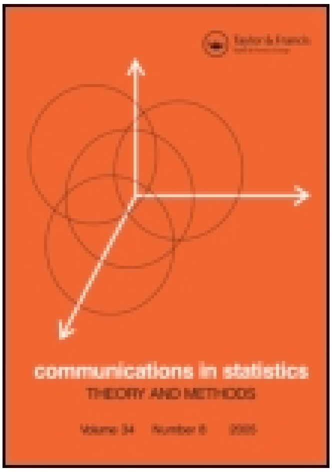
A Generalized Stochastic Restricted Ridge Regression Estimator

ARTICLE *in* POLITICAL COMMUNICATION · OCTOBER 2014

Impact Factor: 1.24 · DOI: 10.1080/03610926.2012.724506



Communications in Statistics - Theory and Methods
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/lsta20

A Generalized Stochastic Restricted Ridge Regression Estimator

M. I. Alheety[a] & B. M. Golam Kibria[b]
[a] Department of Mathematics, Anbar University, Ramadi, Iraq
[b] Department of Mathematics and Statistics, Florida International University, Miami, Florida, USA
Published online: 30 Sep 2014.

Communications in Statistics—Theory and Methods, 43: 4415–4427, 2014

ISSN: 0361-0926 print / 1532-415X online
DOI: 10.1080/03610926.2012.724506

A Generalized Stochastic Restricted Ridge Regression Estimator

M. I. ALHEETY[1] AND B. M. GOLAM KIBRIA[2]

[1]Department of Mathematics, Anbar University, Ramadi, Iraq
[2]Department of Mathematics and Statistics, Florida International University, Miami, Florida, USA

In this article, we introduce a new stochastic restricted estimator for the unknown vector parameter in the linear regression model when stochastic linear restrictions on the parameters hold. We show that the new estimator is a generalization of the ordinary mixed estimator (OME), Liu estimator (LE), ordinary ridge estimator (ORR), (k-d) class estimator, stochastic restricted Liu estimator (SRLE), and stochastic restricted ridge estimator (SRRE). Performance of the new estimator in comparison to other estimators in terms of the mean squares error matrix (MMSE) is examined. Numerical example from literature have been given to illustrate the results.

Keywords Linear regression model; Multicollinearity; Ordinary mixed estimator; Stochastic linear restrictions.

Mathematics Subject Classification 62J05; 62J07.

1. Introduction

We consider the standard multiple linear regression model

$$Y = X\beta + \epsilon, \tag{1}$$

where Y is an $n \times 1$ vector of observations on the response (or dependent) variable, X is an $n \times p$ model matrix of observations on p non-stochastic explanatory variables, β is a $p \times 1$ vector of unknown parameters associated with the p explanatory variables, and ϵ is an $n \times 1$ vector of residuals with expectation $\mathrm{E}(\epsilon) = 0$ and dispersion matrix $\mathrm{Var}(\epsilon) = \sigma^2 I_n$.

If the least squares method is applied to (1), we get the ordinary least squares estimator (OLSE) as

$$\hat{\beta} = S^{-1} X' Y, \tag{2}$$

Received March 5, 2012; Accepted August 14, 2012.
Adress correspondence to B.M. Golam Kibria, Department of Mathematics and Statistics, Florida International University, Miami, FL 33199, USA; E-mail: Kibriag@fiu.edu
Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/lsta.

where $S = X'X$. The OLSE in (2) is unbiased and has minimum variance among all linear unbiased estimators. However, many results have proved that the OLSE is no longer a good estimator when the multicollinearity is present. Due to multicollinearity, estimates of the correlation coefficients can be large in magnitude and their signs can be contrary to intuition. The regression coefficients can be unstable with respect to the correlation coefficients. The correlation matrix has one or more small eigenvalues. Several techniques have been proposed for reducing multicollinearity. Biased estimation as an alternative to OLSE has been recommended in order to obtain some reduction in variance. In addition to model (1), we suppose that, there are some prior information about β in the form of a set of independent stochastic linear restrictions (Theil, 1963)

$$r = R\beta + \epsilon^*, \tag{3}$$

where R is an $q \times p$ non zero matrix with rank $(R) = q < p$, r is an $q \times 1$ known vector which is interpreted as a random variable with $E(r) = R\beta$, and ϵ^* is an $q \times 1$ vector of disturbances with zero mean and variance-covariance matrix $\sigma^2 V$, V is known and positive definite. It is clear that the stochastic restrictions in (3) do not hold exactly but will hold at the mean. Further, it is also assumed that ϵ^* is stochastically independent of ϵ. By unifying the sample and prior information (3) in a common model

$$\begin{pmatrix} Y \\ r \end{pmatrix} = \begin{pmatrix} X \\ R \end{pmatrix} \beta + \begin{pmatrix} \epsilon \\ \epsilon^* \end{pmatrix}, \tag{4}$$

where $E(\epsilon\epsilon^{*\prime}) = 0$ and

$$E\begin{pmatrix} \epsilon \\ \epsilon^* \end{pmatrix} (\epsilon' \ \epsilon^{*\prime}) = \sigma^2 \begin{pmatrix} I & 0 \\ 0 & V \end{pmatrix}.$$

We can use the least squares method for model (4) to get the ordinary mixed estimator (OME) which is introduced by Theill and Goldberger (1961). The OME is defined as follows:

$$\hat{\beta}_{OME} = (S + R'V^{-1}R)^{-1}(X'Y + R'V^{-1}r), \tag{5}$$

where $S = X'X$. Since we assumed the stochastic restrictions are held, i.e., $E(r) - R\beta = 0$, the mixed estimator is unbiased. Considering the model (1), Hoerl and Kennard (1970a,b) proposed the ridge estimator of β:

$$\hat{\beta}(k) = (S + kI)^{-1}X'Y, \quad \text{for } k > 0. \tag{6}$$

Ridge regression methods have been considered by many researchers, beginning with Hoerl and Kennard (1970a,b), followed by Farebrother (1976), Gibbons (1981), Sarkar (1992), Kibria (2003), Saleh (2006), Muniz and Kibria (2010), and very recently Saleh and Kibria (2012), among others. Also, considering model (1), Liu (1993) proposed a new biased estimator of β , called Liu estimator (LE). The LE is defined as

$$\hat{\beta}(d) = (S + I)^{-1}(X'Y + d\hat{\beta}), \text{ for } 0 < d < 1. \tag{7}$$

