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Research article

A MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION FOR PAVEMENT MAINTENANCE WITH CHAOS AND DISCRETEKawther Ahmed ^a, Belal Al-Khateeb ^a, Maher Mahmood ^b^aDepartment of Computer Science, University of Anbar, Ramadi, Anbar, Iraq.^bCivil Engineering Department, University of Anbar, Ramadi, Iraq.**Abstract**

Particle Swarm Optimization (PSO) is a very common algorithm in swarm intelligence algorithms. PSO has been used to solve a lot of problems with one or more goals. Actually, the multi-objectives improvement issues in all real life are combinatorial in nature. Therefore, PSO has been improved to be able to handle very large number of decision variables and reduce or decrease computational complexity. In this work, a chaos multi objective PSO algorithm is improved for solving discrete (binary) optimization issues with crossover operation. The developed Chaos Discrete Multi Objective PSO (CDMOPSO) algorithm is applied to pavement management problem for flexible pavement to get optimal maintenance and rehabilitation plan. The results shown that there is significant improvement in the solutions satisfying pavement conditions and maintenance cost goals. It is required to a very short time of execution by the improved algorithm to reach a very good solution. Also, comparing the convergence of solutions with the rest of the PSO algorithms, it has found that the suggested algorithm is better.

Keywords: chaotic mapping, PSO, multi-objective optimization, pavement, binary PSO.

摘要：粒子群优化算法（PSO）是群体智能算法中非常常见的算法。PSO 已被用于解决一个或多个目标的许多问题。现实，所有现实生活中的多目标改进问题本质上是组合的。因此，PSO 已得到改进，能够处理大量决策变量并降低或降低计算复杂度。在这项工作中，一个混沌多目标 PSO 算法被改进，用于解决交叉操作的离散（二进制）优化问题。将开发的混沌离散多目标 PSO（CDMOPSO）算法应用于柔性路面的路面管理问题，以获得最优的维护和恢复计划。结果表明，满足路面条件和维护成本目标的解决方案有着显著改进。通过改进的算法需要在非常短的执行时间内达到非常好的解决方案。此外，比较解决方案与其他 PSO 算法的收敛性，发现建议的算法更好。

关键词：混沌映射，PSO，多目标优化，路面，二进制 PSO。

I. INTRODUCTION

The Pavement Management System (PMS) strives to maintain all sections of the roadway at a sufficiently high level of service and structural conditions. The PMS for roads requires the use of resources and a reasonable budget in order to provide safe routes to all users. With the passage of time, the condition of the pavement may

deteriorate due to certain conditions such as temperature variations, heavier tracks, repeated adverse weather conditions. Subsequently, early maintenance is requisite to minimize maintenance and rehabilitation costs. There are many researchers use different computational intelligence methods for pavement maintenance decision. Fwa et al. [1] suggested a single-

objective optimization model based on genetic algorithm called "PAVENET" to analyze the pavement maintenance programming problems at the network level. PAVENET was used to examine the effects of resource parameters, network parameters and maintenance-policy parameters. Herabat and Tangphaisankun [2] improved a single and multi-objective optimization model using genetic algorithm to improve the decision-making procedure of the pavement management in Thailand and to identify the appropriate (capability) multi-year intervention programs. Moreira et al. [3] improved a two level procedure to solve pavement optimization issue with three goals: maintenance costs, user costs, and pavement condition. Genetic algorithms were used to solve those multi-objective optimization problems. Terzi and Serin [4] proposed a single objective ant colony for pavement management at the network level. Maximization of the work of pavement intervention, single objective function, was rely for scheduling routine intervention activities. Mahmood [5] proposed a new barebones PSO to solve a detach optimization issues. This algorithm was used for flexible pavement in order to find optimal maintenance decisions. Kawther et al. [6] they used chaotic sequences to obtain the value of inertia weight in multi objective and discrete PSO for pavement maintenance scheduling problem. Crossover operation in (position and velocity) with multi objective PSO algorithm was used. Also, in [7] they used chaotic sequences instead of randomness to initialize the population of multi objective PSO algorithm. Many previous works have depended on the use of single objective PSO for finding pavement maintenance plan, so there is a need for a multi objectives PSO as many issues have more than one objective. The main objective of this research is to improve the algorithm of PSO and to apply it to find an optimal plan of maintenance and rehabilitation of flexible pavements to satisfy two objectives: reduce of treatment cost and reduce the sum of all values of Pavement Condition Index (PCI).

