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Solution of Modified Equal Width Equation Using Quartic Trigonometric-Spline Method

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Abstract.

Using a quartic trigonometric B-spline (QTB-S) scheme, the numerical solution is obtained for modified equal width equation (MEW eq.). The approach based on finite difference scheme with the help of Crank-Nicolson formulation. The finite difference scheme is used for time integration and QTB-S function for space integration. Performance and accuracy of the scheme is validated through testing two problems by using conserved laws and L_∞ and L_2 error norms.

Key word: Quartic trigonometric B-spline (QTB-S), finite difference scheme, modified equal width equation (MEW eq.).

1. Introduction

The MEW eq. single wave is considered here has normalized form.

$$\frac{\partial z}{\partial t} + \psi z^2 \frac{\partial z}{\partial x} - \mu \frac{\partial z}{\partial x t} = 0 \quad x \in [c, d], t \in [0, T] \quad (1)$$

With collocation boundary conditions (BCs)

$$z(c, t) = 0, z(d, t) = 0$$



$$\begin{aligned} z_x(c, t) = z_x(d, t) = 0, \\ z_{xx}(c, t) = z_{xx}(d, t) = 0, \end{aligned} \tag{2}$$

with initial condition (IC)

$$z(x, t) = f(x) \quad , c \leq x \leq d \tag{3}$$

The μ is a positive parameter and ψ is an arbitrary constant, $f(x)$ is a localized disturbance inside the interval $[c, d]$. Islam, S. et al. [1]. Solved the MEW eq. by using quartic B-spline collocation method, and obtained a high efficient and accuracy through comparing their solution with the exact solution. Bulent Saka [2] used quintic B-spline collection schemes for numerical solution for the MEW eq. through an algorithm which is based on Crank-Nicolson formula. Karakoç, S. B. G, et al. [3]. Solved the MEW eq. by using lumped galerkin method. The numerical results they found are equivalent with the exact solution through computing the numerical conserved laws and L_∞ and L_2 error norms. Turabi G. et al. [4] obtained a remarkably successful numerical solution for the (MEW) equation by applying septic b-spline finite element method on the motion of a single solitary wave. Evans, D. J. et al. [5]. Used quadratic B-spline method for solving MEW eq. Esen, A. et al. [6]. Solved MEW eq. by linerizing numerical method based on finite difference scheme. Zaki, S. I. [7] solved MEW eq. numerically via a petrow-galerkin method by using qunitic b-spline finite elements.

2- QTB-S Method

In this section, Firstly, QT basis function is defined as follows.

$$TB^5(x) = \begin{cases} z_p^4(x), & x \in [x_p, x_{p+1}) \\ z_p^3(x)k_p(x_{p+2}) + z_p^2(x)k_p(x_{p+5}) \\ + z_p(x)k_p(x_{p+2})z_p^2(x) + k_p(x_{p+5})z_p^3(x), & x \in [x_{p+1}, x_{p+2}) \\ z_p^2(x)k_p^2(x_{p+4}) + z_p(x)k_p(x_{p+5})z_p(x)k_p(x_{p+1}) \\ + z_p(x)k_p^2(x_{p+3})z_p(x_{p+4}) + k_p(x_{p+5})z_p^2(x_{p+3})k_p(x_{p+1}) \\ + k_p(x_{p+5})z_p(x_{p+1})k_p(x_{p+4})z_p(x_{p+2}) + k_p^2(x_{p+5})z_p^2(x_{p+2}), & x \in [x_{p+2}, x_{p+3}) \\ z_p(x)k_p^3(x_{p+4}) + k_p(x_{p+5})z_p(x_{p+1})k_p^2(x_{p+4}), & x \in [x_{p+3}, x_{p+4}) \\ + k_p^2(x_{p+5})z_p(x_{p+2})k_p(x_{p+4}) + k_p^3(x_{p+5})z_p(x_{p+3}), & x \in [x_{p+4}, x_{p+5}) \\ k_p^4(x_{p+5}), & \\ |0 & otherwise \end{cases}$$

where $z_p(x) = \sin\left(\frac{x - x_p}{2}\right)$, $k_p(x) = \sin\left(\frac{x_p - x}{2}\right)$, $w = \sin(h/2)\sin(h)\sin(3h/2)\sin(2h)$

Due to local support properties of B-spline ,there are only four nonzero basis functions,

$B_{5,p-4}(x_p), B_{5,p-3}(x_p), B_{5,p-2}(x_p)$ and $B_{5,p-1}(x_p)$ included over subinterval $[x_p, x_{p+1}]$.