Liu estimator has been considered by several researchers in several times for different perspectives. To mention a few, Kaciranlar et al. (1999), Yuksel and Akdeniz (2002), Alheety et al. (2008), Alheety and Ramanathan (2009), Yang et al. (2009), and very recently

Kibria (2012). Considering model (1), Sakallioglu and Kaçiranlar (2008) introduced the $(k-d)$ class estimator as a new biased estimator of β

$$\hat{\beta}(k,d) = (S+I)^{-1}(X'Y + d\hat{\beta}(k)), \tag{8}$$

where $k > 0$, $-\infty < d < \infty$. They showed that, the $(k-d)$ class estimator has an advantage over the LE and ORR estimators. Also the $(k-d)$ class estimator is a general estimator which includes OLSE, ORR, and LE estimators. Considering model (4), Hubert and Wijekoon (2006) introduced an alternative Liu estimator of β as

$$\hat{\beta}_{SRD} = F_d\hat{\beta}_{OME}, \tag{9}$$

where $F_d = (S+I)^{-1}(S+dI)$. Yang et al. (2007) introduced the stochastic restricted Liu estimator (SRLE) as:

$$\hat{\beta}_{SRLE}(d) = (S + R'V^{-1}R)^{-1}(F_dX'Y + R'V^{-1}r). \tag{10}$$

If we replace $F_k = (S+kI)^{-1}S$ with F_d, we get the stochastic restricted ridge regression estimator (SRRRE) as follows:

$$\hat{\beta}_{SRRRE}(k) = (S + R'V^{-1}R)^{-1}(F_kX'Y + R'V^{-1}r). \tag{11}$$

In this article, we introduce an alternative stochastic restricted estimator as a generalization of OME, SRLE, SRRRE, LE , ORR, and $(k-d)$ estimators. The organization of the article is as follows. The proposed estimators and their properties are given in Sec. 2. In Sec. 3, the performance of the new estimator compared with other estimators with respect to the mean squares error matrix criteria is given. A numerical example is consider in Sec. 4, while some concluding remarks are presented in Sec. 5.

2. The New Estimator and its Properties

The new proposed estimator is motivated by the following fact.

By using the identity

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + CA^{-1}B)^{-1}DA^{-1},$$

where A, B, C, and D are the positive definite matrices, the OME estimator can be rewritten as follows:

$$\hat{\beta}_{OME} = \hat{\beta} + S^{-1}R'(V + RS^{-1}R')^{-1}(r - R\hat{\beta}).$$

Now, if we replace $\hat{\beta}$ with $\hat{\beta}(k,d)$ we get the new proposed estimator:

$$\hat{\beta}_{SR(k-d)E}(k,d) = \hat{\beta}(k,d) + S^{-1}R'(V + RS^{-1}R')^{-1}(r - R\hat{\beta}(k,d)), \tag{12}$$

which will be called $(SR(k-d)E)$. Since the $(k-d)$ class estimator has advantages over ORR and LE estimators, we hope that, these advantages will inherit to the SR$(k-d)$ estimator. The SR$(k-d)$E is a general estimator which includes OME, SRRE, SRLE, LE, ORR, and $(k-d)$ estimators:

$$\hat{\beta}_{SR(k-d)E}(0,1) = \hat{\beta}_{OME}$$

$$\hat{\beta}_{SR(k-d)E}(0, d) = \hat{\beta}_{SRLE}(d)$$
$$\hat{\beta}_{SR(k-d)E}(k, 1-k) = \hat{\beta}_{\text{SRRE}}(k).$$

If $R = 0$ then

$$\hat{\beta}_{SR(k-d)E}(0, 1) = \hat{\beta}$$
$$\hat{\beta}_{SR(k-d)E}(0, d) = \hat{\beta}(d)$$
$$\hat{\beta}_{SR(k-d)E}(k, 1-k) = \hat{\beta}(k)$$
$$\hat{\beta}_{SR(k-d)E}(k, d) = \hat{\beta}(k, d).$$

The expected value, variance, and bias of the SR($k-d$)E are given as follows:

$$\begin{aligned} E\left(\hat{\beta}_{SR(k-d)E}(k, d)\right) &= A(F_{k,d}S + R'V^{-1}R)\beta \\ Var\left(\hat{\beta}_{SR(k-d)E}(k, d)\right) &= \sigma^2 A\left(F_{k,d}SF'_{k,d} + R'V^{-1}R\right)A' \\ Bias\left(\hat{\beta}_{SR(k-d)E}(k, d)\right) &= \left(A(F_{k,d}S + R'V^{-1}R) - I\right)\beta \\ &= B_1, \end{aligned}$$

where $A = (S + R'V^{-1}R)^{-1}$ and $F_{k,d} = (S + I)^{-1}(S + d(S + kI)^{-1}S)$.