II. PAVEMENT MAINTENANCE

DECISION PROBLEM

A. Problem Parameters

Analyzing maintenance required standards and also some data to do analysis and repair treatment. The highway network is usually designed to equal sections in order to better assess maintenance requirements [8].

The cost of road network includes the costs of reconstruction maintenance and rehabilitation. It is necessary that the expressway system be higher than the minimum required for safety, so it must be rehabilitated even once a year. in next years, cost can be reduced to present value, if a rehabilitation activity is performed, using the following [5]:

The discount rates range is 3 to 5% which recommended by (FHWA) [9].

Where t_i refer to time which expenses money (set in years).

Road authorities can select a rehabilitation action from a list of activities (count on the condition). This list presented in Table 1. Also, for each maintenance activity, the level of trigger must be set. the total duration of the study must be defining by road authorities. Also, the road agency specify the duration of the unit analysis [10].

B. Objective Functions

The road authorities specify the popular objectives for pavement maintenance: select and scheduling treatment to decrease costs, preserve performance of roadway and decrease costs of treatment through the analysis time [8]. multi-objective PSO is apply to program the maintenance activities to achieve optimal decisions in maintenance. multi-objective PSO algorithm is applied for M&R as follows [5].

C. Prediction Model of Pavement Condition

To assessing the general conditions of the roads, Pavement Condition Index (PCI) is used. PCI depending on the results of a visual inspection, specific by the disaster kind, the severity and number. PCI considered a good index of structural integrity, and also allows you to specify the state of maintenance (current and future state). [11]. For main roads in humid frost climatic conditions, an assessment of the future state of the roadway was adopted:

$$PCI = 97.744 - 0.15 X_5 - 0.064 X_4 - 0.515 X_2 + 3.748 X_3 \quad (5)$$

block and alligator) [5].

III. PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) is a swarm intelligence optimization algorithm proposed by Kennedy and Eberhart [12]. PSO depending on the simulation of birds behavior. The beauties of the algorithm are: few adjustable

parameters, simple principle and the fast convergence speed. Therefore, PSO has been widely used in many real life problems [13]. Each possible solution in search space represents by particle. Through each iteration, the particle updates his velocity and position depending on the following equations:

$$V_{i,j}(t+1) = w V_{i,j}(t) + r_1 c_1 [Pbest_{i,j}(t) - X_{i,j}(t)] + r_2 c_2 [Gbest(t) - X_{i,j}(t)] \quad (6)$$

$$X_{i,j}(t+1) = X_{i,j}(t) + V_{i,j}(t+1) \quad (7)$$

Where $X_{(i,j)}(t)$ refer to position of particle i in iteration t , it depending on previous velocity and previous position; c_1 and c_2 are mean learning factors that are fixed numbers; r_1 and r_2 are values two random number between (0,1); $V_{(i,j)}(t)$ refer to velocity of particle i in iteration t ; w refer to inertia weight, which used to control the influence of previous velocities on the current velocity [14]; $Gbest(t)$ refer to the best global position (leader) in iteration t , which guides particles to optimal positions; $[Pbest]_{(i,j)}(t)$ = best local position for j th dimension of particle i , $Pbest$ have a minimum fitness value obtained so far in iteration t .

The performance of each particle (solution) in the swarm is evaluated by the objective function (fitness) of the optimization problem[15][16].

IV. MULTI-OBJECTIVE OPTIMIZATION PROBLEMS

In multi objective scenario, solutions are compared against different goals. Therefore, there are Pareto-optimal solutions (a group of non-dominated solutions instead of single dominated solution. Pareto optimality means that none of the objective can be enhanced without making a negative effect on at least one of the other goals [3]. The storage known as the archive is utilized to store the solutions obtained during the generations and thus used to select the leader of the swarm according to several sort of quality measures [17]. To determine the leader, the sigma method is apply in this research [18].

