Mathematics and Statistics Journal

ISSN-2077-4591
Journal home page: http://www.iwnest.com/MSJ/
2016. 2(2): 1-9

RSEARCH ARTICLE

Using Cubic Trigonometric B-Spline Method to Solve BBM-Burger Equation

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Received 3 March 2016; accepted 26 May 2016; published 16 June 2016

A B S T R A C T

CubicTrigonometric B-spline method is used to solve Benjamin-Bona-Mahony-Burger equation (BBM-Burger) with appropriate initial and boundary conditions, the method based on Crank-Nicolson scheme for time integration and cubic trigonometric b-spline for space integration. The stability of method described by using von Neumann (Fourier) method. Comparisons between exact and suggested solution is used to illustrate the accuracy and efficiency of suggested method.

Keywords: Cubic Trigonometric B-spline method, Benjamin-Bona-Mahony-Burger equation (BBM-Burger, Crank-Nicolson scheme.

INTRODUCTION

Phenomena nonlinear play important roles in engineering problems, physics and applied mathematics, in which each parameter varies depending on different factors. Solving nonlinear equations may guide researchers to know the described process deeply and sometimes to know some facts which are not clearly understood through joint observations [1]. In this paper,collocation method represented by Trigonometric Cubic B-spline is applied to get the numerical solution of BBM-Burgers equation:

$$v_t + \delta v_x + v v_x - v_{xxt} - \lambda v_{xx} = 0 \qquad x \in [a,b]\ , t \in [0,T] \tag{1}$$

with the initial and boundary conditions

$$v(x,0) = f(x) \qquad x \in [a,b] \tag{2}$$

$$v(a,t) = v(b,t) = 0 \tag{3}$$

where δ and λ are constants. In the physical case, the dissipative effect is an alternative model for the Korteweg-de Vries-Burgers (KdVB) equation and which the same as the Burgers equation, while the dispersive effect of (1) is the same as the Benjamin-Bona-Mahony equation [2].During the few years past, many numerical methods have been used by research to solve these equations. Yong-Xue *et al.* [3] used the quadratic B-spline finite element scheme in order to gain approximate solution for the BBM-Burgers equation. Tari, *et al.* [1] used Variational Iteration method (VIM) and Homotopy Pertubation method (HPM) in order to get approximate explicit solution for BBM-Burgers equation. Arora *et al.*[4] used Quartic B-spline collocation scheme to solve

To Cite This Article: Hamad Salih, Ridzuan Yahya, Tawfiq., Using Cubic Trigonometric B-Spline Method to Solve BBM-Burger Equation. **Mathematics and Statistics Journal**, 2(2): 1-9, 2016

this equation .The scheme is based on Crank-Nicolson formulation for time integration and quartic b-spline basis function for space integration. Zarebnia *et al*. in [5] used cubic B-spline collection method for numerical solution of the BBM-Burgers equation. Kamel *et al* [6] used adomian decomposition method (ADM) in order to gain approximate solution of this equation. Fakhari [7] find approximate explicit solution of BBM-Burgers equation by homotopy analysis method (HAM). Omrani *et al*. in [8] used Crank – Nicolson type finite difference method to solve this equation. Ganji *et al*. in [9] used Exp-function method to solve this equation. In this paper, we suggest the Cubic Trigonometric B-spline method to solve nonlinear BBM-Burgers equation; also, we study Von- Neumann stability analysis of this equation.

Cubic Trigonometric B-Spline Collocation Method:

In this section, we define the cubic trigonometric basis function as follows.

$$TB_i^4(x)=\frac{1}{z}\begin{cases} a^3(x_i), & x\in[x_i,x_{i+1}) \\ \mathrm{a}(x_i)\left(\mathrm{a}(x_i)\,\mathrm{b}(x_{i+2})+b(x_{i+3})\,\mathrm{a}(x_{i+1})\right)+b(x_{i+4})\,\mathrm{a}^2(x_{i+1}), & x\in[x_{i+1},x_{i+2}) \\ \mathrm{b}(x_{i+4})\left(\mathrm{a}(x_{i+1})\,\mathrm{b}(x_{i+3})+b(x_{i+4})\,\mathrm{a}(x_{i+2})\right)+a(x_i)\,\mathrm{b}^2(x_{i+3}), & x\in[x_{i+2},x_{i+3}) \\ b^3(x_{i+4}), & x\in[x_{i+3},x_{i+4}] \end{cases} \tag{4}$$