The bias and the variance of an estimator β^* are measured simultaneously by the mean squares error matrix (MSE)

$$MSE(\beta^*) = Var(\beta^*) + Bias(\beta^*)(Bias(\beta^*))'.$$

For this purpose,

$$MSE(\hat{\beta}_{OME}) = \sigma^2 A. \tag{13}$$

$$MSE(\hat{\beta}_{SRLE}(d)) = \sigma^2 A\left(F_d S F'_d + R'V^{-1}R\right)A' + B_2B'_2, \tag{14}$$

$$MSE(\hat{\beta}_{\text{SRRE}}(k)) = \sigma^2 A\left(F_k S F'_k + R'V^{-1}R\right)A' + B_3B'_3, \tag{15}$$

$$MSE(\hat{\beta}_{SR(k-d)E}(k, d) = \sigma^2 A\left(F_{k,d} S F'_{k,d} + R'V^{-1}R\right)A' + B_1B'_1, \tag{16}$$

$$MSE(\hat{\beta}(k, d)) = \sigma^2\left(F_{k,d} S F'_{k,d}\right) + B_4B'_4, \tag{17}$$

where

$$\begin{aligned} B_2 &= (A(F_d S + R'V^{-1}R) - I)\beta, \\ B_3 &= (A(F_k S + R'V^{-1}R) - I)\beta, \\ B_4 &= (F_{k,d}S - I)\beta. \end{aligned}$$

So, it is obvious that $\hat{\beta}_{SR(k-d)E}(k, d)$ is always biased unless $k = 0$ and $d = 1$.

3. Superiority of the New Estimator

Let $\beta_i^* = A_i Y$, $i = 1, 2$ be any two estimators. We know that

$$\begin{aligned} MSE(\beta_1^*) - MSE(\beta_2^*) &= Var(\beta_1^*) - Var(\beta_2^*) + B_1B'_1 - B_2B'_2 \\ &= \sigma^2 D + B_1B'_1 - B_2B'_2, \end{aligned}$$

Table 1
Estimated eigenvalues of Δ_1, Δ_2, Δ_3, and Δ_4 for different values d , when $k_{HK} = 0.0029$

	$k_{HK} = 0.0029$		
d	0.3	0.6	0.9
$\lambda_1(\Delta_1)$	0.5455×10^{-3}	0.34597×10^{-3}	0.9507×10^{-4}
$\lambda_2(\Delta_1)$	0.0116×10^{-3}	0.00688×10^{-3}	0.01766×10^{-4}
$\lambda_3(\Delta_1)$	0.000003×10^{-3}	0.000002×10^{-3}	0.000005×10^{-4}
$\lambda_4(\Delta_1)$	0.00022×10^{-3}	0.00013×10^{-3}	0.00032×10^{-4}
$\lambda_1(\Delta_2)$	0.0000006×10^{9}	0.6132×10^{3}	0.6167×10^{3}
$\lambda_2(\Delta_2)$	0.0000048×10^{9}	0.47561×10^{4}	0.47597×10^{4}
$\lambda_3(\Delta_2)$	0.0000349×10^{9}	0.349319×10^{5}	0.349355×10^{6}
$\lambda_4(\Delta_2)$	5349823435.2	5349895568.2	5349968183.6
$\lambda_1(\Delta_3)$	0.4793757×10^{-8}	0.1241×10^{-7}	0.2287×10^{-7}
$\lambda_2(\Delta_3)$	0.0014×10^{-8}	0.00034×10^{-7}	0.0005×10^{-7}
$\lambda_3(\Delta_3)$	$-0.0000004 \times 10^{-8}$	$-0.00000001 \times 10^{-7}$	0.00002×10^{-11}
$\lambda_1(\Delta_4)$	0.5427×10^{-3}	0.3431×10^{-3}	0.922×10^{-4}
$\lambda_4(\Delta_3)$	0.000004×10^{-8}	0.0000009×10^{-7}	0.00143×10^{-10}
$\lambda_2(\Delta_4)$	0.0116×10^{-3}	0.0068×10^{-3}	0.0171×10^{-4}
$\lambda_3(\Delta_4)$	0.000003×10^{-3}	0.000002×10^{-3}	0.005×10^{-7}
$\lambda_4(\Delta_4)$	0.0002×10^{-3}	0.00013×10^{-3}	0.0003×10^{-4}

Table 2
Estimated eiagenvalues of Δ_1, Δ_2, Δ_3, and Δ_4 for different values of d when $k_{HKB} = 0.01163$

	$k_{HKB} = 0.01163$		
d	0.3	0.6	0.9
$\lambda_1(\Delta_1)$	0.545×10^{-3}	0.346×10^{-3}	0.951×10^{-4}
$\lambda_2(\Delta_1)$	0.011×10^{-3}	0.006×10^{-3}	0.017×10^{-4}
$\lambda_3(\Delta_1)$	0.003×10^{-6}	0.002×10^{-6}	0.0057×10^{-8}
$\lambda_4(\Delta_1)$	0.0002×10^{-3}	0.13×10^{-6}	0.0003×10^{-4}
$\lambda_1(\Delta_2)$	0.61×10^{3}	0.61×10^{3}	0.62×10^{3}
$\lambda_2(\Delta_2)$	0.475×10^{4}	0.476×10^{4}	0.476×10^{4}
$\lambda_3(\Delta_2)$	0.3493×10^{5}	0.3493×10^{5}	0.3494×10^{5}
$\lambda_4(\Delta_2)$	534982295	534989557	534996818
$\lambda_1(\Delta_3)$	0.1917×10^{-7}	0.496×10^{-7}	0.914×10^{-7}
$\lambda_2(\Delta_3)$	0.56×10^{-10}	0.0013×10^{-7}	0.002×10^{-7}
$\lambda_3(\Delta_3)$	-0.0016×10^{-11}	-0.006×10^{-12}	0.0001×10^{-11}
$\lambda_4(\Delta_3)$	0.0017×10^{-10}	0.0037×10^{-10}	0.575×10^{-12}
$\lambda_1(\Delta_4)$	0.534×10^{-3}	0.334×10^{-3}	0.836×10^{-4}
$\lambda_2(\Delta_4)$	0.011×10^{-3}	0.006×10^{-3}	0.015×10^{-4}
$\lambda_3(\Delta_4)$	0.003×10^{-6}	0.0020×10^{-6}	0.05×10^{-8}
$\lambda_4(\Delta_4)$	0.225×10^{-6}	0.00012×10^{-3}	0.0002×10^{-4}