Where $h = (c - d)/n$ and values of $TB_p^5(x)$ and its derivatives at nodal points are required and these derivatives are tabulated in Table 1. [7]. Secondly, It is discussed the QTB-S method for solving the MEW eq. (1).

Table 1: Values of $TB_p^4(x)$ and its derivatives

| X | x_{p-4} | x_{p-3} | x_{p-2} | x_{p-1} | x_p |
|-----------|-----------|-----------|-----------|-----------|-------|
| TB_p | τ_1 | τ_2 | τ_2 | τ_1 | 0 |
| TB_p' | τ_3 | τ_4 | τ_4 | τ_3 | 0 |
| TB_p'' | τ_5 | τ_5 | τ_5 | τ_5 | 0 |
| TB_p''' | τ_6 | τ_7 | τ_7 | τ_6 | 0 |

where

$$\tau_1 = \frac{\sin^3(h/2)}{\sin(2h)\sin(h)\sin(3h/2)}, \tau_2 = \frac{5 + 6\cos(h)}{8\cos^2(h)(1 + 2\cos(h))\cos(h)},$$

$$\tau_3 = \frac{1}{2\sin(h)(1 + 2\cos(h))\cos(h)}, \tau_4 = -\frac{1}{\sin(2h)}, \tau_5 = \frac{1}{\sin(2h)\sin(h)}$$

$$\tau_6 = \frac{\cos(h/2)(1 - 4\cos(h))}{\sin(h)\sin(3h/2)\sin(2h)}, \tau_7 = \frac{1 + 2\cos(h)}{2\sin^2(h/2)\sin(2h)}$$

The solution domain $c \leq x \leq d$ is equally divided by knots x_p into n subintervals $[x_p, x_{p+1}]$, $p = 0, 1, 2, \dots, n - 1$ where $c = x_0 < x_1 < \dots < x_n = d$. Our approach for MEW eq. using QTB-S is to seek an approximate solution as [8]:

$$Z_p^i = \sum_{k=p-3}^{p+1} D_k^i TB_k^4(x) \tag{5}$$

where $D_p(t)$ is a time dependent unknown to be determined where $p = 0, 1, 2, \dots, n$. So as to obtain access to the solution, the values of $B_{5,p}(x)$ and its derivatives at nodal points are required and these derivatives are tabulated using approximate functions (4) and (5), the values at the knots of Z_p^i and their derivatives up to three orders are

$$\begin{cases} (z)^i = \tau D^i + \tau D^i + \tau D^i + \tau D^i, \\ \left(\begin{matrix} z \\ z \\ z \\ z \end{matrix} \right)_{x_p}^i = \begin{matrix} \tau D^{p-4} & \tau D^{p-3} & -\tau D^{p-2} & -\tau D^{p-1} \\ \tau D^{p-4} & -\tau D^{p-3} & -\tau D^{p-2} & \tau D^{p-1} \end{matrix} \\ \left(\begin{matrix} z \\ z \\ z \\ z \end{matrix} \right)_{xx}^i = \begin{matrix} \tau D^{p-4} & \tau D^{p-3} & -\tau D^{p-2} & -\tau D^{p-1} \\ \tau D^{p-4} & \tau D^{p-3} & -\tau D^{p-2} & -\tau D^{p-1} \end{matrix} \\ \left(\begin{matrix} z \\ z \\ z \\ z \end{matrix} \right)_{xxx}^i = \begin{matrix} \tau D^{p-4} & \tau D^{p-3} & -\tau D^{p-2} & -\tau D^{p-1} \\ \tau D^{p-4} & \tau D^{p-3} & -\tau D^{p-2} & -\tau D^{p-1} \end{matrix} \end{cases} \tag{6}$$

