The Selection of Particle Swarm Optimization Learning Factors Values in Solving the Multiple Travelling Salesman Problem

Belal Al-Khateeb¹

1 College of Computer Science and Information Technology, University of Anbar, Ramadi, Iraq, Corresponding Author Email: belal@computer-college.org.

Abstract :This paper addresses the question of whether fixed learning (acceleration) factors are an important factor in the Particle Swarm Optimization (PSO) by testing many selected values for those factors and apply them in solving the Multiple Traveling Salesman Problem (MTSP). Extensive experiments are done and those experiments show that the learning factors are problem dependent, therefore it is recommended to do the same experiments that are done in this paper for each problem that intend to be solved by PSO.

I. Introduction

Particle Swarm Optimization (PSO) is one of the most popular and successful nature-inspired optimization algorithms. PSO is described as a flock of birds flying randomly to find a place of food. The best one becomes the leader of the flock and all other birds must follow it. Each bird represents a solution in a search space called 'particle'. PSO is initialized as random particles, then updating generations to searching of optima [1].

PSO helps to find optimal solutions of optimization problems that have continuous distinctions between variables [2]. With the passage of time, it was developed to deal with discrete problems. The use of PSO algorithm has been successful in most problems that deal with discrete and continuous problems. Therefore, this algorithm has been developed by researchers to be able to solve problems that contain more than one objective (multi-objective)[3].

Many previous researches used metaheuristics and applied artificial techniques in solving real life problems, those researches used parameters selection techniques for the applied algorithms [4][5][6][7][8].

In this research, a set of extensive experiments are done for the first time to find the best learning (acceleration) factors for PSO in order to solve the MTSP efficiently. The designed experiments show that the learning factors are problem dependent therefore; it cannot be fixed for all problems.

The rest of the paper is organized as follows; in section II, related work is presented. PSO is discussed in section III. Section IV presents MTSP. Section V shows the experimental setup for this work. Section VI presents the obtained results and the conclusions are presented in Section VII.

II. Literatures Review

Several researches are done for PSO parameters selection, those researches mostly focused on the choosing the best values for the inertia weight. Feng et.al. proposed an adaptive, easy to implement and low cost inertia weight strategy [9], this strategy depends on particle's position and velocity rather than the number of process iterations. This was done by the illumination of Butterworth filter. The obtained results show that, with careful parameters settings, the proposed strategy was successful and it can be used for many applications.

Bansal et.al. [10] studied 15 popular Inertia Weight strategies and compares their performance on five optimization test problems in order to show the importance of the inertia weight for the PSO exploration and exploitation. The obtained results show that Chaotic Inertia Weight is the best strategy for better accuracy. Random Inertia Weight strategy is best for better efficiency.

Chauhan et.al. [11] proposed a three novel inertia weight strategies, the first strategy is based on the adaptive decreasing of the inertia weight with the iteration number, while the second and third strategy are based on Gompertz function. The obtained results showed that those strategies enhanced the performance quality and convergence rate of PSO.

Maca and Pech [12] presented updating random strategies for inertia weight; those strategies are based on beta distribution. The obtained results show that the presented strategies can enhance the PSO exploration and exploitation.

Harrison et.al. [13] applied 18 inertia weight control strategies and found that only the random selection of the inertia weight can outperform the fixed value inertia weight. For more in depth review for the inertia weight control strategy, readers can refer to the work of Rathore and Sharma [14].

The majority of previous researches didn't consider the selection of the learning (acceleration) factors in PSO, so this paper aims to be the first research to address this issue.

III. Particle Swarm Optimization

The Particle Swarm Optimization (PSO) algorithm is one of computational algorithms that are inspired from animals' behavior such as bird flocks [15] and fish schools [16]. PSO is a population based search algorithm, simple in implementation, effective and considered as a global optimization algorithm [1]. It requires only initialization of mathematical operators. In addition, it is inexpensive in both speed and memory requirements. PSO was developed by Kennedy and Eberhart in 1995. Compared to other evolutionary algorithms, PSO was found to have a unique concept which was a particle (potential solutions) flying in search space, accelerating toward better solutions and has ability to find a feasible solution quickly [17].