Where,

$$\mathrm{a}(x_i)=\sin\left(\frac{x-x_i}{2}\right),\ b(x_i)=\sin\left(\frac{x_i-x}{2}\right),\ z=\sin\left(\frac{h}{2}\right)\sin\left(h\right)\sin\left(\frac{3h}{2}\right)$$

where $h=(b-a)/n$ and $TB_i^4(x)$ is a piecewise cubic trigonometric function with some geometric properties like C^2 continuity, non-negativity and partition of unity [10, 11].The values of $TB_i^4(x)$ and its derivatives at nodal points are required and these derivatives are tabulated in Table 1. Secondly, we discuss the cubic trigonometric B-spline collocation method (CuTBSM) for solving the Benjamin-Bona-Mahony-Burgers equation (1).

Table 1: Values of $TB_i^4(x)$ and its derivatives.

x	x_i	x_{i+1}	x_{i+2}	x_{i+3}	x_{i+4}
TB_i	0	p_1	p_2	p_1	0
TB_i'	0	p_3	0	p_4	0
TB_i''	0	p_5	p_6	p_5	0

where

$$p_1=\frac{\sin^2\left(\frac{h}{2}\right)}{\sin\left(h\right)\sin\left(\frac{3h}{2}\right)},\ p_2=\frac{2}{1+2\cos\left(h\right)},\ p_3=-\frac{3}{4\sin\left(\frac{3h}{2}\right)},\ p_4=\frac{3}{4\sin\left(\frac{3h}{2}\right)}$$

$$p_5=\frac{3\left(1+3\cos\left(h\right)\right)}{16\sin^2\left(\frac{h}{2}\right)\left(2\cos\left(\frac{h}{2}\right)+\cos\left(\frac{3h}{2}\right)\right)},\ p_6=-\frac{3\cos^2\left(\frac{h}{2}\right)}{\sin^2\left(\frac{h}{2}\right)\left(2+4\cos\left(h\right)\right)}$$

Governing Equation and Numerical Method:

This section discusses the cubic trigonometric B-spline collocation method for solving numerically the BBM-Burgers equation. The solution domain $a\le x\le b$ is equally divided by knots x_i into n subintervals $[x_i,x_{i+1}]$, $i=0,1,2,...,n-1$ where $a=x_0<x_1<...<x_n=b$. Our approach for BBM-Burger equation using cubic trigonometric B-spline is to seek an approximate solution as [13]:

$$V_j(x,t)=\sum_{j=-3}^{n-1}D_j(t)\,TB_j^4(x) \tag{5}$$

where $D_j(\mathrm{t})$ is to be determined for the approximated solutions $V_j(x,t)$ to the exact solution at the point (x_j,t_i).The approximations V_j^i at the point (x_j,t_i) over subinterval $[x_i,x_{i+1}]$ can be defined as:

$$V_j^i = \sum_{k=j-3}^{j-1} D_k^i TB_k^4(x) \tag{6}$$

where $j=0,1,2,...,n$. So as to get the approximations to the solution, the values of $B_{3,j}(x)$ and its derivatives at nodal points are required and these derivatives are tabulated using approximate functions (4) and (6), the values at the knots of V_i^j and their derivatives up to second order are:

$$\begin{cases} (V)_j^i = p_1 D_{j-3}^i + p_2 D_{j-2}^i + p_1 D_{j-1}^i, \\ \left(\dfrac{\partial V}{\partial x}\right)_j^i = p_3 D_{j-3}^i + p_4 D_{j-1}^i \\ \left(\dfrac{\partial^2 V}{\partial x^2}\right)_j^i = p_5 D_{j-3}^i + p_6 D_{j-2}^i + p_5 D_{j-1}^i \end{cases} \tag{7}$$

