrm{C}_{j-3}^{n+1} + \hbar_3 C_{j-2}^{n+1} + \hbar_4 C_{j-1}^{n+1} = \Omega_1 C_{j-4}^{n+1} + \Omega_2\, \mathrm{C}_{j-3}^{n+1} + \Omega_3 C_{j-2}^{n+1} + \Omega_4 C_{j-1}^{n+1} \tag{12}$$

where

$$\hbar_1 = \kappa_1 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 - (1+\frac{\lambda\,\Delta t}{2})\kappa_5 \ ,$$
$$\hbar_2 = \kappa_2 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_4 + (1+\frac{\lambda\,\Delta t}{2})\kappa_5 \ ,$$
$$\hbar_3 = \kappa_2 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_4 + (1+\frac{\lambda\,\Delta t}{2})\kappa_5 \ ,$$
$$\hbar_4 = \kappa_1 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 - (1+\frac{\lambda\,\Delta t}{2})\kappa_5 \ ,$$
$$\Omega_1 = \kappa_1 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 + (-1+\frac{\lambda\,\Delta t}{2})\kappa_5 \ ,$$
$$\Omega_2 = \kappa_2 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_4 + (-1+\frac{\lambda\,\Delta t}{2})\kappa_5 ,$$
$$\Omega_3 = \kappa_2 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_4 + (1-\frac{\lambda\,\Delta t}{2})\kappa_5 \ ,$$
$$\Omega_4 = \kappa_1 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 + (-1+\frac{\lambda\,\Delta t}{2})\kappa_5 ,$$

Substitution of $D_j^n = \delta^n \exp(imh\eta)$ into equation (12) where $= \sqrt{-1}$, after simplifying the equation (12), we get

$$\xi = \frac{A_1 + iB_1}{A_2 + iB_2} \tag{13}$$

where

$$A_1 = (\kappa_2 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_4) + (\kappa_1 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 +)(\cos(2\eta h))$$
$$+((\kappa_2 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_4 + 2(-1+\frac{\lambda\,\Delta t}{2})\kappa_5) + (\kappa_1 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 +))(\cos(\eta h))$$
$$B_1 = (((\kappa_1 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_3) - (\kappa_2 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_4))(\sin(\eta h))$$
$$-(\kappa_1 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 + (-1+\frac{\lambda\,\Delta t}{2})\kappa_5)(\sin(2\eta h))$$
$$A_2 = (\kappa_2 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_4) + ((\kappa_1 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_3)(\cos(2\eta h)))$$
$$+((\kappa_2 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_4) + (\kappa_1 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_3))(\cos(\eta h))$$
$$B_2 = (\kappa_1 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 - 2(1+\frac{\lambda\,\Delta t}{2})\kappa_5) - \kappa_2 - \frac{\Delta t}{2}(\alpha+\delta)\kappa_4))(\sin(\eta h))$$
$$-(\kappa_1 + \frac{\Delta t}{2}(\alpha+\delta)\kappa_3 - (1+\frac{\lambda\,\Delta t}{2})\kappa_5)(\sin(2\eta h))$$

simplifying the equation (13), we obtain

$$0 \le \frac{g_5}{-2+g_5} \le \cos(\eta h) - 1$$

So the linear numerical diagram of the (B-B-M-B) equation is unconditionally stable.

5. Test problem and discussion

The efficiency and accurateness of the proposed method can be illustrated in the following two examples in this section with L_∞ and L_2 being the error criteria that are computed by $L_\infty = \max_i \left|\varpi_j^{exact} - \varpi_i^{num}\right|$ and $L_2 = \sqrt{h\left(\sum_i^n \left|\varpi_j^{exact} - \varpi_j^{num}\right|^2\right)}$

The conservation laws apply on equation (1) as follows [9].

$$C_1 = \int_\psi^\tau \varpi(x,t)dx,\ C_2 = \int_\psi^\tau \varpi(x,t)^2 dx,\ C_3 = \int_\psi^\tau [\varpi(x,t)^2 + \frac{1}{3}\varpi(x,t)^3]dx$$

Where C_1, C_2, C_3 match mass, momentum and energy, respectively.

Then for validation, the numerical solutions obtained by testing the QT-BS method of equation (B-B-M-B) (1) are compared with the precise solutions and with the results of numerical methods that have been found in the literature. It was noted that numerical results are expected at different time scales.