The swarm of PSO consists of particles. Each particle represents a potential solution in optimization problem. The particles have two main attributes that are position and velocity. The position of each particle is updated according to its own experience and the experience of its neighbors. The velocity is adjusted to determine the direction that a particle needs to move.

During swarm movement, a particle updates its position depending on new velocity and previous position that obtained by the experiments in search space, while the updating of particle's velocity depends on previous velocity, the local best position (*Pbest*) and the global best position or the leader (*Gbest*). Equations 1 and 2 are used to update the velocity and position respectively [18][19].

$$V_{i,j}(t+1) = wV_{i,j}(t) + r_1c_1[\text{Pbest}_{i,j}(t) - X_{i,j}(t)] + r_2c_2[\text{Gbest}(t) - X_{i,j}(t)](1)$$

$$X_{i,j}(t+1) = X_{i,j}(t) + V_{i,j}(t+1)(2)$$

where V_{ij} is a velocity of *i* particle at iteration *t*; X_{ij} is a position of *i* particle at iteration *t* and it depends on previous position and new velocity, *w* is the inertia weight that is used to control the influence of the previous velocities on the current velocity [20], r_1 and r_2 are two random numbers between (0,1), c_1 and c_2 are learning factors or acceleration factors that are fixed numbers, **Pbest**_{*i*,*j*}(**t**) is the local best particle *i* that have the smallest fitness value obtained so far in one iteration *t*; *Gbest*(*t*) is the particle leader or global best position at generation *t*.

The leader particle in each generation guides other particles to move towards the optimal positions. The performance of each particle in the swarm is evaluated according to objective function or the fitness function of the optimization problem [21][22].

It is assumed that a *j*-dimensions in search space and particles i (potential solutions) has a fitness value F(x) and a velocity V that makes it move in the search space. The process steps of PSO algorithm are shown in the following [23][24]:

Step 1: Initialize a random population (positions X and velocities V of all particles).

Step 2: Assume the local best particles set equals to the positions set such as: *Pbest i*, j = X *i*, *j* and evaluate the fitness value of each particle F(x)i, j (the fitness value measured in different ways according to problem) and then take the best value (either maximum or minimum) from this set to be the global best position (*Gbest*) called the leader.

Step 3: Update the particle's velocity according to equation (1) and then

Update the particle's position according to equation (2).

Step 4: Evaluate the fitness value of each particle with the new position.

Step 5: Compare the current fitness value with the previous position, if the current is better, then *Pbest* i, j = F(x)i, j *Else*

Pbest i, j(t+1) = Pbest i, j(t). Step 6: If Pbest(t+1) is better than Gbest(t) then Gbest(t+1) = Pbest(t+1)

Else

Gbest(t+1) = Gbest(t).

Step 7: If current number of iterations is larger than the maximum number of iterations then stop and return the solution, else go to step 3.

IV. Multiple Traveling Salesman Problem

The Multiple Traveling Salesman Problem (MTSP) is a variation of the Travelling Salesman Problem (TSP). The difference between MTSP and TSP is that in MTSP there are m salesmen, every depot (city) in a given group of n cities is divided into m tours by assigning every of these depots (cities) to a different salesman. The objective is to seek out the minimum cost of the tours in total. The cost can be referred as distance or time [25].

The MTSP is outlined on a graph G = (V, A), where A represents the set of edges and V referred the set of vertices. Let C = (cij) be the cost matrix defined on the group of A. If cij = cji then the cost matrix is symmetric, otherwise it is asymmetric. If the cost matrix satisfies $cij \le cik + ckj$ for *i*, *j*, *k*, then the matrix C satisfies the triangle inequality [26][27].

Among the proposed models for the MTSP within the literature, assignment based mathematical model, therefore tree based mathematical model and a three-index row-based model have been common used [28].