The approximations for the solutions of BBM-Burger equation (1) at t_{j+1} th time level can be given as:

$$\frac{v^{n+1}-v^n}{\Delta t} - \frac{v_{xx}^{n+1}-v_{xx}^n}{\Delta t} + \theta q_j^{n+1} + (1-\theta) q_j^n = 0 \tag{8}$$

where $q_j^n = (vv_x)_j^n + \delta(v_x)_j^n - \lambda(v_{xx})_j^n$ and the subscripts n and $n+1$ are successive time levels, $n=0,1,2,..$ and Δt is the time step. By using the following formula [15]:

$$(\mathrm{vv}_x)^{n+1} = v^{n+1}v_x^n + v^n v_x^{n+1} - v^{n+1}v_x^n \tag{9}$$

The equation (8) with putting the values of nodal values v and derivatives using (7) becomes the following difference equation with variable D_j, $j=-3,...,\mathrm{n}-1$ and noted the equation a Crank-Nicolson when $\theta = \dfrac{1}{2}$

$$wD_{j-3}^{n+1} + w_1 D_{j-2}^{n+1} + w_2 D_{j-1}^{n+1} = zD_{j-3}^n + z_1 D_{j-2}^n + z_2 D_{j-1}^n \tag{10}$$

where

$$\begin{cases} w = p_1\left(1+\dfrac{\Delta t}{2}v_x^n\right) + \dfrac{\Delta t}{2}(v^n+\delta)p_3 + (-1-\dfrac{\Delta t}{2}\lambda)p_5 \\ w_1 = p_2\left(1+\dfrac{\Delta t}{2}v_x^n\right) + (-1-\dfrac{\Delta t}{2}\lambda)p_6 \\ w_2 = p_1\left(1+\dfrac{\Delta t}{2}v_x^n\right) + \dfrac{\Delta t}{2}(v^n+\delta)p_4 + (-1-\dfrac{\Delta t}{2}\lambda)p_5 \\ z = p_1 - \dfrac{\Delta t}{2}\delta p_3 + (-1+\dfrac{\Delta t}{2}\lambda)p_5 \\ z_1 = p_2 + (-1+\dfrac{\Delta t}{2}\lambda)p_6 \\ z_2 = p_1 - \dfrac{\Delta t}{2}\delta p_4 + (-1+\dfrac{\Delta t}{2}\lambda)\mathrm{p}_5 \end{cases} \tag{11}$$

when simplify (10) the system consists of $(N+1)$ linear equation in $(N+3)$ unknown $D^i = [\mathrm{D}_{j-3}^n,...,\mathrm{D}_{j-3}^n]$ at the time level $t=t_{i+1}$, by apply equation (6) to obtain unique solution on the boundary conditions (3) as following:

Mathematics and Statistics Journal, 2(2) April 2016, Pages: 1-9

$$p_1 D^i_{j-3} + p_2 D^i_{j-2} + p_1 D^i_{j-1} = 0 \qquad j = 0$$
$$p_1 D^i_{j-3} + p_2 D^i_{j-2} + p_1 D^i_{j-1} = 0 \qquad j = N \tag{12}$$

From equations (10-12) the system consists $N+3 \times N+3$ in the following form:

$$M_{N+3xN+3}\, D^{n+1}_{1xN+3} = N_{N+3xN+3}\, D^{n}_{1xN+3}$$

where

$$M = \begin{bmatrix} p_1 & p_2 & p_1 & 0 & . & . & 0 \\ w & w_1 & w_2 & 0 & . & . & 0 \\ 0 & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & . & . & w & w_1 & w_2 \\ 0 & 0 & . & . & p_1 & p_2 & p_1 \end{bmatrix}$$

and

$$N = \begin{bmatrix} 0 & 0 & 0 & 0 & . & . & 0 \\ z & z_1 & z_2 & 0 & . & . & 0 \\ 0 & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & . & . & z & z_1 & z_2 \\ 0 & 0 & . & . & 0 & 0 & 0 \end{bmatrix}$$

$$D^n = [\mathrm{D}^n_{-3}, \mathrm{D}^n_{-2}, \ldots, \mathrm{D}^n_{N-1}]$$

Initial state:

The initial vector D^0 is been computed from the initial conditions, the approximate solution V_j^{i+1} at a particular time can be calculated repeatedly the recurrence relation [13]. D^0 can be obtained from initial condition and boundary values of the derivatives of the initial condition as follows [14]:

$$(V_j^0)_x = f_0^{'}(\mathrm{x}_j) \qquad j = 0$$
$$V_j^0 = f_0(\mathrm{x}_j) \qquad \mathrm{j} = 0, 1, \ldots, \mathrm{N} \tag{13}$$
$$(V_j^0)_x = f_n^{'}(\mathrm{x}_j) \qquad j = N$$

Thus the system of equations in (13) can be represented as a matrix of order $N+3 \times N+3$, of the form: $AF^0 = d$

where

$$A = \begin{bmatrix} p_3 & 0 & p_4 & 0 & . & . & 0 \\ p_1 & p_2 & p_1 & 0 & . & . & 0 \\ 0 & . & . & . & . & . & 0 \\ . & . & . & . & . & . & .\,. \\ . & . & . & 0 & . & . & . \\ 0 & 0 & . & . & p_1 & p_2 & p_1 \\ 0 & 0 & . & . & p_3 & 0 & p_4 \end{bmatrix}$$

$$F^0 = [\mathrm{D}^0_{-3}, \mathrm{D}^0_{-2}, \ldots, \mathrm{D}^0_{N-1}]^T$$
$$d = [f^{'}(\mathrm{x}_0), f(\mathrm{x}_0), f(\mathrm{x}_1), \ldots f(\mathrm{x}_N), f^{'}(\mathrm{x}_N)]^T$$

Stability Analysis:

In this section we investigate the stability analysis of the proposed scheme by using Von Neumann method. This approach studied by many researchers [12, 13].

Now, substituting the approximate solution V, we have

$$D_j^n = \delta^n \exp(\mathrm{imh}\,\mu) \tag{14}$$

where $i=\sqrt{-1}$, μ is the mode number and h is the element size. To apply this method, we have linearized the nonlinear term vv_x by consider v as constant as z in equation (8) we get the equation:

$$\begin{aligned} &a\delta^{n+1}\exp(\mathrm{i(m-3)\,h}\,\mu)+b\delta^{n+1}\exp(\mathrm{i(m-2)\,h}\,\mu)+c\delta^{n+1}\exp(\mathrm{i(m-1)\,h}\,\mu)\\ &=a_1\delta^{n}\exp(\mathrm{i(m-3)\,h}\,\mu)+b_1\delta^{n}\exp(\mathrm{i(m-2)\,h}\,\mu)+c_1\delta^{n}\exp(\mathrm{i(m-1)\,h}\,\mu)\end{aligned} \tag{15}$$

where

$$\begin{cases} a = p_1+\dfrac{\Delta t}{2}p_3(\delta+z)+(-1-\lambda\dfrac{\Delta t}{2})\,\mathrm{p}_5\\ b = p_2+(-1-\lambda\dfrac{\Delta t}{2})\,\mathrm{p}_6\\ c = p_1+\dfrac{\Delta t}{2}p_4(\delta+z)+(-1-\lambda\dfrac{\Delta t}{2})\,p_5\\ a_1 = p_1-\dfrac{\Delta t}{2}p_3(\delta+z)+(\lambda\dfrac{\Delta t}{2}-1)\,p_5\\ b_1 = p_2-(\lambda\dfrac{\Delta t}{2}-1)\,p_6\\ c_1 = p_1-\dfrac{\Delta t}{2}p_4(\delta+z)+(\lambda\dfrac{\Delta t}{2}-1)\,p_5 \end{cases} \tag{16}$$

Divided both sides of (15) by $\exp(\mathrm{i(m-2)\,h}\,\mu)$ we get

$$\begin{aligned}&\delta^{n+1}(\mathrm{a}\exp(\mathrm{i(-1)\,h}\,\mu)+b+c\exp(\mathrm{ih}\,\mu)\\ &=\delta^{n}(\mathrm{a}_1\exp(\mathrm{i(-1)\,h}\,\mu)+b_1+c_1\exp(\mathrm{ih}\,\mu)\end{aligned} \tag{17}$$

