MODIFIED QUASI SIMPS0N 'S 3/8 RULE FOR SOLVING SYSTEM OF INTEGRAL EQUATION OF THE SECOND KIND LINEAR

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Abstract:
Actually, it is possible to solve systems of integral equation by using many approaches. However, in this study, the modified quasi Simpson's 3/8 rule used   to find the numerical solution of a system   of linear Volterra integral equations of the second kind. This method solves systems of linear Volterra integral equations of   the second kind in more accurate way than the modified   trapezoidal   rule. Some indicative examples are given to elaborate the accuracy of this method .

Key words : MODIFIED QUASI SIMPS0N 'S 3/8 , INTEGRAL EQUATION , SECOND KIND LINEAR

1. Introduction:

In the   present study , the modified trapezoidal   method   used   for   solving system of   linear integral equations of the second kind ,[7].Furthermore , some numerical rules are used for solving system of the one – dimensional linear volterra integral equations, [4].

Now we consider the following system of linear Volterra integral equations of the second kind :

$$r = 1,2,\ldots,m \ \ldots\ldots\ldots\ldots\ (\ 1.1\ )\quad u_r(x) = f_r(x) + \sum_{s=1}^{m}\lambda_{r,s}\int_{a}^{x} k_{r,s}(x,y)u_s(y)dy,$$

where $a \le x \le b$, $\lambda_{r,s}$ is a real number, f_r , $k_{r,s}$ are given continuous functions and u_1 , u_2 , … , u_m are function that must be determined   by systems of linear   volterra integral equations   of   the unknown the second kind.

There   are   many   methods   for solving   the   one – dimensional   volterra integral equations. These methods depend on the structure of the one–dimensional integral equations,[3].They   introduce   numerical solutions   for   one – dimensional volterra linear integral equations of the second kind , [2]. In this paper the repeated modified quasi Simpson's 3/8 rule

$$\int_{a}^{b} f(x)dx = \frac{3h}{80}\left[13f(a) + 27\sum_{j=1}^{\frac{n}{3}}[f(x_{3j-1}) + f(x_{3j-2})] + 26\sum_{j=1}^{\frac{n}{3}-1} f(x_{3j}) + 13f(b)\right] +$$

$$\frac{3h^2}{40}\left[f^{/}(a) - f^{/}(b)\right] \ \cdots\cdots\cdots\quad (\ 1.2\ )\quad ,\ [\ 1\ ]$$

and repeated modified trapezoidal rule

$$..... (1.3)\int_a^b f(x)dx = \frac{h}{2}\left[f(a)+2\sum_{j=1}^{n-1} f(x_j)+f(b)\right]+\frac{h^2}{12}\left[f'(a)-f'(b)\right]-\frac{(b-a)h^{(4)}}{720}f^{(4)}(\xi)$$

to find the numerical solution for the system given by equation (1.1).To do this, one assumes the used   and fr/(x) exist for all r,

s = 1,2,…,m. For further information on $\frac{\partial k_{r,s}(x,y)}{\partial x},\frac{\partial k_{r,s}(x,y)}{\partial y}$ function formula   (1.3), See, [6],[5].

2.The   Modified   Quasi Simpson's 3/8 Rule.

Now , we approximate the integral that appear in the right hand side of equation (1.1) at x = xi , the repeated modified quasi Simpson's 3/8 rule to get : i = 0,1,…, n, by

$$u_r(x_0) = f_r(x_0),$$

$$u_r(x_i) = f_r(x_i) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{3h}{80}\left[13k_{r,s}(x_i,x_0)u_s(x_0)+27\sum_{j=1}^{\frac{i}{3}}\left\{k_{r,s}(x_i,x_{3j-1})u_s(x_{3j-1})+k_{r,s}(x_i,x_{3j-2})u_s(x_{3j-2})\right\}+\right.\right.$$

$$26\sum_{j=1}^{\frac{i}{3}-1}k_{r,s}(x_i,x_{3j})u_s(x_{3j})+13k_{r,s}(x_i,x_i)u_s(x_i)\right]+$$