V. DISCRETE PARTICLE SWARM OPTIMIZATION

To solve a discrete problem, Kennedy and Eberhart in 1997 A.D, introduced a discrete version of PSO algorithm [19]. The most popular kind of discrete PSO is a binary. In binary PSO, the position of each particle will be 0 or 1 [20]. The position will update during the following equations [21] [22]:

$$V_{i,j}(t) = sig(V_{i,j}(t)) \quad (8)$$

$$sig(V_{i,j}) = \frac{1}{1+e^{-V_{i,j}(t)}} \quad (9)$$

Where sig refer to sigmoid function and $V_{(i,j)}(t)$ is the velocity of every particle in every iteration.

To get either (0 or 1) in the positions of all particles, the position equation is changed to:

Where $rand$ = quasi-random number (0, 1).

VI. CROSSOVER OPERATION

There are many kinds of crossovers, such as one-point crossover operation which is used in this paper. Crossover is used as a try to prevent falling in local optima. in this paper, the crossover point in the middle of the solution is used.

VII. CHAOTIC SEQUENCES FOR LEARNING FACTOR (c1, c2)

Chaotic is a nonlinear phenomenon. Chaotic is easy of implementation and also has a particular ability to avert trapped in local optima [23]. In this research, the chaos map is employed to initialize the learning factor (c_1 , c_2) to improve the diversity and control the value of learning factor in multi-objective PSO. Also, the logistic map is used because it considers the common kind of chaos map.

$$Z_{n+1} = \mu Z_n(1 - Z_n) \quad n = 1,2,3 \dots \dots \quad (11)$$

Where $Z_1, Z_2, Z_3, \dots, Z_n$ are chaos series used to initialize the population; μ refer to control parameter= 4 [24].

VIII. CHAOTIC WITH PARTICLE SWARM OPTIMIZATION FOR DISCRETE PROBLEM

To achieve our hypothesis, PSO with Chaotic and Discrete is implemented and developed. The following steps are considered to implement the suggested algorithm:

- 1- Initialize random particles (100 velocities and 100 positions). Each particle, consists of a number of dimensions (50 dimensions).
- 2- For discrete problem, the direct (continuous) representation of positions is converted to indirect (binary) representation.

- 3- Initialize the best local position, assuming that it is equal the initial position of the particle.
- 4- Using the objective function to evaluate all solutions. The first one is for PCI value (using equations 4 and 5) and the second one is for cost (using equations 1, 2, 3).
- 5- As in Pareto front, select the best solutions. Then, save the solutions in A (external archive).
- 6- If A is full (in this paper, archive capacity is determined to store only 20 solutions) then Crowding Distance Computation (CDC) [21][22] is applied to specify the deletion of solutions from the archive. CDC steps are:

a- Get the number of non-dominated solutions from the archive.

b- For all particles, initialize array of distance (Initialize with zero).

c- For each objective function, find the solutions which have the lowest and highest fitness values. The two objectives (M) are always selected because they are given an infinite CDC values.

Set $CDC(1, M) = CDC(\text{end}, M) =$
infinity value.

d- The solution sorted in ascending order. Then, for each particle, calculated the distance between upper and lower particles from this particle. CDC of particle i it equal to sum of distances of particles $(i-1)$ and $(i+1)$ divided on the subtraction between the minimum value and maximum value.

e- CDC array must be sort in descending. Then chosen 20% of particles which have the highest CDC.

- 7- Choose the particle leader or the global best position. In this research, to select the leader from solutions, the sigmoid method is applied. The equation of sigmoid method is:

$$\sigma = \frac{f_1^2 - f_2^2}{f_1^2 + f_2^2} \quad (12)$$

f_1, f_2 are fitness values for the first objective function and the second objective

function respectively. The sigmoid method steps are:

a- Calculate $\sigma(j)$ for all members in A using equation (12).

b- Using Euclidian distance (Dist) between $\sigma(1)$ and $\sigma(i)$ to calculate $\sigma(i)$. Then, apply Euclidian distance between $\sigma(j)$ and $\sigma(i)$ (tempDist).

c- If $\text{tempDist} \leq \text{Dist}$ then $\text{Dist} = \text{tempDist}$. Otherwise, Dist will not change. The particle leader has the lowest (Dist) [18].

