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Scaling Group Transformation under the Effect of Thermal Radiation Heat Transfer of a Non-Newtonian Power-Law Fluid over a Vertical Stretching Sheet with Momentum Slip Boundary Condition

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Abstract

An analysis is conducted to study the problem of heat transfer with thermal radiation effect of a non-Newtonian power-law fluid over a vertical stretching sheet with momentum slip boundary condition. The partial differential equations governing the physical situation are converted into a set of nonlinear coupled ordinary differential equations using scaling group of transformation. These equations are then solved numerically using the Runge–Kutta–Fehlberg fourth-fifth order numerical method under Maple 13. The radiation parameter leads to increased velocity and temperature, but it decreases the rate of heat transfer and skin friction coefficient for both Newtonian and non-Newtonian fluid. The velocity decreases, whereas the temperature increases with momentum slip parameter for Newtonian and non-Newtonian. The skin friction coefficient decreases with both momentum slip and power-law parameter. The rate of heat transfer falls with momentum slip, but rises with power-law parameter. Good agreement is found between the numerical results of this paper with published results both for Newtonian and non-Newtonian fluid.

Keywords Momentum slip boundary condition, Non-Newtonian fluid, Power law fluid, Scaling group transformation, Thermal radiation.

1. Introduction

The study of non-Newtonian fluid flows has gained considerable interest for their numerous engineering applications. Details of the behavior of non-Newtonian fluid for both steady and unsteady flow situations, along with mathematical models can be found in the books by Astarita and Marrucci [6] and Bohme [14]. Over recent years, applications of non-Newtonian fluids in many industrial processes have been increasing. Many particulate slurries, multiphase mixtures, pharmaceutical formulations, cosmetics and toiletries, paints, biological fluids, and food items are examples of non-Newtonian fluids (Schowalter, [38]; Bird et al., [12]; Crochet et al. [18]; Shenoy and Mashelkar, [39]; Andersson and Irgens, [4], Irvine and Karni, [26], and Postelnicu and Pop, [33]).

Non-Newtonian fluids have received the attention of numerous investigators due to their diverse applications. Acrivos was investigated boundary-layer flows for such fluids in 1960[2] and since then, a large number of related studies have been conducted because of the importance and presence of such fluids in chemicals, polymers, molten plastics and others. Cheng [16] studied non-Newtonian power-law fluids with coupled heat and mass transfer via natural convection from a vertical truncated cone in a saturated porous medium. Ishak et al. [27] examined the steady boundary-layer flow past a moving wedge on a flat plate in a non-Newtonian power-law fluid. Mukhopadhyay [30] presented the effect of buoyancy and thermal radiation on unsteady boundary layer flow and heat transfer on a permeable stretching sheet embedded in a porous medium. Abdul Hakeem and Sathiyanathan [1] described the flow of an unsteady free convection past an infinite vertical plate with time dependent suction under the effects of radiation. Tai and Char [40] studied the effects of the radiation parameter on the free convection flow of a non-Newtonian power law fluid along a vertical plate in a porous medium. Hayat et al. [22] studied the effect of radiation on the magnetohydrodynamic (MHD) mixed convection stagnation point flow in a porous medium. Cortell [17] examined the thermal radiation effects on the steady flow and heat transfer of a non-Newtonian power-law fluid past a semi-infinite, non-isothermal, porous flat plate, subject to suction. Molla et al. [29] investigated the influence of thermal radiation on a steady two-dimensional natural convection laminar flow of an optically thick (it is a measure of transparency) and incompressible fluid along a vertical flat plate with streamwise surface temperature, i.e., wall temperature varies with the axial distance from the leading edge of the plate. More recently, Pal and Talukdar [32] examined the effects of thermal radiation on the free convective flow and mass transfer of a viscous incompressible fluid past an infinite vertical plate. Gonzalez et al. [20] calculated heat transfer via natural convection and surface thermal radiation in a square open cavity receiver with large temperature differences and variable properties.