The approximations for the solutions of MEW eq. (1) at t_{p+1} th time level can be given as

$$\left[\frac{(z - z_x)^{n+1} - (z - z_x)^n}{\Delta t} + \psi \left[\frac{(z^2 z_x)^{n+1} + (z^2 z_x)^n}{2} \right] \right] = 0 \tag{7}$$

where $n = 0, 1, \dots$ and Δt is the time step. The nonlinear term in eq. (7) approximated using the Taylor series [1]:

$$(z^2 z_x)^{n+1} = (z^n)^2 z_x^{n+1} + 2z^n z_x^n z_x^{n+1} - 2(z^n)^2 z_x^n \tag{8}$$

By using the values of z and derivatives it in equation (7) get the following difference equation with variable $D_p, p = -3, \dots, n-1$ and a Crank-Nicolson when $\theta = \frac{1}{2}$

$$a_1 D^{n+1}_{p-4} + a_2 D^{n+1}_{p-3} + a_3 D^{n+1}_{p-2} + a_4 D^{n+1}_{p-1} = b_1 D^n_{p-4} + b_2 D^n_{p-3} + b_3 D^n_{p-2} + b_4 D^n_{p-1} \tag{9}$$

where

$$\begin{aligned} a_1 &= (2(1 + \Delta t \psi z^n z_x^n) \tau_1 + \Delta t \psi (z^n)^2 \tau_3 - 2\mu \tau_5), & b_1 &= 2(\tau_1 + \frac{\Delta t \psi (z^n)^2 \tau_3}{2} - \mu \tau_5) \\ a_2 &= (2(1 + \Delta t \psi z^n z_x^n) \tau_2 + \Delta t \psi (z^n)^2 \tau_4 + 2\mu \tau_5), & b_2 &= 2(\tau_2 + \frac{\Delta t \psi (z^n)^2 \tau_4}{2} + \mu \tau_5) \\ a_3 &= (2(1 + \Delta t \psi z^n z_x^n) \tau_2 - \Delta t \psi (z^n)^2 \tau_4 + 2\mu \tau_5), & b_3 &= (\tau_2 + \frac{\Delta t \psi (z^n)^2 \tau_4}{2} + \mu \tau_5) \\ a_4 &= (2(1 + \Delta t \psi z^n z_x^n) \tau_1 - \Delta t \psi (z^n)^2 \tau_3 - 2\mu \tau_5), & b_4 &= (\tau_1 - \frac{\Delta t \psi (z^n)^2 \tau_3}{2} - \mu \tau_5) \end{aligned} \tag{10}$$

when simplifying (9) the system, It will consist of $(F+1)$ linear equation in $(F + 4)$ unknown $D^i = [D^n_{p-3}, \dots, D^n_{F-1}]$ at the time level $t = t_{p+1}$. In order to get the unique solution to the system, these three equations are added by getting it from boundary conditions; the system consists $(F + 4) \times (F + 4)$ in the below form:

$$U_{(F+4) \times (F+4)} D^{n+1}_{1 \times F+4} = O_{(F+4) \times (F+4)} D^n_{1 \times F+4}$$

From the initial conditions and its derivatives, It is computed the initial vector by using this approximate solution

$$\left\{ \begin{aligned} (z^0) &= f'(x) & p &= 0 \\ (z^0) &= f''(x) & p &= 0 \\ (z^0) &= f(x) & p &= 0, 1, \dots, F \\ (z^0) &= f'(x) & p &= F \end{aligned} \right. \tag{11}$$

From eq. (11) obtain system consist $(F + 4) \times (F + 4)$ of the form

$$AS^0 = d$$

3. Numerical Example and discussion

In this section, two examples are given in this section with and error norms are

$$\text{calculated by } L_{\infty} = \max_i |z_i - Z_i| \text{ and } L_2 = \sqrt{h \left(\sum_i^n |z_i - Z_i|^2 \right)} .$$

The conservation laws apply on equation (1) as follows [11]

$$C_3 = \int_c^d [z(x, t)^2 + \frac{1}{3}z(x, t)^3] dx,$$

$$C_2 = \int_c^d z(x, t)^2 dx,$$

$$C_1 = \int_c^d z(x, t) dx,$$

Where C_3, C_2, C_1 correspond mass, and energy, momentum and mass respectively.