The three-index row-based model for the MTSP is as follows: Let *n* be the number of cities to be visited, and *m* be the number of salesmen (we assume $n \ge 3m+1$). Then the variable *xij* is defined as follows [28]:

$$x_{ij} = \begin{cases} 1, & if \ edge(i,j) is \ used \ in \ tour, \\ 0, & otherwise. \end{cases}$$

Goal function:

$$minimize \sum_{(i,j) \in A} c_{ij} x_{ij}$$
(3)

Constraints:

$$\sum_{j=2}^{n} x_{1j} = m$$
(4)

$$\sum_{j=2}^{n} x_{j1} = m$$
(5)

$$\sum_{i=1}^{n} x_{ij} = 1, j = 2, ..., n$$
(6)

$$\sum_{j=1}^{n} x_{ij} = 1, i = 2, ..., n$$
(7)

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \le |S| - 1, S V - 1, S \ne 0$$
(8)

$$x_{ij} = 0 \vee 1, (i,j) \in A$$
(9)

In this model, constraints (6), (7) and (8) satisfy the assignment problem constraints. Constraints (4) and (5) ensure the comeback of each salesman to their starting point. Constraint (8) is used to prevent sub-tours [29].

V. Experimental Setup

For the purpose of investigating our hypothesis, PSO algorithm that is described in section III is implemented in order to solve the MTSP using different datasets, those datasets can be downloaded from (http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/tsp/). The total number of cities are randomly distributed among groups; the number of cities in each group is more than three cities and less than total number of cities /2. All the particles in the group have the same start and end city. The following settings are used for all the experiments:

- PSO population is 30.
- Number of Salesmen is 5.
- Number of Iteration is 500.
- Inertia weight value is 0.8.
- Velocity values are in the range [-6,6].
- MTSP datasets are Pr76, Pr152, Pr299 and Pr439.

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- Number of experiments for each dataset is 5 for each learning factors value.

VI. Results and Discussion

PSO algorithm is executed four times, each one with a dataset; the obtained results are shown in tables 1 thru 4. The values for each table is

-					Table 1: Res				
	c1	c2	Experiment	-	Experiment3	Experiment4	Experiment5	Min	Average
	0.5	0.5	260827	279298	262871	265383	283070	260827	270289.8
	0.5	1	295216	287779	301077	296319	293417	287779	294761.6
_	0.5	1.5	269613	287071	301642	300602	295586	269613	290902.8
_	0.1	0.1	245053 285261	255735 293550	244447 290234	237546 275592	252501 287320	237546 275592	247056.4 286391.4
_	1 2	1 2	303282	293330	290234		287320	273392 281213	
	2	2	290958	293243	293462	281213 285556	284266	269957	291493.6 287076.2
_	4	4	290338	290232	295884	283330	298078	291330	294029.8
-	0.1	0.5	269232	246857	263200	277621	279035	246857	267189
_	0.1	1	292862	286805	285969	281363	274708	274708	284341.4
	0.1	1.5	282938	279327	283531	281501	288995	279327	283258.4
-	0.1	2	273676	252504	284740	285089	289609	252504	277123.6
-	0.1	2.5	287137	294241	282402	281776	289601	281776	287031.4
	0.1	3	298487	278909	282960	279937	296257	278909	287310
	0.1	3.5	294911	289567	274959	295764	287723	274959	288584.8
-	0.1	4	291093	300722	285270	304412	281992	281992	292697.8
_	0.5	0.1	224440	232307	224681	244014	242270	224440	233542.4
F	1	0.1	246757	232311	233193	236904	234365	232311	236706
F	1.5	0.1	245311	244852	240318	246764	222589	222589	239966.8
	2	0.1	245429	240349	241911	252918	246917	240349	245504.8
	2.5	0.1	234574	239267	242979	236061	241821	234574	238940.4
	3	0.1	235888	237351	252707	233883	243097	233883	240585.2
	3.5	0.1	237446	250389	241200	231188	256820	231188	243408.6
	4	0.1	240967	217831	238871	218836	254645	217831	234230
	0.5	0.5	260827	279298	262871	265383	283070	260827	270289.8
					Table 2: Rest	ults of Pr152.			
c1	c2	Ex	periment1	Experiment2	Experiment3	Experiment4	Experiment	t5 Min	Average
0.5	0.5	42	3701	413966	443990	424888	434061	41396	
1	1	43	2201	409107	419764	438566	424918	40910	07 424911.2
1.5	1.5	43	0314	441548	447056	411702	431589	41170	02 432441.8
2	2	43	6253	451234	436193	450271	428341	42834	440458.4
2.5	2.5	45	3176	457128	455017	455231	456653	45317	6 455441
3	3	44	4735	448248	455977	448193	449947	44473	35 449420
3.5	3.5		6109	465903	443291	443867	434082	43408	
4	4		9091	436718	465811	474392	446595	43671	
0.1	0.1			389793	384693	401416	362889	36139	
0.1	0.1 361399 0.5 436589			434275	416377	435849	452027	41637	
0.1	1			444468	444124	438575	432027	40577	
	1 405770 1.5 431624			444408		438373	438195	40377	
					439456				
0.1	2	_	3520	438003	464836	415045	425735	41504	
0.1	2.5 440430			440120	468245	431403	449121	43140	
0.1	3 458033			454300	449803	443909	449278	44390	
0.1	3.5 439999			434864	454008	454008	452686	43486	
0.1	4	_	5721	466299	435892	457800	448980	43572	
0.5	0.1	38	7979	392751	361645	384078	394221	36164	
1	0.1	37	4037	375733	331473	370619	382359	33147	3 366844.2
1.5	0.1	39	1013	392766	381083	394322	391028	38108	33 390042.4
2	0.1	37	1601	376201	381688	370594	382907	37059	376598.2
2.5	0.1	_	9471	356977	378839	387807	351549	35154	
3	0.1	_	1948	373678	382779	357588	360112	35758	
3.5	0.1	_	2825	376146	323696	360520	3805112	32369	
4	0.1	_	1166	383257	399074	350562	380390	35056	
•	5.1	55		200207	<i>C7701</i> T	550502	500570	55050	372007.0