We can rewrite equation (17) as following

$$\delta=\frac{X_2-iY}{X_1-iY} \tag{18}$$

Where

$$X_1=(2p_1+2(-1-\frac{\Delta t}{2}\lambda)\,\mathrm{p}_5)\cos(\mathrm{h}\,\mu)+(\mathrm{p}_2+(-1-\frac{\Delta t}{2}\lambda)\,\mathrm{p}_6$$

$$X_2=(2p_1+2(\frac{\Delta t}{2}\lambda-1)\,\mathrm{p}_5)\cos(\mathrm{h}\,\mu)+(\mathrm{p}_2+(\frac{\Delta t}{2}\lambda-1)\,\mathrm{p}_6$$

$$Y=\Delta t p_4(\delta+z)\sin(\mathrm{h}\,\mu)$$

We note that $X_2\le X_1$ so $|\delta|^2=\delta\bar{\delta}=\dfrac{X_2^2+Y_2^2}{X_1^2+Y_2^2}\le 1$

Therefore the linearized numerical scheme for BBM-Burger equation is unconditionally stable.

Numerical Illustrations:

To illustrate the accuracy and efficiency of proposed method, two examples are given in this section with L_∞ and L_2 error norms are calculated by $L_\infty=\max\limits_i|u_i-v_i|$ and $L_2=\sqrt{h\left(\sum\limits_i^n|u_i-v_i|^2\right)}$. Then compare the numerical solutions obtained by test cubic trigonometric B-spline collocation method for BBM-Burger equation

(1) with the exact solutions and those numerical methods which were exiting in literature. Numerical results are computed at different time levels.

Example 1:

Consider the BBM problem [5] with $\delta = 1.0$ and $\lambda = 0$,

$$v_t + v_x + vv_x - v_{xxt} = 0 \qquad x \in [-40, 60],\ t \in [0, T]$$

with initial conditions

$$v(x,0) = f(x) = 3c\,\mathrm{sech}^2\left(\frac{1}{2}\sqrt{\frac{c}{1+c}}(x - x_0)\right), \ -40 \le x \le 60$$

and boundary conditions as follows:

$$v(-40,t) = 0, v(60,t) = 0$$

The exact solution of this problem is

$$u(x,t) = 3c\,\mathrm{sech}^2\left(\frac{1}{2}\sqrt{\frac{c}{1+c}}(x - (1+c)t - x_0)\right)$$

with $c = 0.1$ and $x_0 = 0$. The proposed method is applied to calculate the numerical solutions of BBM-Burger equation (1)-(3) with $\Delta t = 0.01$ at several values of t. The absolute errors at different time levels with $h = 1/300$ and $\Delta t = 0.01$ is illustrated in Table 2. Figure 1 illustrates a comparison between the exact and approximate solutions at $T = 1.0$ with $h = 1/300, \Delta t = 0.01$.Figure 2. illustrates the comparison between exact and numerical solutions at different time levels with $h = 1/300, \Delta t = 0.01$. We see that the numericalresults of this problem by proposed method are more accurate than the results obtained by cubic B-spline method (CuBS) which developed by Zarebnia and Parvaz [5].

Table 2: Absolute errors at different time-levels for Example 1

	Present method			CuBS [5]		
x	$t=0.5$	$t=1.0$	$t=1.50$	$t=0.5$	$t=1.0$	$t=1.5$
-40	5.88E-06	4.98E-06	4.22E-06	-----------	-----------	-----------
-30	-1.64E-07	-2.75E-07	-3.46E-07	2.12E-04	1.99E-04	5.42E-05
-20	-4.51E-06	-7.23E-06	-8.79E-06	1.06E-03	1.00E-03	2.72E-03
-10	-4.12E-04	-6.35E-04	-7.25E-04	3.57E-03	3.52E-03	1.00E-03
0	-7.71E-04	-3.00E-03	-6.46E-03	7.45E-04	1.20E-03	5.59E-04
10	6.67E-04	1.66E-03	3.06E-03	3.91E-03	4.06E-03	1.25E-03
20	7.15E-06	1.82E-05	3.50E-05	1.36E-03	1.50E-03	5.09E-04
30	2.32E-07	5.51E-07	9.84E-07	2.81E-04	3.12E-04	1.07E-05
40	1.11E-08	2.61E-08	4.62E-08	-----------	-----------	-----------
50	5.42E-10	1.28E-09	2.26E-09	-----------	-----------	-----------
60	1.97E-10	2.32E-08	2.74E-08	-----------	-----------	-----------