$$\frac{3h^2}{40}\left[k_{r,s}(x_i,x_0)u_s'(x_0)+J_{r,s}(x_i,x_o)u_s(x_o)-k_{r,s}(x_i,x_i)u_s'(x_i)-J_{r,s}(x_i,x_i)u_s(x_i)\right]\right\}, i = 3,6,...,n$$

and

$$u_r(x_i) = f(x_i) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{h}{2}\left[k_{r,s}(x_i,x_0)u_s(x_0)+2\sum_{j=1}^{i-1}k_{r,s}(x_i,x_j)u_s(x_j)+k_{r,s}(x_i,x_i)u_s(x_i)\right]+\right.$$

$$\frac{h^2}{12}\left[k_{r,s}(x_i,x_0)u_s'(x_0)+J_{r,s}(x_i,x_0)u_s(x_0)-k_{r,s}(x_i,x_i)u_s'(x_i)-J_{r,s}(x_i,x_i)u_s(x_i)\right]\right\}, i \neq 3,6,...,n$$

i=1,2   ,   … ,n. the   above   system   of   equations   consists   of   and $\ell = 0,i$,

$$J_{r,s}(x_i,y_\ell) = \frac{\partial k_{r,s}(x_i,y_\ell)}{\partial y}$$
where
equations with 2m(n+1) unknowns namely, ur(xi) , i=0,1,…,n , u/r(xi). Next,we have to find u/r(xi),   2m(n+1)

, r= 1,2,…,m. to do this , we must differentiate equation (1.1) with respect to x to get i=0,1,…,n

$$u_s'(x) = f_r'(x) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\int_a^x H_{r,s}(x,y)u_s(y)\,dy + k_{r,s}(x,x)u_s(x)\right\},\; a\leq x\leq b,\; r=1,2,...,m .... (2.1)$$

, r = 1,2,…,m. It is easy to check any solution of equation (1.1) is a solution

$$H_{r,s}(x,y) = \frac{\partial k_{r,s}(x,y)}{\partial x}$$
where equation (2.2) . By evaluating equation (2.1) at x = xi , i = 0,1,…,n , one can get   of

$$u_s'(x_i) = f_r'(x_i) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\int_a^{x_i} H_{r,s}(x_i,y)u_s(y)\,dy + k_{r,s}(x_i,x_i)u_s(x_i)\right\},\; i=o,1,...,n,\; r=1,2,...,m .... (2.2)$$

:   Next, to solve equation (2.2), one must consider three cases :

exists   for   each   r , s = 1,2,…,m , we approximate   the   integral   that

$$\frac{\partial^2 k_{r,s}(x,y)}{\partial x \partial y}$$
Case (1):- If

appear in the right hand side of the integral equation (2.2)with the repeated modified quasi Simpson's 3/8 rule to obtain :

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$$u_r{}'(x_0) = f_r{}'(x_0) + \lambda_{r,s}\, k_{r,s}(x_o,x_0)\, u_s(x_0),$$

$$u_r{}'(x_i) = f_r{}'(x_i) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{3h}{80}\left[13\,H_{r,s}(x_i,x_0)u_s(x_0) + 27\sum_{j=1}^{\frac{i}{3}}\left\{H_{r,s}(x_i,x_{3j-1})u_s(x_{3j-1}) + H_{r,s}(x_i,x_{3j-2})u_s(x_{3j-2})\right\} + \right.\right.$$

$$26\sum_{j=1}^{\frac{i}{3}-1} H_{r,s}(x_i,x_{3j})u_s(x_{3j}) + 13\,H_{r,s}(x_i,x_i)u_s(x_i)\Big] +$$

$$\frac{3h^2}{40}\left[H_{r,s}(x_i,x_0)u_s{}'(x_0) + L_{r,s}(x_i,x_o)u_s(x_o) - H_{r,s}(x_i,x_i)u_s{}'(x_i) - L_{r,s}(x_i,x_i)u_s(x_i)\right] + k_{r,s}(x_i,x_i)u_s(x_i)\Bigg\},$$