Then, by comparing the numerical solutions, it is obtained by testing the QTB-S method for MEW eq. (1) with the exact solutions and those numerical methods which were existing in previous literature. Numerical results are computed at different time levels.

Example 3.1

The IC for the MEW eq. is given by $z(x, 0) = B \operatorname{sech}(k(x - x_0))$ and BCs $z(0, t) = 0, z(80, t) = 0$ taken from the exact solution $z(x, t) = B \operatorname{sech}(k(x - ct - x_0))$ with the parameters c is the wave velocity, $c = \frac{\psi B^2}{6}, k = \sqrt{\frac{1}{\mu}}$ and $c=0, d=80$. Table 2. Shows the

error norms and errors at different time levels and C_1, C_2, C_3 with $\Delta t = 0.2$, $\Delta x = 0.1, B=0.25$, and $x_0 = 30$ compare the result that get with Saka, B. [2] and Evans [6] and $t = 20$ found more accurate for quartic trigonometric B-spline. Figures 1 shows the single solitary wave solutions at $t = 0, 20$ and $B=0.25$. Figures 2 at different value to $B=0.25, 0.5, 0.75$ and 1 . Shows the single solitary wave solutions at $t = 20$.

Table 2 Error for single at different times.

| T | Δt | C_1 | C_2 | C_3 | $L_{\infty} \times 10^4$ | $L_2 \times 10^4$ |
|----|------------|----------|----------|----------|--------------------------|-------------------|
| 0 | 0.2 | 0.785398 | 0.166666 | 0.005208 | 0 | 0 |
| 5 | | 0.785417 | 0.166709 | 0.005210 | 0.005687 | 0.008376 |
| 10 | | 0.784639 | 0.166478 | 0.005194 | 0.011401 | 0.016708 |
| 15 | | 0.783098 | 0.165993 | 0.005162 | 0.017107 | 0.025068 |
| 20 | | 0.780851 | 0.165285 | 0.005115 | 0.022814 | 0.033396 |

| | | | | | | |
|--------------|------|----------|----------|----------|-----------|----------|
| 20[2] | | 0.781668 | 0.166434 | 0.005194 | 1.744330 | 1.958879 |
| 20[6] | | 0.785398 | 0.166474 | 0.005208 | 2.576377 | 2.701647 |
| 0 | 0.05 | 0.785398 | 0.166666 | 0.005208 | 0 | 0 |
| 5 | | 0.785002 | 0.166573 | 0.005202 | 0.0056111 | 0.008384 |
| 10 | | 0.784008 | 0.166207 | 0.005177 | 0.0112608 | 0.016765 |
| 15 | | 0.782158 | 0.165590 | 0.005137 | 0.0169151 | 0.025148 |
| 20 | | 0.779612 | 0.164751 | 0.005082 | 0.0225604 | 0.033538 |
| 20[6] | | 0.785398 | 0.166474 | 0.005208 | 2.569972 | 2.692812 |

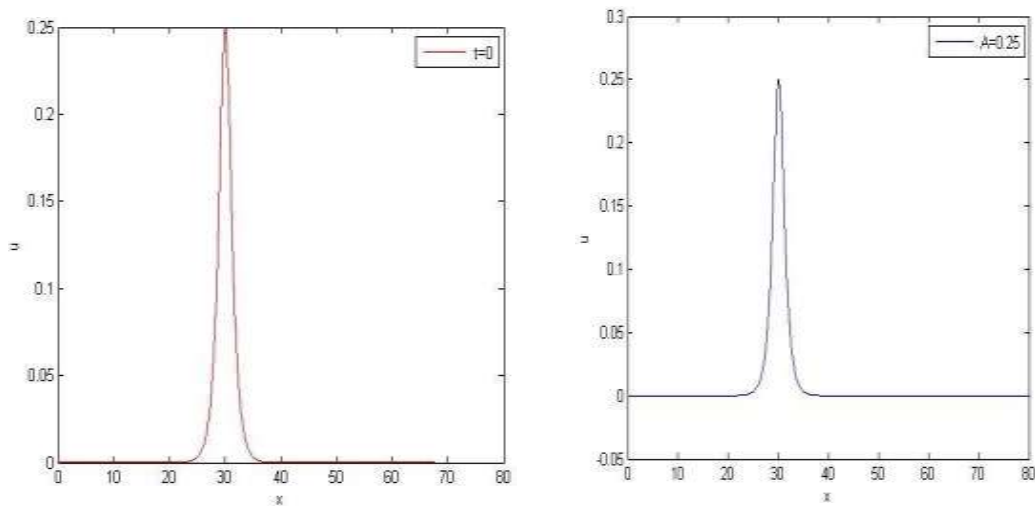


Figure 1 .single solitary solutions at $t = 0, 20$ and $B=0.25$.