Table 3
Estimated eiagenvalues of Δ_1, Δ_2, Δ_3, and Δ_4 for different values of d when $k_{LW} = 0.00798$

	$k_{LW} = 0.00798$		
d	0.3	0.6	0.9
$\lambda_1(\Delta_1)$	0.545×10^{-3}	0.345×10^{-3}	0.951×10^{-4}
$\lambda_2(\Delta_1)$	0.01×10^{-3}	0.006×10^{-3}	0.017×10^{-4}
$\lambda_3(\Delta_1)$	0.003×10^{-6}	0.002×10^{-6}	0.05×10^{-8}
$\lambda_4(\Delta_1)$	0.0002×10^{-3}	0.0001×10^{-3}	0.0003×10^{-4}
$\lambda_1(\Delta_2)$	0.61×10^{3}	0.61×10^{3}	0.62×10^{3}
$\lambda_2(\Delta_2)$	0.475×10^{4}	0.476×10^{4}	0.476×10^{4}
$\lambda_3(\Delta_2)$	0.3493×10^{5}	0.3493×10^{5}	0.3494×10^{5}
$\lambda_4(\Delta_2)$	534982295	534989557	534996818
$\lambda_1(\Delta_3)$	0.131×10^{-7}	0.34×10^{-7}	0.628×10^{-7}
$\lambda_2(\Delta_3)$	0.0003×10^{-7}	0.0009×10^{-7}	0.0015×10^{-7}
$\lambda_3(\Delta_3)$	-0.001×10^{-11}	-0.004×10^{-12}	0.001×10^{-12}
$\lambda_4(\Delta_3)$	0.0012×10^{-10}	0.0025×10^{-10}	0.0039×10^{-12}
$\lambda_1(\Delta_4)$	0.537×10^{-3}	0.338×10^{-3}	0.872×10^{-4}
$\lambda_2(\Delta_4)$	0.01×10^{-3}	0.006×10^{-3}	0.016×10^{-4}
$\lambda_3(\Delta_4)$	0.0031×10^{-6}	0.002×10^{-6}	0.005×10^{-7}
$\lambda_4(\Delta_4)$	0.0002×10^{-3}	0.0001×10^{-3}	0.0003×10^{-3}

where $D = A_1A_1' - A_2A_2'$. If we want to know whether $\Delta = MSE(\beta_1^*) - MSE(\beta_2^*)$ is a positive definite (p.d.) or not, we may confine ourselves to the following fact.

Let $\hat{\beta}_j = A_jY$, $j = 1, 2$ be two linear estimators of β. Suppose that $D = Cov(\hat{\beta}_1) - Cov(\hat{\beta}_2)$ is p.d. then $\Delta = MSE(\hat{\beta}_1) - MSE(\hat{\beta}_2)$ is n.n.d. if and only if $b_2'(D + b_1b_1')^{-1}b_2 \leq 1$, where b_j denotes the bias vector of $\hat{\beta}_j$ (see Trenkler and Toutenburg, 1990). Thus, showing Δ is a p.d. it is reduced to the matrix type $\theta A - cc'$ is p.d. when A is p.d. Now, we will give some lemmas which are used in order to be compared with any two estimators.

Lemma 3.1. *Let* A *be a p.d. matrix,* c *be an non zero vector, and θ be a positive scaler. Then $\theta A - cc'$ is p.d. if and only if $c'A^{-1}c < \theta$.*

Lemma 3.2. *Let $\hat{\beta}_j^* = A_jY$, $j = 1, 2$ be two homogeneous linear estimator of β such that $D = A_1A_1' - A_2A_2'$ is p.d.. If $B_2'D^{-1}B_2 < \sigma^2$ then Δ is p.d.*

Lemma 3.3. *Let $\hat{\beta}_j^* = A_jY$, $j = 1, 2$ be two homogeneous linear estimator of β such that $D = A_1A_1' - A_2A_2'$ is p.d.. then Δ is p.d. if and only if $B_2'(\sigma^2D + B_1B_1')^{-1}B_2 < 1$.*

Lemma 3.4. *Let* B *be a p.d. matrix and* A *be a n.n.d. matrix, and $\Lambda = diag\left(\lambda_i^B(A)\right)$ is the diagonal matrix of the eigen values of A in the matrix B. Then there exists a non singular matrix* W *such that $B = W'W$ and $A = W'\Lambda W$.*

Lemma 3.5. *(Yang et al., 2009). Suppose A is a real symmetric matrix, P is a matrix then $A \geq 0 \Leftrightarrow \forall P$, $P'AP \geq 0 \Leftrightarrow$ each eigenvalue of A is non negative.*