where $i = 3,6,...,n$, $r = 1,2,...,m$,

and

$$u_r(x_i) = f(x_i) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{h}{2}\left[H_{r,s}(x_i,x_0)u_s(x_0) + 2\sum_{j=1}^{i-1}H_{r,s}(x_i,x_j)u_s(x_j) + H_{r,s}(x_i,x_i)u_s(x_i)\right] + \right.$$

$$\frac{h^2}{12}\left[H_{r,s}(x_i,x_0)u_s{}'(x_0) + L_{r,s}(x_i,x_0)u_s(x_0) - H_{r,s}(x_i,x_i)u_s{}'(x_i) - L_{r,s}(x_i,x_i)u_s(x_i)\right] + k_{r,s}(x_i,x_i)u_s(x_i)\Bigg\},$$

where $i \neq 3,6,...,n$, $r = 1,2,...,m$,

$$L_{r,s}(x_i,y_\ell) = \frac{\partial H_{r,s}(x_i,y_\ell)}{\partial y} , \quad \ell = 0,i , \quad i = 1,2,...,n , \quad r,s = 1,2,...,m.$$
and

By solving the above equations with the equation (1.1) altogether, one can get the system of linear equations. does not exist for each r, s = 1 , 2 , ... ,m, we approximate the integral
$$\frac{\partial^2 k_{r,s}(x,y)}{\partial x \partial y}$$
Case (2) :- if

appear in the right hand side of the integral equation (2.2) with the repeated Simpson's 3/8 rule to that obtain:

$$u_r{}'(x_0) = f_r{}'(x_0) + \lambda_{r,s}\, k_{r,s}(x_o,x_0)\, u_s(x_0),$$

$$u_r{}'(x_i) = f_r{}'(x_i) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{3h}{8}\left[H_{r,s}(x_i,x_0)u_s(x_0) + 3\sum_{j=1}^{\frac{i}{3}}\left\{H_{r,s}(x_i,x_{3j-1})u_s(x_{3j-1}) + H_{r,s}(x_i,x_{3j-2})u_s(x_{3j-2})\right\} + \right.\right.$$

$$2\sum_{j=1}^{\frac{i}{3}-1}H_{r,s}(x_i,x_{3j})u_s(x_{3j}) + H_{r,s}(x_i,x_i)u_s(x_i)\Big] + k_{r,s}(x_i,x_i)u_s(x_i)\Bigg\}, i = 3,6,....,n, r = 1,2,...,m.$$

$$u_r{}'(x_i) = f_r{}'(x_i) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{h}{2}\left[H_{r,s}(x_i,x_0)u_s(x_0) + 2\sum_{j=1}^{i-1}H_{r,s}(x_i,x_j)u_s(x_j) + H_{r,s}(x_i,x_i)u_s(x_i)\right] + k_{r,s}(x_i,x_i)u_s(x_i)\right\},$$

$$i \neq 3,6,...,n , r = 1,2,...,m.$$

By solving the above equations with the equation (1.1) altogether, one can get the system of linear equations.

does not exist for some p, q =1 ,2 , … ,m , we approximate the integral

$$\frac{\partial^2 k_{r,s}(x, y)}{\partial x \partial y}$$

Case(3):- if that appear in the right hand side of the integral equation(2.2) with the repeated

$$\int_a^{x_i} \frac{\partial k_{p,q}(x, y)}{\partial x \partial y} u_q(y)\, dy$$

Simpson's 3/8 rule and the other integrals that appear in the right hand side of the integral equation (2.2) with the repeated modified quasi Simpson's 3/8 rule to obtain:

$$u_r'(x_0) = f_r'(x_0) + \lambda_{r,s}\, k_{r,s}(x_o, x_0)\, u_s(x_0),$$

$$u_p'(x_0) = f_p'(x_0) + \lambda_{p,s}\, k_{p,s}(x_o, x_0)\, u_s(x_0),$$

$$u_r'(x_i) = f_r'(x_i) + \sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{3h}{80}\left[13 H_{r,s}(x_i, x_0)u_s(x_0) + 27\sum_{j=1}^{\frac{i}{3}}\{H_{r,s}(x_i, x_{3j-1})u_s(x_{3j-1}) + H_{r,s}(x_i, x_{3j-2})u_s(x_{3j-2})\}\right. \right. +$$