Example 5.1

Consider the (B-B-M-B) problem [3] with $\delta = 1.0$ and $\lambda = 1.0$,

$$\varpi_t + \varpi_x + \varpi\varpi_x - \varpi_{xxt} - \varpi_{xx} = 0 \qquad x \in [-12,12]\ , t \in [0,T]$$

with initial condition

$$\varpi(x,0) = g(x) = \sec h^2\left(x/4\right),\ -12 \le x \le 12$$

boundary conditions as follows:

$$\varpi(-12,t) = \sec h^2\left(-3 - t/3\right), \varpi(12,t) = \sec h^2\left(3 - t/3\right)$$

The precise solution of this problem is $\varpi(x,t) = \sec h^2\left(x/4 - t/3\right)$. The QT-BS method is employed to calculate the numerical solutions of this problem. For the purpose of comparison, the numerical results obtained in this paper are found to be more accurate as compared to CuBS [4]. The absolute errors at different time levels with $h = 1/200$ and $\Delta t = 0.01$ at different time levels in Table 2 and Table 3. Figuer1 shows a graph of approximate space-time and a good agreement with their precise solutions at $0.2 \le t \le 1$. Figuer2 depicts the error at T=2 and A=200.

Table 2: Absolute errors for Example5.1

x	Present method			CuBS [4]		
	$t=0.2$	$t=0.5$	$t=0.7$	$t=0.2$	$t=0.5$	$t=0.7$
-12	2.1164-16	6.9389E-17	1.1015E-16	3.33E-11	2.23E-10	3.33E-11
-10	1.2931E-03	2.6551E-03	3.2522E-03	2.29E-02	1.98E-02	1.79E-02
-5	6.0445E-03	1.4880E-02	2.0122E-02	2.56E-01	2.24E-01	2.06E-01
0	1.7425E-02	3.9343E-02	4.9853E-02	9.78E-01	9.33E-01	8.97E-01
5	5.816E-03	1.2295E-02	1.4373E-02	3.19E-01	3.80E-01	4.23E-01
10	1.6463E-03	4.8198E-03	7.4331E-03	3.04E-02	3.97E-02	4.72E-02
12	0	0	3.4694E-18	2.00E-10	6.66E-11	2.66E-10

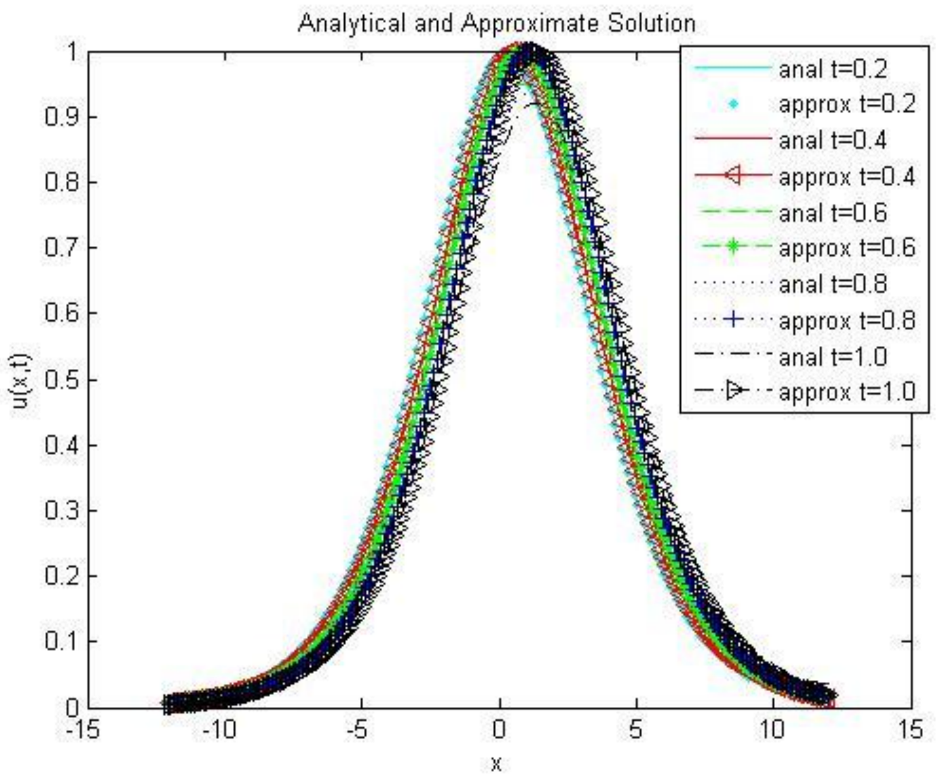
Table 3: Absolute errors for Example5.1

x	Present method			CuBS [4]		
	$t=1.0$	$t=1.5$	$t=2.0$	$t=1.0$	$t=1.5$	$t=2.0$
-12	1.06E-16	4.1373E-16	1.3097E-16	1.33E-11	1.26E-10	8.66E-11