Table 1: Results of Pr76.

					ults of Pr299			
c1	c2	Experiment1	Experiment2	Experiment3	Experiment4	Experiment5	Min	Average
0.5	0.5	276118	260863	266901	269739	277477	260863	270219.6
1	1	275467	283840	285975	275407	285975	275407	281332.8
1.5	1.5	272859	277479	274610	271864	280109	271864	275384.2
2	2	288484	282434	292640	265244	279463	265244	281653
2.5	2.5	288096	282434	278420	271564	284711	271564	281045
3	3	290514	281846	290173	265071	284761	265071	282473
3.5	3.5	286087	292480	285594	284072	289301	284072	287506.8
4	4	270214	282669	292036	299057	288128	270214	286420.8
0.1	0.1	251891	260695	247910	237456	253309	237456	250252.2
0.1	0.5	278542	268080	269522	262481	258759	258759	267476.8
0.1	1	282375	274842	275939	273800	270957	270957	275582.6
0.1	1.5	278851	279277	283326	281587	286174	278851	281843
0.1	2	280132	273068	268620	278098	264661	264661	272915.8
0.1	2.5	273084	287611	297792	287309	276188	273084	284396.8
0.1	3	288606	282164	285607	294023	272498	272498	284579.6
0.1	3.5	291400	277743	277620	283504	274461	274461	280945.6
0.1	4	273606	282384	274589	282630	278429	273606	278327.6
0.5	0.1	248475	267648	242726	256840	240547	240547	251247.2
1	0.1	252426	248797	251207	249571	248421	248421	250084.4
1.5	0.1	253174	243635	258306	254861	257449	243635	253485
2	0.1	255079	254205	251472	253848	248869	248869	252694.6
2.5	0.1	256164	259166	254863	253064	243562	243562	253363.8
3	0.1	238266	256857	246677	245178	257281	238266	248851.8
3.5	0.1	264195	262273	257175	264861	244477	244477	258596.2
4	0.1	248948	256807	254860	249581	246985	246985	251436.2
				Table 4: Res	ults of Pr439.			
c1	c2	Experiment1	Experiment2	Experiment3	Experiment4	Experiment5	Min	Average
6								
0.5	0.5	-	733768	699029	698286	712214	698286	•
0.5	0.5	721391 751439	733768 721865	699029 715257	698286 728167	712214 719793	698286 715257	712937.6 727304.2
		721391 751439	721865	715257	728167		715257	712937.6 727304.2
1 1.5	1 1.5	721391	721865 720692	715257 741865	728167 737615	719793	715257 720692	712937.6 727304.2 733601.4
1 1.5 2	1 1.5 2	721391 751439 733236 740163	721865 720692 713860	715257 741865 728987	728167 737615 714920	719793 734599 737974	715257 720692 713860	712937.6 727304.2 733601.4 727180.8
1 1.5	1 1.5	721391 751439 733236 740163 740940	721865 720692 713860 758284	715257 741865 728987 733230	728167 737615 714920 758005	719793 734599 737974 728082	715257 720692 713860 728082	712937.6 727304.2 733601.4 727180.8 743708.2
1 1.5 2 2.5 3	1 1.5 2 2.5 3	721391 751439 733236 740163 740940 731050	721865 720692 713860 758284 740125	715257 741865 728987 733230 755457	728167 737615 714920 758005 739346	719793 734599 737974 728082 752685	715257 720692 713860 728082 731050	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6
1 1.5 2 2.5 3 3.5	1 1.5 2 2.5 3 3.5	721391 751439 733236 740163 740940 731050 738115	721865 720692 713860 758284 740125 731132	715257 741865 728987 733230 755457 751683	728167 737615 714920 758005 739346 748155	719793 734599 737974 728082 752685 747685	715257 720692 713860 728082 731050 731132	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354
1 1.5 2 2.5 3 3.5 4	1 1.5 2 2.5 3 3.5 4	721391 751439 733236 740163 740940 731050	721865 720692 713860 758284 740125 731132 754337	715257 741865 728987 733230 755457 751683 747045	728167 737615 714920 758005 739346 748155 728554	719793 734599 737974 728082 752685	715257 720692 713860 728082 731050 731132 728554	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6
$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ \end{array} $	1 1.5 2 2.5 3 3.5	721391 751439 733236 740163 740940 731050 738115 754337 666807	721865 720692 713860 758284 740125 731132 754337 650325	715257 741865 728987 733230 755457 751683 747045 673785	728167 737615 714920 758005 739346 748155 728554 676923	719793 734599 737974 728082 752685 747685 766588 671880	715257 720692 713860 728082 731050 731132 728554 650325	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944
1 1.5 2 2.5 3 3.5 4	1 1.5 2 2.5 3 3.5 4 0.1	721391 751439 733236 740163 740940 731050 738115 754337	721865 720692 713860 758284 740125 731132 754337	715257 741865 728987 733230 755457 751683 747045	728167 737615 714920 758005 739346 748155 728554	719793 734599 737974 728082 752685 747685 766588	715257 720692 713860 728082 731050 731132 728554	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2
$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ \end{array} $	1 1.5 2 2.5 3 3.5 4 0.1 0.5	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233	721865 720692 713860 758284 740125 731132 754337 650325 708297	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069	728167 737615 714920 758005 739346 748155 728554 676923 727938	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874
$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1 \end{array} $	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ \end{array} $	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731233	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6
$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ \end{array} $	1 1.5 2 2.5 3 3.5 4 0.1 0.5 1 1.5 2	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731233 731320	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2
$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	1 1.5 2 2.5 3 3.5 4 0.1 0.5 1 1.5 2 2.5	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731233 731320 756208	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8
$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	1 1.5 2 2.5 3 3.5 4 0.1 0.5 1 1.5 2 2.5 3	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731233 731320 756208 742448	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ \end{array} $	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731233 731320 756208 742448 752241	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ \end{array} $	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448 745264	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241 735735	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180 750412	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002 744999	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731233 731320 756208 742448 752241 764778	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448 735735	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4 748237.6