$$26\sum_{j=1}^{\frac{i}{3}-1} H_{r,s}(x_i, x_{3j})u_s(x_{3j}) + 13 H_{r,s}(x_i, x_i)u_s(x_i)\Big] +$$

$$\frac{3h^2}{40}\Big[H_{r,s}(x_i, x_0)u_s'(x_0) + L_{r,s}(x_i, x_o)u_s(x_o) - H_{r,s}(x_i, x_i)u_s'(x_i) - L_{r,s}(x_i, x_i)u_s(x_i)\Big] + k_{r,s}(x_i, x_i)u_s(x_i)\Big\},$$

where $i = 3,6,...,n$, $r = 1,2,...,m$, $r \neq p$,

$$u_p'(x_i) = f_p'(x_i) + \sum_{\substack{s=1\\s\neq q}}^{m}\lambda_{p,s}\left\{\frac{3h}{80}\left[13H_{p,s}(x_i, x_0)u_s(x_0) + 27\sum_{j=1}^{\frac{i}{3}}\Big[H_{p,s}(x_i, x_{3j-1})u_s(x_{3j-1}) + H_{p,s}(x_i, x_{3j-2})u_s(x_{3j-2})\Big]\right. \right. +$$

$$26\sum_{j=1}^{\frac{i}{3}-1} H_{p,s}(x_i, x_{3j})u_s(x_{3j}) + 13 H_{p,s}(x_i, x_i)u_s(x_i)\Big] +$$

$$\frac{3h^2}{40}\Big[H_{p,s}(x_i, x_0)\, u_s'(x_0) + L_{p,s}(x_i, x_0)\, u_s(x_0) - H_{p,s}(x_i, x_i)\, u_s'(x_i) - L_{p,s}(x_i, x_i)\, u_s(x_i)\Big] + k_{p,s}(x_i, x_i)u_s(x_i)\Big\} +$$

$$\frac{3h}{8}\lambda_{p,q}\left[H_{p,q}(x_i, x_0)u_q(x_0) + 3\sum_{j=1}^{\frac{i}{3}}\Big[H_{p,q}(x_i, x_{3j-1})u_q(x_{3j-1}) + H_{p,q}(x_i, x_{3j-2})u_q(x_{3j-2})\Big] \right. +$$

$$26\sum_{j=1}^{\frac{i}{3}-1} H_{p,q}(x_i, x_{3j})u_q(x_{3j}) + H_{p,q}(x_i, x_i)u_q(x_i)\Big] + \lambda_{p,q}\, k_{p,q}(x_i, x_i)u_q(x_i), \quad i = 3,6,...,n,$$

and

$$u_p'(x_i) = f_p'(x_i) + \sum_{\substack{s=1\\s\neq q}}^{m}\lambda_{p,s}\left\{\frac{h}{2}\left[H_{p,s}(x_i, x_0)u_s(x_0) + 2\sum_{j=1}^{i-1} H_{p,s}(x_i, x_j)u_s(x_j) + H_{p,s}(x_i, x_i)u_s(x_i)\right] \right. +$$

$$\frac{h^2}{12}\Big[H_{p,s}(x_i, x_0)u_s'(x_0) + L_{p,s}(x_i, x_0)u_s(x_0) - H_{p,s}(x_i, x_i)u_s'(x_i) - L_{p,s}(x_i, x_i)u_s(x_i)\Big] + k_{p,s}(x_i, x_i)u_s(x_i)\Big\} +$$

$$\lambda_{p,q}\left\{\frac{h}{2}\left[H_{p,q}(x_i,x_0)u_q(x_0)+2\sum_{j=1}^{i-1}H_{p,q}(x_i,x_j)u_q(x_j)+H_{p,q}(x_i,x_i)u_q(x_i)\right]+k_{p,q}(x_i,x_i)u(x_i)\right\}, i\neq 3,6,...,n.$$

where $L_{r,s}(x,y_\ell)=\dfrac{\partial H_{r,s}(x,y_\ell)}{\partial y}=\dfrac{\partial^2 k_{r,s}(x,y_\ell)}{\partial x\partial y}$ *and* $L_{p,s}(x,y_\ell)=\dfrac{\partial H_{p,s}(x,y_\ell)}{\partial y}=\dfrac{\partial^2 k_{p,s}(x,y_\ell)}{\partial x\partial y},$