Non-Newtonian flows with slip boundary condition have been studied by other researchers (Tanner, [41]; Roux, [35]; Ariel et al., [5]; and Hayat et al., [23]). The study of the non-Newtonian flows with slip boundary has become active in recent because of the wide application of such fluids in food engineering, petroleum production, power engineering, and in polymer melt and polymer solutions used in plastic processing industries (Postelnicu and Pop, [33]). Sahoo and Do [37] investigated the effects of partial slip on the steady flow of an incompressible, electrically conducting third-grade fluid over to a stretching sheet taking into account magnetic field. Turkyilmazoglu [42] investigated the MHD slip flow of an electrically conducting, viscoelastic fluid past a stretching surface. Sahoo [36] investigated the effects of the slip parameter on the steady flow of an incompressible, electrically conducting third grade fluid over a stretching sheet. Very recently, Zheng et al. [44] discussed the effect of slip with the MHD flow of an incompressible, generalized Oldroyd-B fluid induced by an accelerating plate. Bagai and Nishad [11] investigated the similarity solutions for the problem of free convection flow over a non-isothermal horizontal plate embedded in porous media in the presence of internal heat generation with non-Newtonian power-law fluid.

Group theory was introduced by Lie to find symmetries of differential equations (Bluman and Kumei, [13]; Ibragimov, [25]; Atalik, [7]). The invariance of partial differential equations under Lie symmetry groups has been shown to possibly result in similarity or invariant solutions, which remain invariant under some subgroup of the full symmetry group admitted by the governing system (Bluman and Kumei, [25]; Atalik, [7]). This approach has been applied to a large number of differential equation systems in fluid mechanics and mathematical physics (Bluman and Kumei, [13]; Ibragimov, [25]; Atalık, [7]).

Scaling group of transformation is a special form of Lie group analysis. This method unifies almost all known exact integration techniques for both ordinary and partial differential equations (Cebeci and Bradshaw, [15], Avramenko et al. [8], Dolap and Pakdemirli, [19]). Scaling group of transformation is used to similarity transformations of the equations governing the problem. Similarity transformations are utilized to transform the non-linear partial differential equations into ordinary differential equations with corresponding boundary conditions.

In this paper, the problem of flow and heat transfer of a non-Newtonian power-law fluid over a vertical stretching sheet is investigated numerically. The effects of momentum slip condition and thermal radiation are taken into account. The obtain similarity equations were solved numerically to show the effects of the governing parameters, namely, the power-law index, momentum slip, and thermal radiation parameters, on dimensionless velocity and temperature profiles.

2. Governing transport equations

Let us consider the case of a steady two-dimensional flow and heat transfer of a non-Newtonian power law fluid over a solid vertical stretching sheet. We consider the effects of hydrodynamic slip condition and thermal radiation. Further, a Cartesian coordinate system $(\bar{x}, \bar{y})$ is used, where $\bar{x}$ and $\bar{y}$ are the coordinates measured along and normal to the surface, respectively. Under the above assumptions, the partial differential equations in dimensional form governing the problem and the corresponding boundary conditions are given by: (Prasad et al. [34]), (Molla et al. [29]).

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{-K}{\rho}\frac{\partial}{\partial \bar{y}}\left(-\frac{\partial \bar{u}}{\partial \bar{y}}\right)^{n} \pm g\beta\left(T - T_{\infty}\right), \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{16\sigma_1 T_{\infty}^3}{3\rho c_p k_1}\frac{\partial^2 T}{\partial \bar{y}^2}. \tag{3}$$

The appropriate boundary conditions are

$$\bar{u}=\bar{u}_w+\bar{u}_{slip},\bar{v}=0,T=T_w=T_\infty+A\left(\frac{\bar{x}}{L}\right)\quad \text{at}\quad \bar{y}=0,$$
$$\bar{u}=0,T\to T_\infty\quad \text{as}\quad \bar{y}\to\infty. \tag{4}$$

In the above equations, $\bar{u}_w=\frac{U_r}{L}\bar{x}$ is the velocity of the plate, $\bar{u}_{slip}=a\left(-\frac{\partial\bar{u}}{\partial\bar{y}}\right)^{n-1}\frac{\partial\bar{u}}{\partial\bar{y}}$ is the slip velocity, $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, respectively, K is the consistency coefficient of the fluid, n is the power law index, ρ is the fluid density, g is the acceleration due to gravity, β is the coefficient of thermal expansion, a is the momentum slip factor with dimension $(\text{velocity})^{-1}$, A is a constant with paper dimension, .