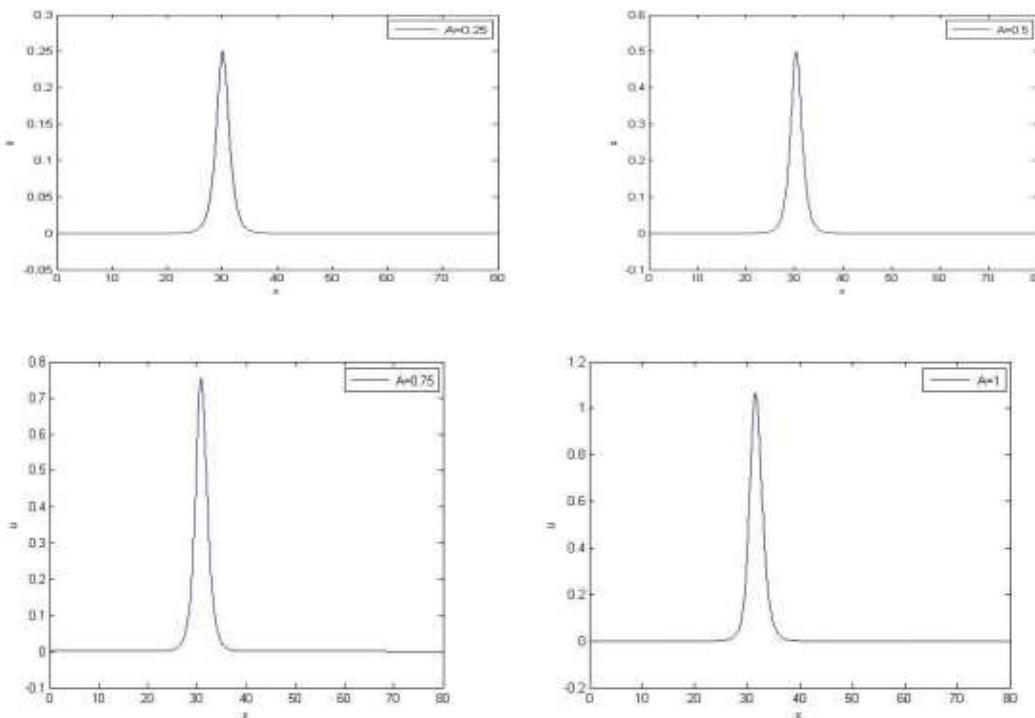


Figure 2 .Single solitary solutions at $t = 20$ for different values of B.

Example 3.2.

The exact solution $z_t + 3z^2z_x - z_{xxt} = 0$

With IC

$$z(x, 0) = B \operatorname{sech}(k(x - x_0))$$

And BCs are obtained from exact solution with parameter $c=0$, $d=70$, with $\Delta t = 0.05$, $\Delta x = 0.1$, $x_0 = 30$ table 2. Shows the error norms and errors at different time and various value to B and calculating conservation properties of the MEW eq. To mass, momentum and energy C_1 , C_2 , C_3 .

Table 3 Error for single at different times and various values to A.