$\ell=0,i$, $i=1,2,...,n$, $r,s=1,2,...,m.$

By solving the above equations with the equation (1.1) altogether, one can get the system of linear equations.

3. Numerical Examples:

In this section we give some numerical examples to illustrate the above method for solving systems of linear volterra integral equations of the second kind

. Also, we compare this method with modified trapezoidal rule method for solving system of linear volterra integral equations

Example (1),[7]:

Consider the following system of linear volterra integral equations of the second kind:

.

$$u_1(x)=(-\frac{1}{2}x^2+\frac{1}{4}x+1)e^{2x}+(x+\frac{1}{4})e^{-2x}-\frac{3}{4}x-\frac{1}{4}+\int_o^x xy\,u_1(y)\,dy+\int_o^x (x+y)\,u_2(y)\,dy,$$

$$u_2(x)=(2x^2+x+\frac{5}{4})e^{-2x}-\frac{1}{4}e^{2x}-\frac{1}{2}x^2+\int_0^x (x-y)u_1(y)dy+\int_0^x (x+y)^2 u_2(y)dy,\ 0\le x\le 1$$

we solve this example numerically by using the composite modified quasi Simpson's 3/8 rule . To do this , , i = 0, 1, … ,9 . we subdivide the interval [0,1] into 9 subintervals such that xi = $\frac{i}{9}$

Here's $k_{11}(x,y)=xy$, $k_{12}(x,y)=x+y$, $k_{21}(x,y)=x-y$, $k_{22}(x,y)=(x+y)^2$,

$$\frac{\partial^2 k_{11}(x,y)}{\partial x\partial y}=1,\qquad \frac{\partial^2 k_{12}(x,y)}{\partial x\partial y}=0,\qquad \frac{\partial^2 k_{21}(x,y)}{\partial x\partial y}=0,\quad and\quad \frac{\partial^2 k_{22}(x,y)}{\partial x\partial y}=2$$

Table (1) represents the exact and the numerical solutions of example (1) at specific points for n = 9

Nodes x_i	Exact Solutions		Modified Trapezoidal		Modified Quasi Simpson's 3/8 Rule	
	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$
x = 0	1	1	1	1	1	0
x = 0.11111111111111	1. 24882886900168	0. 80073740291681	1. 24884939142863	0. 80073742524729	1. 24884939142863	0.80073742524729
x = 0.22222222222222	1. 55962349760678	0. 64118038842995	1. 55962408801108	0. 61181101976591	1. 55962408801108	0.64118110197659
x = 0.33333333333333	1. 94773404105468	0. 51341711903259	1. 9477342626525	0. 51341909763927	1. 94773404433254	0. 3134171767883
x = 0.44444444444444	2.43242545428721	0. 41111229050719	2. 43242462502882	0. 41111616404039	2. 43242480184683	0.41111602722440
x = 0.55555555555556	3.03773177751748	0. 32919298780791	3. 03772951997607	0. 32919964926517	3. 03772974835500	0.32919944641561
x = 0.66666666666667	3.79366789468318	0. 26359713811573	3. 7936639166085	0. 26360807970410	3. 7936639166085	0.26359909125401
x = 0.77777777777778	4.73771785964308	0. 21107208779109	4. 73771138295678	0. 21109004083595	4. 73771138295678	0.21108734673334
x = 0.88888888888889	5.91669359066433	0. 16901331540607	5. 91668175312201	0. 16904361933690	5. 91668440476481	0.16903939481635
x = 1	7. 38905609893065	0. 13533528323661	7. 38904532827186	0. 13538908681802	7. 38905746371846	0.13535405723657