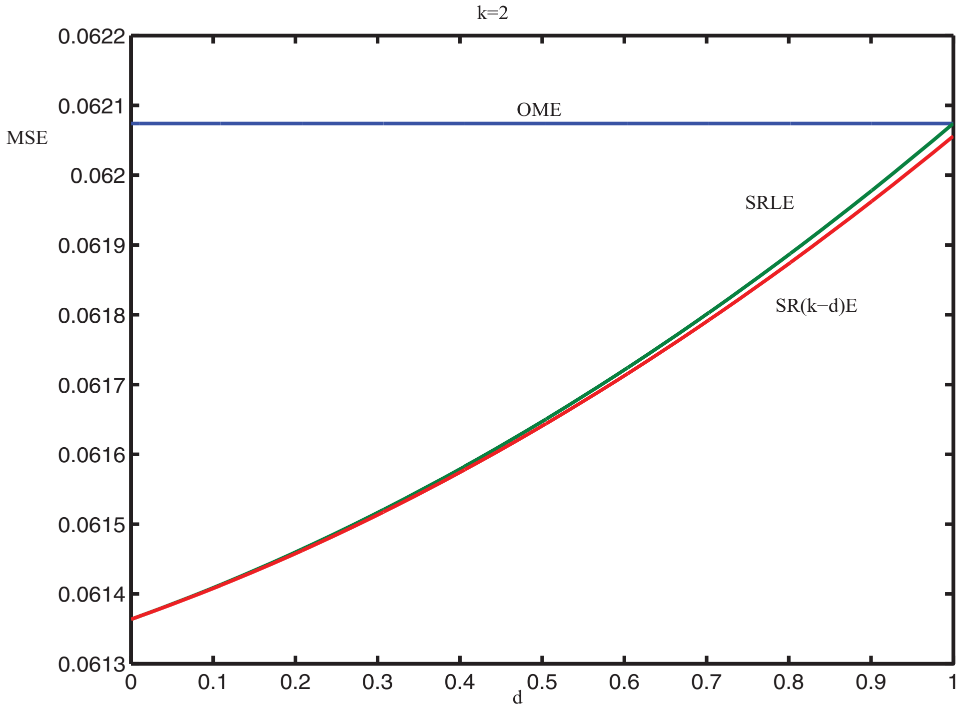


Figure 1. Various MSE of the proposed estimator compared to others for different values of d when $k = 2$..

Let us consider the difference between the proposed $SR(k-d)E$ and other estimators:

$$\Delta_1 = MSE(\hat{b}_{OME}) - MSE\big(\hat{b}_{SR(k-d)E}(k,d)\big) = \sigma^2 D_1 - B_1 B_1',$$

$$\Delta_2 = MSE(\hat{b}(k,d)) - MSE\big(\hat{b}_{SR(k-d)E}(k,d)\big) = \sigma^2 D_2 + B_4 B_4' - B_1 B_1',$$

$$\Delta_3 = MSE(\hat{b}_{SRLE}(d)) - MSE\big(\hat{b}_{SR(k-d)E}(k,d)\big) = \sigma^2 D_3 + B_2 B_2' - B_1 B_1',$$

$$\Delta_4 = MSE(\hat{b}_{SRRR}(k)) - MSE\big(\hat{b}_{SR(k-d)E}(k,d)\big) = \sigma^2 D_4 + B_3 B_3' - B_1 B_1',$$

where

$$D_1 = A - A\big[F_{k,d} S F_{k,d}' + R' V^{-1} R\big] A',$$
$$D_2 = F_{k,d} S^{-1} F_{k,d}' - A\big[F_{k,d} S F_{k,d}' + R' V^{-1} R\big] A',$$
$$D_3 = A\big[F_d S F_d' + R' V^{-1} R\big] A' - A\big[F_{k,d} S F_{k,d}' + R' V^{-1} R\big] A',$$
$$D_4 = A\big[F_k S F_k' + R' V^{-1} R\big] A' - A\big[F_{k,d} S F_{k,d}' + R' V^{-1} R\big] A'.$$

Now we can start showing the performance of the new estimator compared with others:

In order to prove that Δ_1, Δ_2, Δ_3, and Δ_4 are n.n.d., we need to apply Lemma 3.3, that means, we need to show under which condition D_1, D_2, D_3, and D_4 will be p.d. We can rewrite D_1 as follows:

$$D_1 = A\big(S - F_{k,d} S F_{k,d}'\big) A'.$$

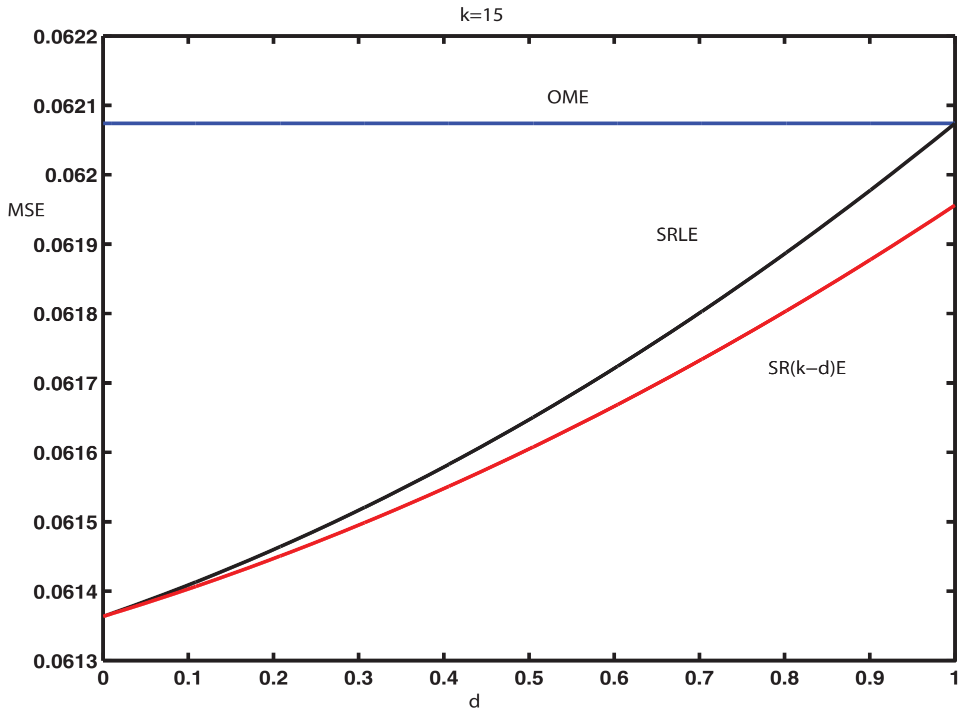


Figure 2. Various MSE of the proposed estimator compared to others for different values of d when $k = 15$.