8- Update the particles's velocity by using equation (6). The value of w is decreased linearly from 0.9 to 0.4 according to the following equation:

$$W = W_{max} - \frac{W_{max} - W_{min}}{T_{max}} T \quad (13)$$

Where T_{max} is the maximum number of iterations; T is the current iteration [25]. The r_1, r_2 are two random numbers between (0,1). To find the values of $c1$ and $c2$, the chaotic logistic map as in equation (11) is used.

9- Apply crossover (one-point) in velocity. Then, used a controlled mutation to keep the velocity in the range (-6, 6) which recommended by Kennedy and Eberhart [12].

10- Update the particle's position using equations (8) (9) (10). Then, applied mutation operator. mutation is applying by select randomly number of particles and changing his positions.

11- Apply crossover (one-point) in positions. Then, applied mutation operator. The mutation is done by changing the position that has the value of 1, of some randomly selected particles.

12- For the new particles, evaluate its fitness value. Then, save a new solutions to compare it with the previous solution.

13- Update the local best solution by: [26].

$$pbest_i(t+1) = \begin{cases} pbest_i(t), & \text{if } F(pbest_i(t)) < F(x_i(t+1)) \\ x_i(t+1), & \text{otherwise} \end{cases} \quad (14)$$

14- Pareto is applying again to choose the best solution. Then, save a new solutions and their positions in an external archive.

15- Checked the external archive again as in step 6.

16- New leader must set for current iteration as in step 7.

17- Repeat steps 8 to 16 until arrive to the maximum number of iterations. The maximum iteration is 100.

IX. COMPROMISE SOLUTION

To respond to the subjectivity of decision makers, a fuzzy membership function is adopted to replicate authority preferences and identify the compromise solution from the optimal set of Pareto. Considering a non-dominated solution Y_k in the archive, the satisfactory degree of Y_k for the i th objective function F_i is calculated by a membership function as follows:

$$\mu_i^k = \begin{cases} 1, & F_i(Y_k) \leq F_i^{min} \\ \frac{F_i^{max} - F_i(Y_k)}{F_i^{max} - F_i^{min}}, & F_i^{min} < F_i(Y_k) < F_i^{max} \\ 0, & F_i(Y_k) \geq F_i^{max} \end{cases} \quad (15)$$

where F_i^{max} and F_i^{min} are maximum and minimum of i th fitness function F_i . Normalized membership function μ^k of Y_k by:

$$\mu^k = \frac{\sum_{i=1}^M \mu_i^k}{\sum_{k=1}^{|A|} \sum_{i=1}^M \mu_i^k} \quad (16)$$

Where $|A|$ refer to element number of archive, M refer to number of objective. The compromise solution is the solution that have a maximum of μ^k in the archive A [26].

X. RESULTS

To find the optimum maintenance plan for 5 sections of pavement over 10 years, the propose Chaos with Discrete Multi Objective PSO (CDMOPSO) is applied. Figure 1 show the obtained results. The compromise solution in the archive that have the maximum membership value (μ) is chosen (as the optimum maintenance solution for the roadway). For 100 generations, the optima maintenance of compromise solution shows in table 2. the value of PCI values is $3.1484e+09$ while the cost is 180.24 and the time taken by the algorithm to reach the final results is about two minutes. Compared with time achieved by DBB-MOPSO algorithm [10], the time

obtained from CDMOPSP considering very short. Comparison at execution time shown in figure 2. The obtained result shows large enhancement in the pavement performance and cost of maintenance, compared with obtained results in the previous research [7] ($5.87E+10$ for PCI values, 399.25 for maintenance cost). The result show that the propose algorithm able to increase the diversity of the solutions and thus increase the search space to find more suitable solutions. Figures (3 to 6) show the convergence pattern with number of generations. Table 3 shows the solutions that obtained in 100 generations with CDMOPSO.

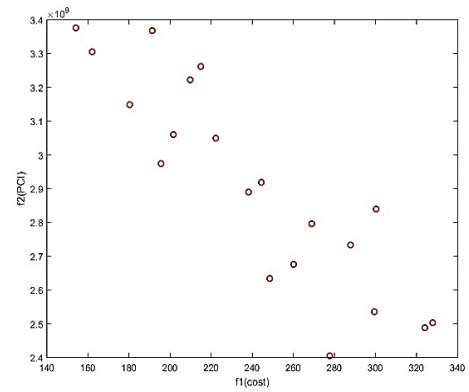


Figure 1. The Solutions in 100 Generations.