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ \end{array} $	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448 745264 644250	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241 735735 668097	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180 750412 661381	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002 744999 658860	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731320 756208 742448 752241 764778 649496	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448 735735 644250	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4 748237.6 656416.8
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1 \end{array} $	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448 745264 644250 647539	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241 735735 668097 649022	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180 750412 661381 644349	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002 744999 658860 636959	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731320 756208 742448 752241 764778 649496 664793	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448 735735 644250 636959	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4 748237.6 656416.8 648532.4
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ \end{array} $	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448 745264 644250 647539 661769	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241 735735 668097 649022 679815	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180 750412 661381 644349 651355	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002 744999 658860 636959 642531	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731320 756208 742448 752241 764778 649496 664793 659875	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448 735735 644250 636959 642531	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4 748237.6 656416.8 648532.4 659069
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ \end{array} $	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448 745264 644250 647539 661769 675051	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241 735735 668097 649022 679815 668607	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180 750412 661381 644349 651355 671714	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002 744999 658860 636959 642531 651675	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731320 756208 742448 752241 764778 649496 664793 659875 652325	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448 735735 644250 636959 642531 651675	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4 748237.6 656416.8 648532.4 659069 663874.4
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448 745264 644250 647539 661769 675051 636674	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241 735735 668097 649022 679815 668607 653138	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180 750412 661381 644349 651355 671714 643268	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002 744999 658860 636959 642531 651675 640645	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731320 756208 742448 752241 764778 649496 664793 659875 652325 666329	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448 735735 644250 636959 642531 651675 636674	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4 748237.6 656416.8 648532.4 659069 663874.4 648010.8
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$\begin{array}{c} 1 \\ 1.5 \\ 2 \\ 2.5 \\ 3 \\ 3.5 \\ 4 \\ 0.1 \\ 0.5 \\ 1 \\ 1.5 \\ 2 \\ 2.5 \\ 3 \\ 3.5 \\ 4 \\ 0.1$	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448 745264 647539 661769 675051 636674 656751	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241 735735 668097 649022 679815 668607 653138 661341	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180 750412 661381 644349 651355 671714 643268 661604	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002 744999 658860 636959 642531 651675 640645 669651	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731320 756208 742448 752241 764778 649496 664793 659875 652325 666329 654244	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448 735735 644250 636959 642531 651675 636674 654244	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4 748237.6 656416.8 648532.4 659069 663874.4 648010.8 660718.2
$\begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	$ \begin{array}{c} 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.5\\ 1\\ 1.5\\ 2\\ 2.5\\ 3\\ 3.5\\ 4\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1\\ 0.1$	721391 751439 733236 740163 740940 731050 738115 754337 666807 695233 728326 739077 741839 759642 761002 730448 745264 644250 647539 661769 675051 636674	721865 720692 713860 758284 740125 731132 754337 650325 708297 721911 726468 744669 724148 768932 752241 735735 668097 649022 679815 668607 653138	715257 741865 728987 733230 755457 751683 747045 673785 684157 733069 724886 733507 733117 756175 752180 750412 661381 644349 651355 671714 643268	728167 737615 714920 758005 739346 748155 728554 676923 727938 721935 729639 742866 743989 734257 753002 744999 658860 636959 642531 651675 640645	719793 734599 737974 728082 752685 747685 766588 671880 709905 714129 731320 756208 742448 752241 764778 649496 664793 659875 652325 666329	715257 720692 713860 728082 731050 731132 728554 650325 684157 714129 724886 731320 724148 734257 730448 735735 644250 636959 642531 651675 636674	712937.6 727304.2 733601.4 727180.8 743708.2 743732.6 743354 750172.2 667944 705106 723874 730260.6 738840.2 743420.8 752562.8 748022.4 748237.6 656416.8 648532.4 659069 663874.4 648010.8