Therefore D_1 is p.d. if and only if $S - F_{k,d}SF'_{k,d}$ is p.d. Because S is p.d., there exists an orthogonal matrix T such that $T'T = TT' = I$ and $T'ST = \Lambda = diag\{\lambda_1, \ldots, \lambda_p\}$ where Λ is the diagonal matrix whose elements are the eigen values of the matrix S. Hence, $T'S - F_{k,d}SF'_{k,d}T = diag\left\{\gamma_1, \ldots, \gamma_p\right\}$, where

$$\gamma_i = \lambda_i \left(1 - \left(\frac{\lambda_i(\lambda_i + k + d)}{(\lambda_i + k)(\lambda_i + 1)}\right)^2\right).$$

Therefore, D_1 is p.d. if $\gamma_i > 0, \forall i$. That means D_1 is p.d. if and only if

$$\frac{\lambda_i^2 (\lambda_i + k + d)^2}{(\lambda_i + k)^2 (\lambda_i + 1)^2} < 1 \Leftrightarrow$$

$$d < 1 + \frac{k}{\lambda_i}.$$

After applying Lemma 3.1, we can state the following theorem.

Theorem 3.1. Under the linear regression model with the stochastic restrictions (3), for $d < 1 + \frac{k}{\lambda_i}$, then Δ_1 is n.n.d. if and only if

$$B_1' D_1^{-1} B_1 < \sigma^2.$$

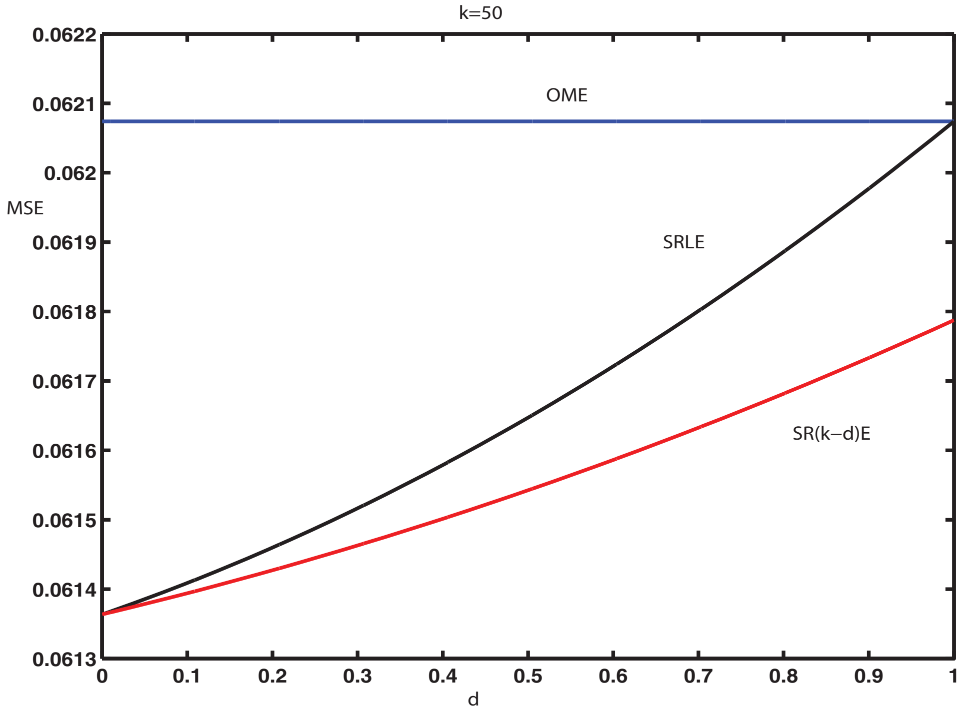


Figure 3. Various MSE of the proposed estimator compared to others for different values of d when $k = 50$..

Let $D_2 = B - C$, where $B = F_{k,d}S^{-1}F'_{k,d}$ and $C = A[F_{k,d}SF'_{k,d} + R'V^{-1}R]A'$. By applying Lemma 3.4 we can derive the necessary and sufficient condition for D_2 to be n.n.d. Since B is p.d. and C is a symmetric matrix, then there exists a non singular matrix W such that $I_p = WBW'$ and $WCW' = \Lambda$. If $B - C$ is n.n.d matrix, $WBW' - WCW' = I - \Lambda$ is n.n.d., then $1 - \lambda_i \geq 0$. So, we get $\lambda_{max}(B^{-1}C) \leq 1$. Now, suppose that $\lambda_{max}(B^{-1}C) \leq 1$. Since B is p.d. and C is a symmetric matrix, then $\lambda_p \leq \frac{x'Cx}{x'Bx} \leq \lambda_1$, where $\lambda_1(B^{-1}C) \geq \ldots \geq \lambda_p(B^{-1}C)$ are the roots of $|C - \lambda B| = 0$. From that we get $x'Cx \leq x'Bx$. So, $B - C$ is an n.n.d. matrix. Therefore, it is obvious that D_2 is n.n.d if and only if $\lambda_{max}(B^{-1}C)$. Now we are ready to give the following theorem.