Example (2),[7]:
Consider the following system of linear volterra integral equations of the second kind:

$$u_1(x)=\sin x-\frac{2}{5}x^{\frac{5}{2}}+x+\int_0^x (x-y)^{\frac{3}{2}}\left(u_1(y)+u_2(y)\right)dy,$$

$$u_2(x)=-\sin x+\frac{2}{5}(1-x)^{\frac{5}{2}}-x+\frac{3}{5}+\int_0^x (1-x+y)^{\frac{3}{2}}\left(u(y)+u(y)\right)dy,\ \ 0\le x\le 1$$

we solve this example numerically by using the composite modified quasi Simpson's 3/8 rule . to do this , i = 0, 1, … ,9 . we subdivide the interval [0,1] into 9 subintervals such that $x_i = \dfrac{i}{9}$. Here $k_{11}(x , y) = k_{12}(x , y) = (x - y)^{3/2}$ and $k_{21}(x , y) = k_{22}(x , y) = (1-x + y)^{3/2}$,

$$\frac{\partial^2 k_{11}(x, y)}{\partial x \partial y} = \frac{\partial^2 k_{12}(x, y)}{\partial x \partial y} = \frac{-3}{4(x - y)^{\frac{1}{2}}} \quad and \quad \frac{\partial^2 k_{21}(x, y)}{\partial x \partial y} = \frac{\partial^2 k_{22}(x, y)}{\partial x \partial y} = \frac{-3}{4(1 - x + y)^{\frac{1}{2}}}$$

Table (2) represents the exact and the numerical solutions of example (2) at specific points for n = 9

Nodes x_i	Exact Solutions		Modified Trapezoidal		Modified Quasi Simpson's 3/8 Rule	
	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$
x = 0	0	1	0	1	0	1
x = 0.11111111111111	0. 2219937392106	0. 77800626037894	0. 22189085896263	0. 77801048748531	0. 22189085896263	0. 77801048748531
x = 0.22222222222222	0.44261996567834	0. 55738003432166	0. 44251542458257	0. 55737508783952	0. 44251542458257	0. 55737508783952
x = 0.33333333333333	0.66052803012949	0. 339471969987051	0. 66042196245848	0. 33945813898064	0. 66044068349694	0. 33945760912229
x = 0.44444444444444	0.87440080797280	0. 12559919202720	0. 87429231971092	0. 12557707003956	0. 8742939457174	0. 12557886508212
x = 0.55555555555556	1.08297094132742	-0. 08297094132742	1. 08285873285773	-0. 08300073194773	1. 08285895229119	-0. 08299907007052
x = 0.66666666666667	1. 28503646973640	-0. 28503646973640	1. 28491892585430	-0. 38511007708479	1. 28493825316104	-0. 28507340973712
x = 0.77777777777778	1.47947565392451	-0. 47947565392451	1. 47935085918747	-0. 47951953684871	1. 47935144859839	-0. 47952005566370
x = 0.88888888888889	1.66526081018955	-0. 66526081018955	1. 66512654146858	-0. 66531266536194	1. 66512727868642	-0. 6653131707032
x = 1	1.84147098480790	-0. 84147098480790	1. 84132468733486	-0. 84164753917704	1. 8413447359289	-0. 84162671487513

Example (3), [7]:
Consider the following system of linear volterra integral equations of the second kind:

$$u_1(x) = -\frac{1}{60}x^6 - \frac{5}{4}x^4 + \frac{1}{5}(3 - 3x)^{\frac{5}{2}} - \frac{1}{5}(3 - x)^{\frac{5}{2}} - 1 + \int_o^x (3 - x - 2y)^{\frac{3}{2}} u_1(y)\, dy + \int_o^x (-x + y)^3\ u_2(y)\, dy,$$

$$u_2(x) = -\frac{1}{6}x^2 + \frac{1}{8}x^4 + \frac{25}{18}x^3 - \frac{1}{4}x^2 + 10x - 5 + \int_0^x \frac{(x^2 + y^2)}{3} u_1(y)dy + \int_0^x \frac{(x^2 - y + 4)}{2} u_2(y)dy, \; 0 \le x \le 1$$