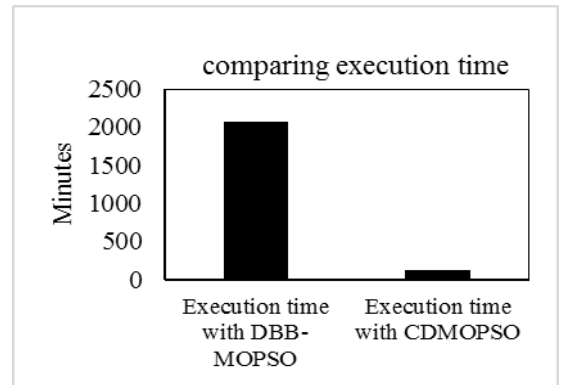


Figure 2. The Execution Time.

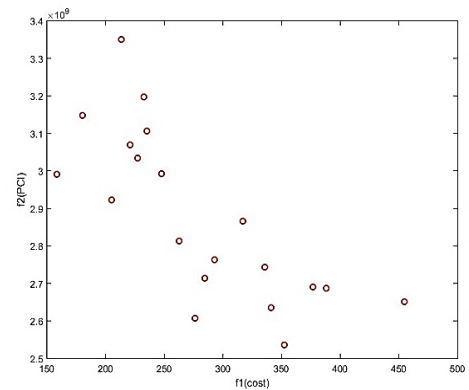


Figure 3. The Solutions in 20 Generations.

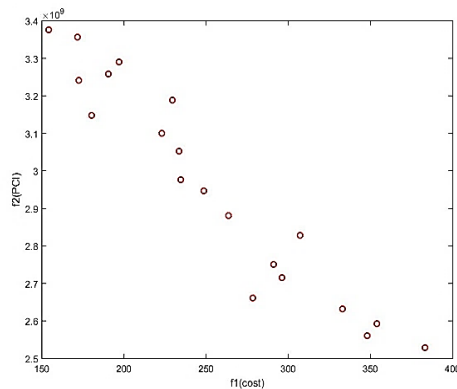


Figure 4. Solutions in 40 Generations.

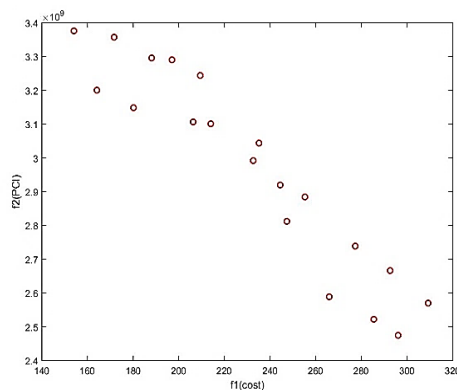


Figure 5. The Solutions in 60 Generations.

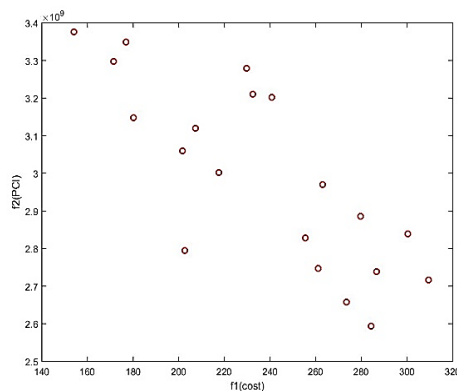


Figure 6. The Solutions in 80 Generations.

XI. CONCLUSIONS AND FUTURE WORKS

The propose algorithm CDMOPSO is applied to find the best maintenance and rehabilitation schedule for the roadway for two objectives: minimizing the total cost of rehabilitation of the roadway and minimizing the sum of PCI values. The execution time is short in the improved algorithm.

The use of the ergodicity characteristics of chaotic variables instead of specific values for c_1 and c_2 with a crossover operation makes it possible to obtain a better diversity

of solutions. This is further enhancement in the performance of the improved algorithm.

In future works, different evolutionary algorithms can be used to conduct additional exam of the enhanced algorithm performance. As this study only addresses an unconstrained optimization problem, the CDMOPSO algorithm can be perform on constrained problems in future work to examine the effects of apply another type of chaotic map on PSO.

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