We introduce now the following dimensionless variables

$$x=\frac{\bar{x}}{L},\ y=\frac{\bar{y}}{L}\text{Re}^{\frac{1}{n+1}},u=\frac{\bar{u}}{U_r},v=\frac{\bar{v}}{U_r}\text{Re}^{\frac{1}{n+1}},\ \theta=\frac{T-T_\infty}{T_w-T_\infty}, \tag{5}$$

where $\text{Re}=\frac{U_r^{2-n}L^n}{\gamma}$ is the Reynolds number based the characteristic length L.

The dimensionless forms of the governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n\pm\lambda x\theta, \tag{7}$$

$$\frac{\theta}{x}u+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{\text{Re}^{\frac{2}{1+n}}}{LU_r}\left(\alpha+\frac{16\sigma_1T_\infty^3}{3\rho c_p\kappa_1}\right)\frac{\partial^2\theta}{\partial y^2}. \tag{8}$$

Boundary conditions (4) become

$$u=x+\frac{aU_r^{n-2}\text{Re}^{\frac{1}{n+1}}}{L^n}\left(-\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial u}{\partial y},v=0,\ \theta=1\quad \text{at}\ y=0,$$
$$u=0,\ \theta=0\qquad \text{as}\ y\to\infty. \tag{9}$$

Here $\lambda=\frac{g\beta AL^2}{U_r^2}$ is the buoyancy or mixed convection parameter.

We introduce stream function ψ which is defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ to reduce the number of equations and number of dependent variables into Eqs. (6-8). Note that continuity equation (6) is satisfied automatically.

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y}-\frac{\partial \psi}{\partial x}\frac{\partial^2 u}{\partial y^2}=n\left(-\frac{\partial^2 \psi}{\partial y^2}\right)^{n-1}\left(\frac{\partial^3 \psi}{\partial y^3}\right)\pm \lambda x\theta, \tag{10}$$

$$\frac{\theta}{x}\frac{\partial \psi}{\partial y}+\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}=\frac{\mathrm{Re}^{\frac{2}{1+n}}}{LU_r}\left(\alpha+\frac{16\sigma_1 T_\infty^3}{3\rho c_p \kappa_1}\right)\frac{\partial^2 \theta}{\partial y^2}. \tag{11}$$

Boundary conditions (9) become

$$\frac{\partial \psi}{\partial y}=x+\frac{aU_r^{n-2}\,\mathrm{Re}^{\frac{1}{n+1}}}{L^n}\left(-\frac{\partial^2 \psi}{\partial y^2}\right)^{n-1}\frac{\partial^2 \psi}{\partial y^2},\ -\frac{\partial \psi}{\partial x}=0\ ,\ \theta=1 \ \text{ at } y=0,$$
$$\frac{\partial \psi}{\partial y}=0\ ,\ \theta=0 \qquad \text{as} \qquad y\to\infty. \tag{12}$$

We consider following scaling group of transformations which is a special form of Lie group analysis (Jalil et al. [28], Mukhopadhyay and Layek [31], Aziz et al. al. [10])

$$\Gamma: x=e^{-\varepsilon c_1}x^*,\ y=e^{-\varepsilon c_2}y^*,\ \psi=e^{-\varepsilon c_3}\psi^*,\ \theta=e^{-\varepsilon c_4}\theta^*. \tag{13}$$

Here ε is the parameter of the group Γ and c_i , $(i=1,2,3,4)$ are arbitrary real numbers.