-10	6.9002E-04	4.0285E-03	3.7824E-03	1.54E-02	1.19E-02	9.16E-03
-5	3.0994E-03	3.5237E-02	4.0018E-02	1.82E-01	1.49E-01	1.23E-01
0	8.9008E-03	5.2923E-02	2.8860E-02	8.38E-01	7.33E-01	6.31E-01
5	3.0181E-03	5.2377E-03	4.5119E-2	4.87E-01	5.89E-01	6.76E-01
10	7.7837E-04	2.1844E-02	3.3271E-02	6.05E-02	8.86E-02	1.25E-01
12	0	0	0	1.13E-09	-3.33E-10	-1.01E-16

Table 4: Invariants for N=200 at different time

t	C_1	C_2	C_3
0.2	7.9572	5.2432	5.1590
0.4	7.9515	5.1574	5.0775
0.8	7.9356	4.9472	4.9248
1.0	7.9251	4.9220	4.8530
1.5	7.8901	4.767	4.6850
2.0	7.8396	4.5860	4.5302



Figuer.1. Approximate solution and exact solution at different value to time level $0.2 \le t \le 1$**. problem5.1**.

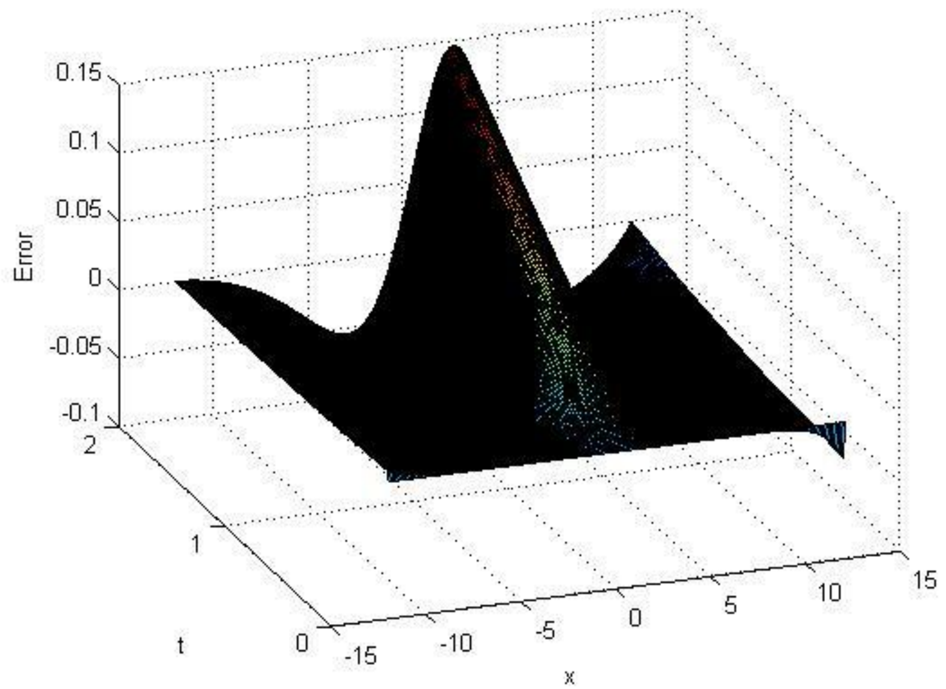


Figure.2.Erroer plot at T=2 and N=200 of problem 5.1

Example 5.2
Consider the B-B-M- B inhomogeneous problem [1] with $\delta = 1.0$ and $\lambda = 1.0$, $x \in [0, \pi]$
$$\varpi_t + \varpi_x + \varpi\varpi_x - \varpi_{xxt} - \varpi_{xx} = e^{(-t)}[\cos(x) - \sin(x) + 0.5e^{(-t)}\sin(2x)$$
initial conditions
$$\varpi(x,0) = \sin(x)$$

The boundary conditions are taken from the precise solution $\varpi(x,t) = e^{(-t)}\sin(x)$. The result for L_∞ and L_2 errors for N=121 and different value to T in table 5 are comparison with [1]. The numerical results obtained in this paper are found to be more accurate. In table 6, we take different values to N and T=10 with $\Delta t = 0.01$ and the result is comparison with [1]. We illustrate and show that the QT-BS is more accurate than the method suggested by Arora and Omarani. Figure 3 shows approximate and a good agreement with their exact solutions T.