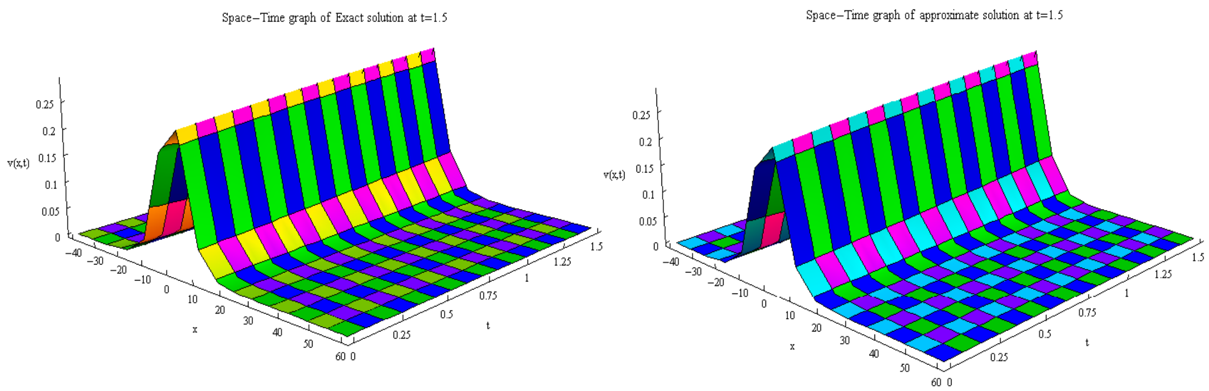

Fig. 1: Spatial-time exact and approximate solution for Example 1 with $n = 300$, $\Delta t = 0.01$