Theorem 3.2. *Under the linear regression model with the stochastic restrictions (3), then D_2 is n.n.d. if and only if $\lambda_{max}(B^{-1}C) \leq 1$. From Theorem 3.2 and after applying Lemma 3.3, we can give the following theorem.*

Theorem 3.3. *Under the linear regression model with the stochastic restrictions (3), if $\lambda_{max}(B^{-1}C) \leq 1$ then Δ_1 is n.n.d. if and only if,*

$$B'_1\left(\sigma^2 D_2 + B_4 B'_4\right)^{-1} B_1 < 1.$$

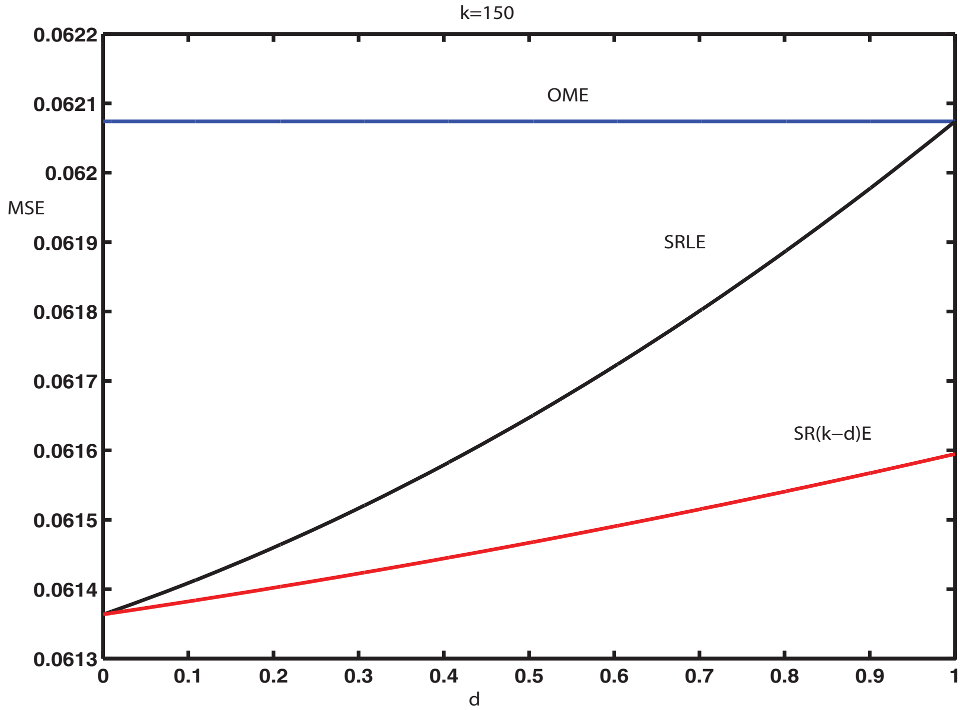


Figure 4. Various MSE of the proposed estimator compared to others for different values of d when $k = 150$..

As done in D_1, $D_3 = A(F_d S F'_d - F_{k,d} S F'_{k,d})A'$ and $T'(F_d S F'_d - F_{k,d} S F'_{k,d})T = diag\{\tau_1, ..., \tau_p\}$. Therefore, D_3 is p.d. if and only if $\tau_i > 0, i = 1, ..., p$.

$$\tau_i = \frac{(\lambda_i + d)^2\lambda_i - (\lambda_i + \frac{d\lambda_i}{\lambda_i + k})^2\lambda_i}{(\lambda_i + 1)^2} > 0$$
$$\Leftrightarrow \frac{(\lambda_i + d)}{(\lambda_i + 1)} - \frac{\lambda_i(\lambda_i + k + d)}{(\lambda_i + k)(\lambda_i + 1)} > 0.$$

Let k be defined to be fixed. Therefore,

$$\tau_i > 0 \Leftrightarrow d > 0.$$

Now we can present the following theorem

Theorem 3.4. *Under the linear regression model with the stochastic restrictions (3). For $d > 0$, then Δ_3 is n.n.d. if and only if,*

$$B'_1\left(\sigma^2 D_3 + B_2 B'_2\right)^{-1} B_1 < 1.$$

Since, the proof of the D_4 to be p.d. is similar to the proof of D_3, therefore, we state the following theorem.

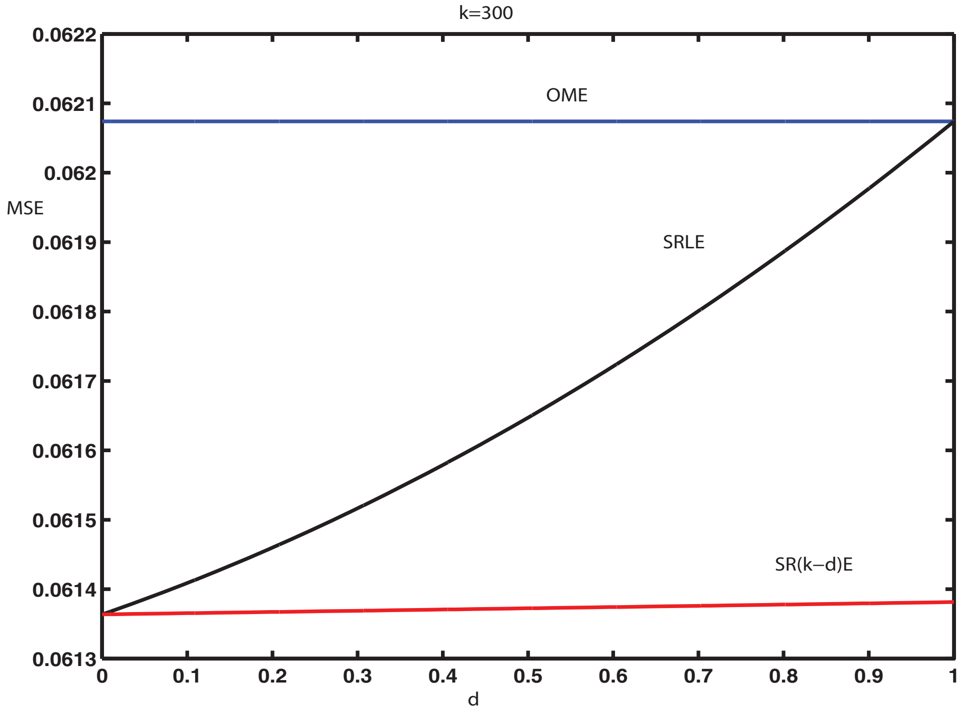


Figure 5. Various MSE of the proposed estimator compared to others for different values of d when $k = 300$..