For invariant of the differential equations and boundary conditions, (i.e. the structure of the equations before and after the transformations will be remained same), we must have.

$$c_2=\left(\frac{n-1}{2n}\right)c_3\ ,\ c_1=\left(\frac{n+1}{2n}\right)c_3\ ,\ c_4=0. \tag{14}$$

The characteristic equation is

$$\frac{dx}{\frac{n+1}{2n}x}=\frac{dy}{\frac{n-1}{2n}y}=\frac{d\psi}{\psi}=\frac{d\theta}{0}. \tag{15}$$

Solving (15), we have

$$\eta=y\,x^{\frac{1-n}{1+n}},\ \psi=x^{\frac{2n}{1+n}}f(\eta),\ \theta=\theta(\eta). \tag{16}$$

Using (16), Eqs. (10) - (11) becomes

$$n\left(-f''\right)^{n-1}f'''-f'^2+\frac{2n}{n+1}ff''+\lambda\theta=0, \tag{17}$$

$$\left(1+\frac{4}{3}R\right)\theta''+\mathrm{Pr}_m\left(\frac{2n}{n+1}f\theta'-\theta f'\right)=0, \tag{18}$$

Subject to the boundary conditions

$$f'=1+\delta\left(-f''\right)^{n-1}f'',\ f=0\ ,\ \theta=1 \qquad \text{at} \qquad \eta=0,$$
$$f'=0\ ,\ \theta=0 \qquad \text{as} \qquad \eta\to\infty. \tag{19}$$

Here primes denote differentiation with respect to η.

The modified Prandtl number (for power law fluids) is $\Pr_m = \frac{LU_r \operatorname{Re}^{\frac{-2}{1+n}}}{\alpha} x^{\frac{2n-2}{n+1}}$, the radiation parameter is $R = \frac{4\sigma_1 T_\infty^3}{\kappa \kappa_1}$, $\delta = \frac{aU_r^{n-2} \operatorname{Re}^{\frac{n}{1+n}}}{L^n} x^{\frac{2n}{1+n}}$, δ is slip parameter.

Expressions for the quantities of physical interests, the skin friction factor and the rate of heat transfer can be found from the following definitions

$$C_{f\bar{x}} = \frac{-K}{\rho \bar{u}_w^2(\bar{x})} \left[\left(-\frac{\partial \bar{u}}{\partial \bar{y}} \right)^{n-1} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right) \right]_{\bar{y}=0}, Nu_{\bar{x}} = \frac{-\bar{x}}{T_w - T_\infty} \left(\frac{\partial T}{\partial \bar{y}} \right)_{\bar{y}=0}, \tag{20}$$

Using (5) and (16) we have from (20)

$$C_{f_{\bar{x}}} \operatorname{Re}_{\bar{x}}^{\frac{1}{1+n}} = \left[-f''(0) \right]^{n-1} f''(0),\ Nu_{\bar{x}} \operatorname{Re}_{\bar{x}}^{\frac{-1}{1+n}} = -\theta'(0). \tag{21}$$

where $\operatorname{Re}_{\bar{x}} = \frac{\rho \bar{u}_w^{2-n} \bar{x}^n}{K}$ is the local Reynolds number.

3. Results and discussions

The ordinary differential Eqs. (17) and (18) with the boundary conditions (19) have been solved using the Runge–Kutta–Fehlberg fourth- fifth numerical method as proposed by Aziz [9]. Numerical results are obtained to study the effect of the various values of the radiation parameter R, slip parameter δ, power law index n and mixed convection parameter λ on dimensionless velocity, temperature, shear stress coefficient and rate of heat transfer. The numerical results of the dimensionless velocity $f'(\eta)$ and the temperature $\theta(\eta)$ are shown in Figs. 2-9. Our present results have also been compared with those of the corresponding published data by Hamad et al. [21] and Hayat et al. [24] for Newtonian fluid $n=1$ in the absence of mixed convection $\lambda = 0$. The present results are compatible with those of the published data. The comparison results are given in **Table** 1. We also compare our results with Andersson and Besb [3] and Xu and Laio [43] for non-Newtonian fluid in **Table** 2 and found good agreement. The numerical results of skin friction coefficient $-f''(0)$ and the heat transfer rate at the wall $-\theta'(0)$ are given in **Tables 3, 4**. We notice from **Table 3** that the increase of the slip parameter δ decreases the rate of heat transfer and skin friction coefficient. Also, one can see that the increase of power law index parameter n increases the heat transfer rate at the wall and decreases the skin friction coefficient. We notice from **Table 4** skin friction coefficient and the heat transfer rate at the wall increase while power law index parameter increases, and decreases with the increase of the radiation parameter.