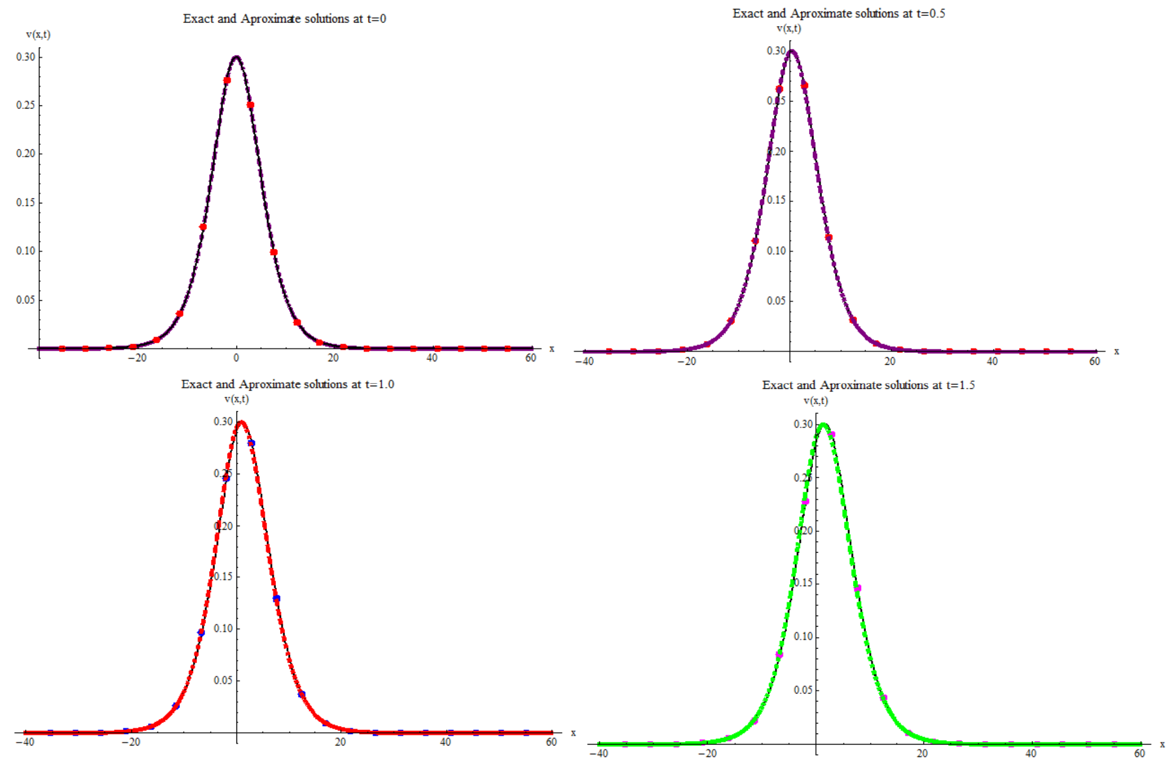

Fig. 2: The approximate and exact solutions for Example 1 at different time levels.

Example 2:

Consider the Benjamin-Bona-Mahony Burger problem [5] with $\delta = 1.0$ and $\lambda = 1.0$,

$$v_t + v_x + vv_x - v_{xxt} - v_{xx} = 0 \qquad x \in [-12, 12],\ t \in [0, T]$$

with initial conditions

$$v(x,0) = f(x) = \operatorname{sech}^2(x/4),\ -12 \le x \le 12$$

and boundary conditions as follows:

$$v(-12,t) = \operatorname{sech}^2(-3 - t/3),\ v(12,t) = \operatorname{sech}^2(3 - t/3)$$

The exact solution of this problem is $u(x,t) = \operatorname{sech}^2(x/4 - t/3)$.

The cubic trigonometric B-spline method is employed to compute the numerical solutions of this problem. The absolute errors at different time levels with $h = 1/200$ and $\Delta t = 0.01$ are given in Table 3 and Table 4. Figure 3 illustrates the numerical solutions with different time levels. Figure 4 illustrates the space-time exact and approximate solutions at $T = 2.0$ and found to be similar in comparison to CuBS [5].

Table 3: Absolute errors at different time-levels for Example 2

x	Present method			CuBS [5]		
	$t=0.2$	$t=0.5$	$t=0.7$	$t=0.2$	$t=0.5$	$t=0.7$
-12	6.50E-16	-3.80E-16	4.11E-16	3.33E-11	-2.23E-10	-3.33E-11
-10	-1.41E-03	1.44E-03	-3.75E-03	2.29E-02	1.98E-02	1.79E-02
-5	-1.34E-02	-3.02E-02	-3.93E-02	2.56E-01	2.24E-01	2.06E-01
0	1.39E-02	2.18E-02	1.87E-02	9.78E-01	9.33E-01	8.97E-01
5	2.99E-03	1.30E-02	2.38E-02	3.19E-01	3.80E-01	4.23E-01
10	-1.30E-03	-3.53E-03	-3.04E-03	3.04E-02	3.97E-02	4.72E-02
12	-1.73E-18	0.000000	-3.46E-18	2.00E-10	6.66E-11	-2.66E-10

Table 4: Absolute errors at different time-levels for Example

x	Present method			CuBS [5]		
	$t=1.0$	$t=1.5$	$t=2.0$	$t=1.0$	$t=1.5$	$t=2.0$
-12	1.06E-15	-3.57E-16	6.58E-16	1.33E-11	-1.26E-10	8.66E-11
-10	-4.53E-03	-5.13E-03	-5.14E-03	1.54E-02	1.19E-02	9.16E-03
-5	-5.01E-02	-6.16E-02	-6.67E-02	1.82E-01	1.49E-01	1.23E-01
0	3.14E-03	-4.45E-02	1.01E-02	8.38E-01	7.33E-01	6.31E-01

5	4.61E-02	9.92E-02	1.66E-02	4.87E-01	5.89E-01	6.76E-01
10	-7.63E-03	-1.12E-02	-1.25E-02	6.05E-02	8.86E-02	1.25E-01
12	0.000000	-6.93E-18	0.000000	1.13E-09	-3.33E-10	-1.01E-16