| t | B | C_1 | C_2 | C_3 | L_2 | L_∞ |
|-------|------|----------|----------|----------|----------|------------|
| 0 | 0.25 | 0.785398 | 0.166666 | 0.005208 | 0.0 | 0.0 |
| 5 | | 0.785100 | 0.166573 | 0.005202 | 0.008384 | 0.005611 |
| 10 | | 0.784008 | 0.166207 | 0.005177 | 0.016765 | 0.011260 |
| 15 | | 0.782158 | 0.169151 | 0.005137 | 0.025148 | 0.165590 |
| 20 | | 0.796125 | 0.164751 | 0.005082 | 0.033538 | 0.022560 |
| 20[2] | | 0.785398 | 0.166667 | 0.005208 | 0.275368 | 0.328530 |
| 20[5] | | 0.784954 | 0.166476 | 0.005199 | 0.290516 | 0.249825 |
| 0 | 0.5 | 1.570796 | 0.666667 | 0.083333 | 0.0 | 0.0 |
| 5 | | 1.561702 | 0.661130 | 0.081850 | 0.066792 | 0.045629 |
| 10 | | 1.532771 | 0.643763 | 0.077356 | 0.133671 | 0.089389 |
| 15 | | 1.494190 | 0.623506 | 0.072255 | 0.202022 | 0.130250 |
| 20 | | 1.453792 | 0.605354 | 0.067785 | 0.272280 | 0.175605 |
| 20[2] | | 1.570796 | 0.666666 | 0.083333 | 0.640123 | 0.920874 |
| 0 | 1.0 | 3.141592 | 2.666666 | 1.333333 | 0.0 | 0.0 |
| 5 | | 2.975453 | 2.540387 | 1.193472 | 0.517206 | 0.328195 |
| 10 | | 2.772068 | 2.451067 | 1.092882 | 1.027194 | 0.615067 |
| 15 | | 2.696522 | 2.462922 | 1.096435 | 1.422060 | 0.809032 |
| 20 | | 2.697919 | 2.529398 | 1.155016 | 1.677453 | 0.909200 |
| 20[4] | | 3.141592 | 2.666666 | 1.333333 | 1.446540 | 0.836017 |

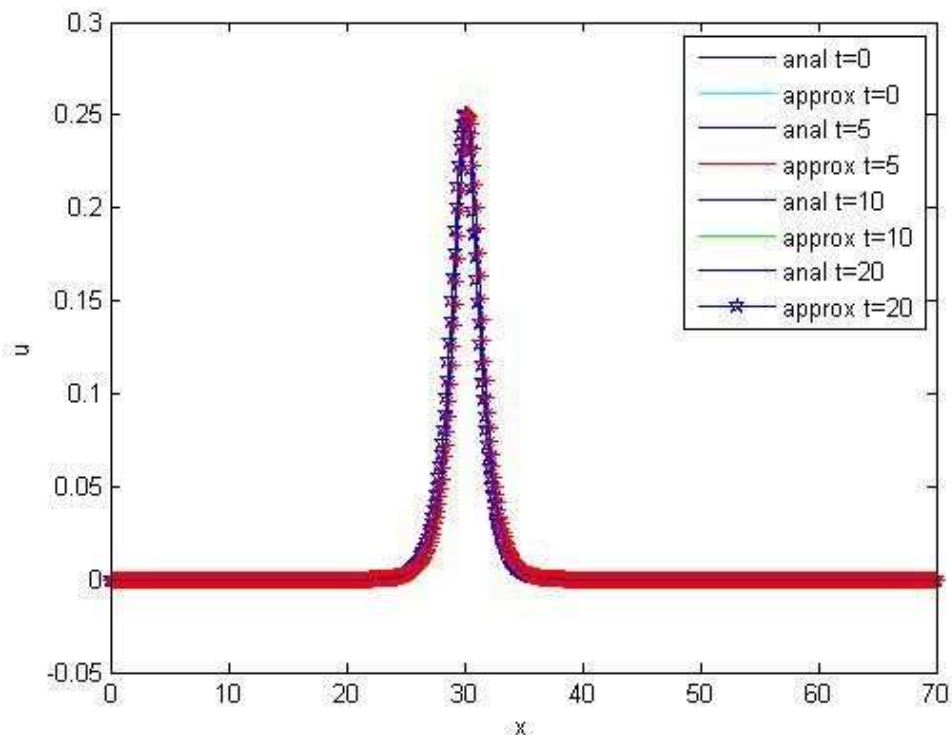


Figure 3. Approximate solution for Single solitary wave and exact solution at $0 \leq t \leq 20$ and $B=0.25$

4- Conclusion

QTB-S method are presented for solving MEW equation. Two test problems are used to show efficiency and accuracy performance of the method through comparing the results that got it and some other published numerical methods. This method is obtained to be unconditionally stable by Applying the von Neumann method.

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