The effects of thermal radiation parameter on the profile of dimensionless velocity and temperature for (a) Newtonian fluid $(n=1)$, (b) non–Newtonian fluid $(n=0.3)$ profiles are shown in Figs. 2 and 3, respectively. We notice that the dimensionless velocity and temperature increases as R increases for two cases; of Newtonian and non-Newtonian.
Figs. 4 and 5 illustrate the effects of mixed convection parameter on the profile of dimensionless velocity and temperature for (a) Newtonian fluid $(n=1)$, (b) non-Newtonian fluid $(n=0.4)$. We see that the velocity profiles increases as the mixed convection parameter increases, but we note that from Fig. 5, the temperature decreases when A increases for two case of Newtonian and non-Newtonian.
Figs. 6 and 7 depict the effects of the slip parameter δ on the dimensionless velocity and temperature profiles, respectively. It is clear from these figures that the velocity decreases with the increase of the slip parameter, while the temperature is increased with the increase of the slip parameter, both for Newtonian and non-Newtonian.
Figs. 8 and 9 depict the effects of fluid power law index parameter n on the dimensionless velocity and temperature profiles. We notice that the velocity and temperature decreases when an increase in n. It is found that a cross flow occurs at $\eta = 1.6336$ (approx).

4. Conclusions

The heat transfer characteristics of a non-Newtonian power law fluid over a vertical stretching sheet with effects of the thermal radiation and hydrodynamic linear slip parameter is studied numerically in this paper. Using scaling transformation the governing equations were converted to a coupled non-linear system of ordinary differential equations, which was then solved numerically by Runge–Kutta–Fehlberg fourth-fifth order numerical method. From the our analysis we found that

- R leads to increases the dimensionless velocity and the temperature both for Newtonian and non-Newtonian, while the rate of heat transfer at the wall and skin friction coefficient decrease.
- δ leads to decrease the dimensionless fluid velocity, increase the temperature both for Newtonian and non-Newtonian. The increase of the slip parameter δ decreases skin friction coefficient and the wall heat transfer rate.
- λ leads to increase the fluid velocity, decrease the dimensionless temperature both for Newtonian and non-Newtonian fluids.
- n leads to decrease of the dimensionless temperature and velocity.

Acknowledgments
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Tables and Graphs

Table 1. Comparison of skin friction coefficient $-f''(0)$ for various values of slip parameter δ when $n=1$

δ	Hayat et al. [22]	Hamad et al. [21]	Present results
0.0	1.000000	1.00000000	1.00000000
0.1	0.872082	0.87208247	0.87208247
0.2	0.776377	0.77637707	0.77637707
0.5	0.591195	0.59119548	0.59119548
1.0	0.430162	0.43015970	0.43015970
2.0	0.283981	0.28397959	0.28397959
5.0	0.144841	0.14484019	0.14484019
10.0	0.081249	0.08124198	0.08124198
20.0	0.043782	0.04378834	0.04378834
50.0	0.018634	0.01859623	0.01859623
100.0	0.009581	0.00954997	0.00954997

Table 2. Comparison of $-f''(0)$ for various values of *n* with Xu and Liao (2009) and Andersson and Bech (1992) for the boundary conditions without slip $(a=0)$, radiation $(R=0)$ and mixed convective parameter $(\lambda=0)$.

n	Andersson and Bech [3]	Xu and Lia [43]	Current results
0.4	1.273		1.27387
0.6	1.096		1.09582
0.8	1.029	1.02853	1.02844
1.0	1.000	1.00000	1.00000

Table 3. Values of $-f''(0)$ and $-\theta'(0)$ for $\Pr_m = 0.72$, $\lambda = 0, R = 0.4$ and various values of n and δ

n	$-f''(0)$		$-\theta'(0)$	
	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.1$	$\delta = 0.2$
0.5	0.961323	0.805565	0.569228	0.550200
1	0.872570	0.777700	0.601634	0.581256