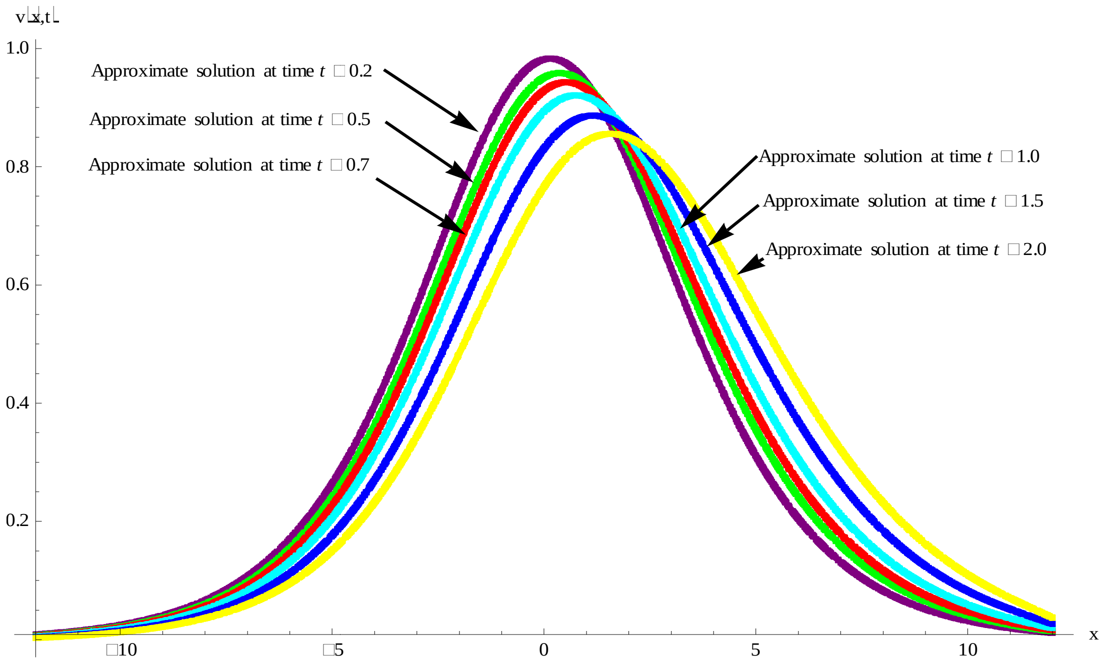

Fig. 3: The approximate solutions for Example 2 at different time levels.

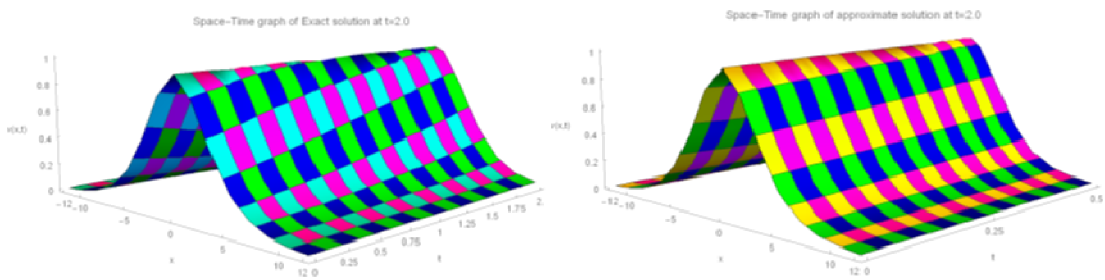

Fig. 4: Spatial-time exact and approximate solution for Example 2 with $n = 200$, $\Delta t = 0.01$

Conclusions:

In this paper cubic trigonometric B-spline method is proposed to find the solution of Benjamin-Bona-Mahony Burger equation. The comparison between the numerical and exact solution are illustrated and shows that the cubic trigonometric B-spline is more accurate than the method suggested by Zarebnia etal. in [5]. The von Neumann method used to check stability analysis of the proposed method to get unconditionally stable.

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