Theorem 3.5. *Let k considered to be fixed. Under the linear regression model with the stochastic restrictions (3). For* $d < 1 + \frac{1}{\lambda_i} - (\lambda_i + k)$*, then* Δ_4 *is n.n.d. if and only if,*

$$B_1'(\sigma^2 D_4 + B_3 B_3')^{-1} B_1 < 1.$$

4. Numerical Example

To illustrate the performance of the proposed estimator in the MSE, a numerical example is given. We consider the widely used Portland cement data, which has been analyzed by many researchers, among them like Alheety and Gore (2008) and Alheety et al. (2009) are notable. By using Lemma 3.5 we can get Δ_i, $i = 1, \ldots 4$ is n.n.d if and only if each eigenvalue of Δ_i is non negative. Following Kibria (2003), we have estimated the ridge parameter k and given them in Tabels 1 to 3. From Tables 1–3, we observed that the numerical results are corresponding to the theoretical results except Δ_3 where the condition is not satisfied. Figures 1–3, clearly showed the advantage of considering $(k - d)$ estimator to use $k > 1$ which is adjusted by d.

5. Some Concluding Remarks

A generalized stochastic restricted ridge regression estimator for the unknown vector parameter in the linear regression model is proposed. The ordinary mixed estimator (OME), Liu estimator (LE), ordinary ridge estimator (ORR), (k-d) class estimator, stochastic restricted Liu estimator (SRLE) and stochastic restricted ridge estimator (SRRE) are special cases of the proposed estimator. Performance of the new estimator in comparison to other estimators in terms of the mean squares error matrix (MMSE) is discussed. Numerical example from literature have been given to illustrate the results of this article. Our wish is that the findings of this article will be useful for practitioners.

Acknowledgments

We are thankful to the referees for their valuable comments and suggessions, which certainly helped to improve the quality and presentation of the article.

References

Alheety, M. I., Gore, S. D. (2008). A new estimator in multiple linear regression. *Model Assist. Statist. Applic.* 3(3):187–201.

Alheety, M. I., Gore, S. D., Kibria, B. M. G. (2008). A new biased estimator for the restricted linear regression model. *J. Statist. Theory Appl.* 7(3):293–302.

Alheety, M. I., Ramanathan, T. V., Gore, S. D. (2009). On the distribution of stochastic shrinkage parameters of Liu-type estimators. *Braz. J. Probab. Statist.* 3(23).

Alheety, M. I., Ramanathan, T. V. (2009). Confidence Interval for Shrinkage Parameters in Ridge Regression. *Commun. Statist. Theor. Meth.* 38(19):3489–3497.

Farebrother, R. W. (1976). Fruther results on the mean square error of ridge regression. *J. Roy. Statist. Soci.* B.38: 284–250.

Gibbons, D. G. A. (1981). Simulation study of some ridge estimators. *J. Amer. Statist. Assoc.* 76: 131–139.

Hoerl, A. E., Kennard, R. W. (1970a). Ridge regression: Biased estimation for non-orthogonal problem. *Technometrics* 12: 55–67.

Hoerl, A. E., Kennard, R. W. (1970b). Ridge regression: application for non-orthogonal problem. *Technometrics* 12: 69–82.

Hubert, M. H., Wijekoon, P. (2006). Improvement of the Lie estimator in linear regression model. *Statisti. Pap.* 74: 471–479.

Kaciranlar, S., Sakallioglu, S., Akdeniz, F., Styan, G. P. H., Werner, H. J. (1999). A new biased estimator in linear regression and a detailed analysis of the widely-analyzed dataset on Portland cement. *Sankhya B* 61: 443–459.

Kibria, B. M. G. (2003). Performance of some new ridge regression estimators. *Commun. Statist. Simul. Computat.* 32: 419–435.

Kibria, B. M. G. (2012). On some liu and ridge type estimators and their properties under the ill-conditioned Gaussian linear regression model. *J. Statist. Computat. Simulat.* 82(1):1–17.

Liu, K. (1993). A new class of biased estimate in linear regression. *Communications in Statistics-Theory and Methods* 22, 393–402.

Muniz, G., Kibria, B. M. G. (2009). On some ridge regression estimators: An empirical comparisons. *Commun. Statist. Simulat. Computat.* 38(3):621–630.

Rao, C., Toutenburg, H. (1995). *Linear Models: Least Squares and Alternatives*. Springer Series in Statistics. Springer.

Saleh, A. K. Md. E. (2006). *Theory of Preliminary Test and Stein-type Estimation with Applications*. New York: John Wiley.

Saleh, K. Md. E., Kibria, B. M. G. (2012). On some ridge regression estimators: a nonparametric approach. *J. Nonparametric Statist.* 23(3):819–851.

Sakallioglu, S., Kaciranlar, S. (2008). A new biased estimator based on ridge estimation. *Statist. Pap.* 49(4):669–689.

Sarker, N. (1992). A new estimator combining the ridge regression and the restricted least squares method of estimation. *Commun. Statist. A* 21(7):1987–2000.
Theil, H. (1963). On the use of incomplete prior information in regression analysis. *J. Amer. Statist. Assoc.* 58: 401–414.
Theil, H., Goldberger, A. S. (1961). On pure and mixed estimation in econometrics. *Int. Econ. Rev.* 2: 65–78.
Yang, H., Chang, X., Liu, D. (2009). Improvement of the Liu estimator in weighted mixed regression. *Commun. Statist. Theor. Metho.* 38: 285–292.
Yuksel, G., Akdeniz, F. (2001). Properties of some new preliminary test Liu estimators and comparisons with the usual preliminary test estimators. *J. Statist. Res.* 35(2):45–56.