we solve this example numerically by using the composite Simpson's 3/8 rule to this $\int_o^x (3 - x - 2y)^{\frac{3}{2}} u_1(y)\, dy$ integral,
and other integrals by using the modified quasi Simpson's 3/8 rule . To do , i = 0, 1, … ,9 . we subdivide the interval [0,1] into 9 subintervals such that $x_i = \dfrac{i}{9}$

$$k_{11}(x, y) = (3 - x - 2y)^{\frac{3}{2}}, \; k_{12}(x, y) = (y - x)^3, \; k_{21}(x, y) = \frac{(x^2 + y^2)}{3} \quad and \quad k_{22}(x, y) = \frac{(x^2 - y + 4)}{2},$$

$$\frac{\partial^2 k_{11}(x, y)}{\partial x \partial y} = \frac{-3}{2(3 - x - 2y)^{\frac{1}{2}}}, \quad \frac{\partial^2 k_{12}(x, y)}{\partial x \partial y} = 6(x - y), \quad and \quad \frac{\partial^2 k_{21}(x, y)}{\partial x \partial y} = \frac{\partial^2 k_{22}(x, y)}{\partial x \partial y} = 0$$

Table (3) represents the exact and the numerical solutions of example (3) at specific points for n = 9

Nodes x_i	Exact Solutions		Modified Trapezoidal		Modified Quasi Simpson's 3/8 Rule	
	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$
x = 0	1	-5	1	-5	1	-5
x = 0.11111111111111	1	-4. 98765432098765	0. 999999744137100	-4. 9876543214144	0.999999744137100	-4. 9876543242144
x = 0.22222222222222	1	-4. 95061728395062	0. 99999959181404	-4.25061728853062	0.99999954181404	-4.25061728853062
x = 0.33333333333333	1	-4. 88888888888889	0. 99999930971351	-4.88888890650520	0.99999943468232	-4. 88888890876938
x = 0.44444444444444	1	-4. 80246913580247	0. 99999933973798	-4.80240919473750	0.99999938236548	-4. 80246919615093
x = 0.55555555555556	1	-4. 64135802469136	0. 999999913827138	-4.69135816995950	0.99999939065581	-4. 69135816650165
x = 0.66666666666667	1	-4. 55555555555557	0. 99999936627144	-4. 5555586007242	0.99999990322345	-4. 55555583969343
x = 0.77777777777778	1	-4. 39506172839506	0. 999999456046670	-4.39506239588574	0.9999995628623	-4. 39506236200500
x = 0.88888888888889	1	-4. 20987654320989	0. 9999984549989	-4.20987800641883	0.99999812206309	-4. 20987806190103
x = 1	1	-4	0. 9997569473680	-4.00002419438480	0.99970570014155	-4. 00002033667935

Note:- one can notice in the above examples that (n=9) but it is not fixed value . so that , it is possible to take (n) to equal any number of multi (3) .

4.   Conclusions   and Recommendations:

From the present study , we can conclude that the composite modified quasi Simpson's 3/8 rule can be used to solve systems of the one – dimensional volterra integral equations of the second kind and also the repeated modified quasi Simpson's 3/8 rule gave more accurate results than   the repeated modified trapezoidal rule . We will use this method to solve the multi – dimensional integral equations.

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قاعدة شبه سمبسون ٨١٣ المطورة لحل أنظمة من المعادلات التكاملية الخطية من النوع الثاني

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الخلاصة

يمكن حل منظومة من المعادلات التكاملية في عدة طرق . لكن في هذا البحث ، سوف نستخدم قاعدة شبه سمبسون ٨١٣ المطورة لإيجاد الحل العددي لمنظومة من معادلات فولتيرا التكاملية الخطية من النوع الثاني . هذه الطريقة تقوم بحل منظومة من معادلات فولتيرا التكاملية الخطية من النوع الثاني بدقة أكثر من طريقة شبه المحرف المطورة . ثم إعطاء بعض الأمثلة الإيضاحية لبيان دقة هذه الطريقة .