Table 3: Results of Pr299.

The obtained results show that even when applying PSO on the same problem then the performance will vary depending on the dataset and on the values of the learning factors (c1 and c2). As in table 1 it was found that the best average value is obtained when c1=0.5 and c2=0.1, while in table 2 the best average value is obtained when c1=3.5 and c2=0.1, in table 3, when c1=3 and c2=0.1, the best average value is obtained when c1=2.5 and c2=0.1.

The obtained results strongly support the aim of this paper as it was found that even for the same problem, different values for the learning factors (c1 and c2) can lead to better results. One another interesting thing to notice from the results is that for all the used datasets, c2=0.1 gave the best average. This can give an indication about fixing the value c2 and experiment only c1.

VII. Conclusions and Future Work

In this paper, extensive experiments are done in order to show the importance of selecting the best values for the PSO learning factors, those experiments are applied on MTSP using four different datasets. Each dataset is applied to a PSO with different values of the learning factors, and is executed for five times.

The obtained results are strongly supported the aim of this paper as it was found that it is important to carefully choose the values for the learning factors in PSO even for the same problem rather than use fixed values. In addition, it was found that, for MTSP, fixing c2 value to 0.1 gave better results than varying it.

For the possible directions for future work it is recommended to do more experiments with more MTSP dataset, also, it is recommended to apply the same settings that are used in this paper to other problems.

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