Table 5: L_2 and L_∞ Errors at N=121 and different time-levelsa for Example5.2

Error	**present method**				**QuBS [1]**	
	t=1	t=2	t=4	t=10	$t=2$	t=10
L_∞	1.25801E-3	1.10980E-3	3.83346E-4	2.89577E-6	2.83801E-3	4.05946E-6
L_2	1.25667E-3	1.1099E-3	3.78937E-4	2.64010E-6	1.72970E-3	4.07835E-6

Table 6: L_2 Error at T=10 and different value for N Example5.2

N	present method	QuBs[1]	[2]
10	1.0275E-4	1,7147E-4	2.200E-2
20	2.5540E-5	5.6341E-5	5.000E-3

80	2.6413E-6	7.2635E-6	3.329E-4
320	2.6826E-6	8.163E-7	2.076E-5

Table 7: Invariants for N=121 at different time

T	C_1	C_2	C_3
0.2	1.6370	1.0529	0.6954
0.4	1.3410	0.7065	0.4650
0.6	1.0985	0.4741	0.3090
0.8	0.8999	0.3182	0.2038
1.0	0.7372	0.2135	0.1332

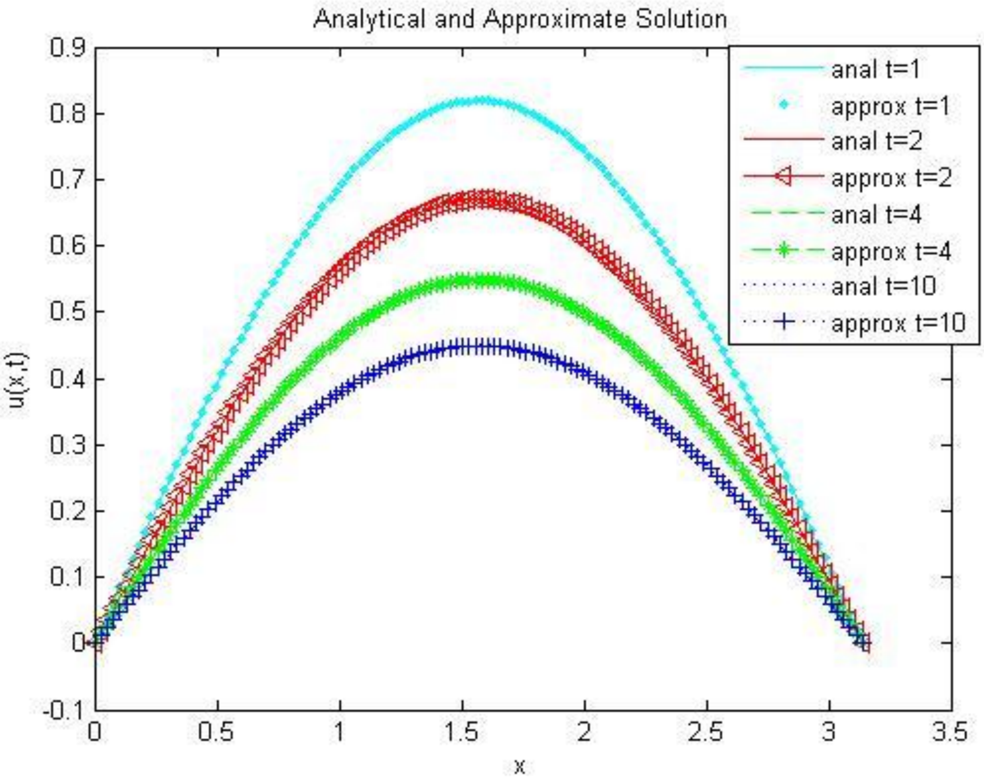


Figure.3. Approximate solution and exact solution at different value to T.problem5.2.

6. Conclusions

The quartic trigonometric B-spline method is proposed to solve the BBM-Berger equation in the present work. Applying this method is occurred by taking two examples and then compare the results with cubic B-spline method as in example one. The obtained results from the proposed method are given more accuracy than the cubic B-spline method but in example two the comparison is made with both quartic B-spline and finite difference methods. Then, the founded results from the proposed methods give high accuracy and efficiency than both of quartic B-spline and finite difference methods. Finally, the obtained results from the proposed method are validated with the von Neumann method for the purpose of checking the stability and getting unconditionally stable.

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