Table 4. Values of $-f''(0)$ and $-\theta'(0)$ for $\Pr_m = 0.72$, $\lambda = 1, \delta = 0.4$ and various values of n and R

n	$-f''(0)$		$-\theta'(0)$	
	$R = 0.1$	$R = 0.5$	$R = 0.1$	$R = 0.5$
0.5	0.168252	0.143965	0.768472	0.633806
1	0.301614	0.269028	0.814779	0.668703

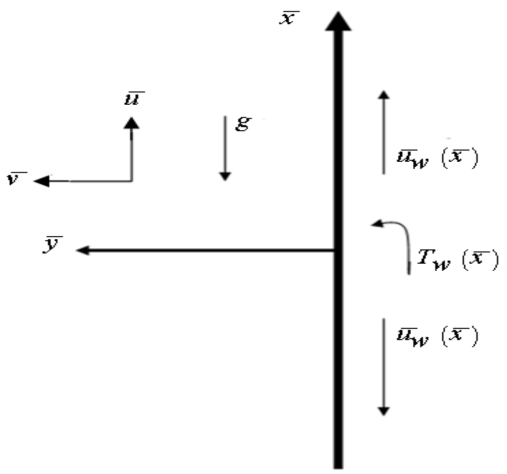
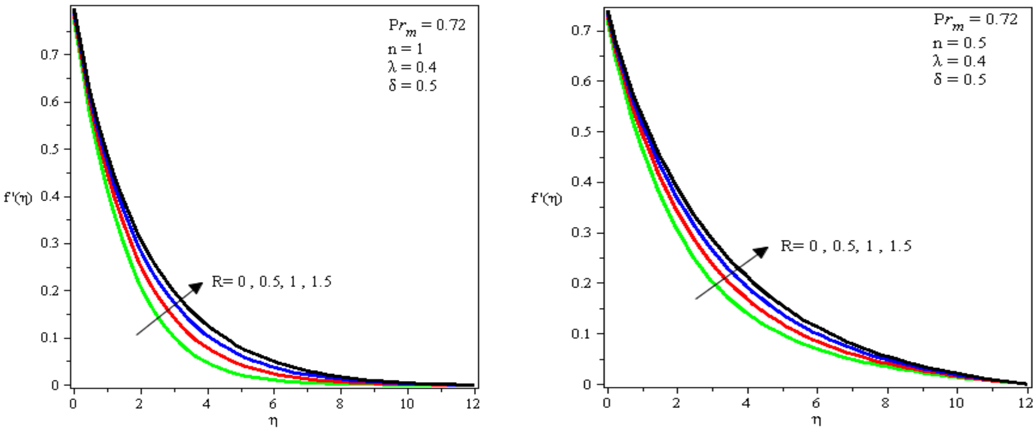


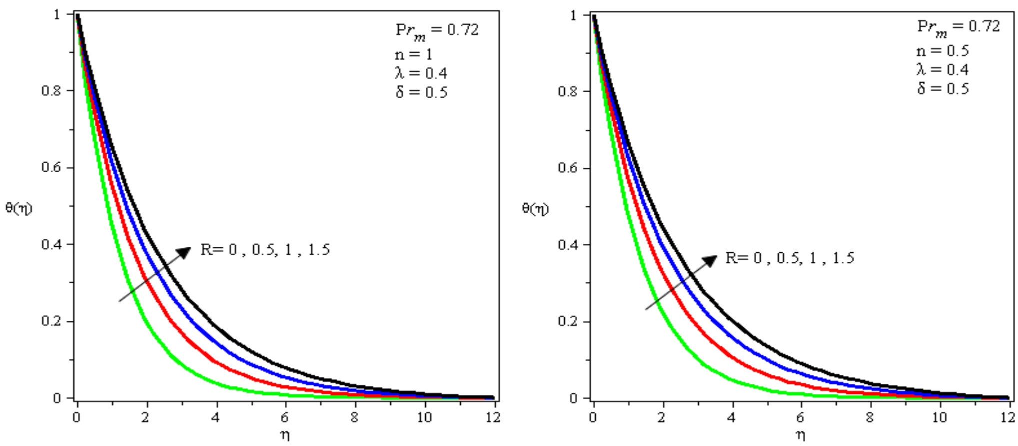
Fig. 1. Flow configuration and coordinate system



(a)

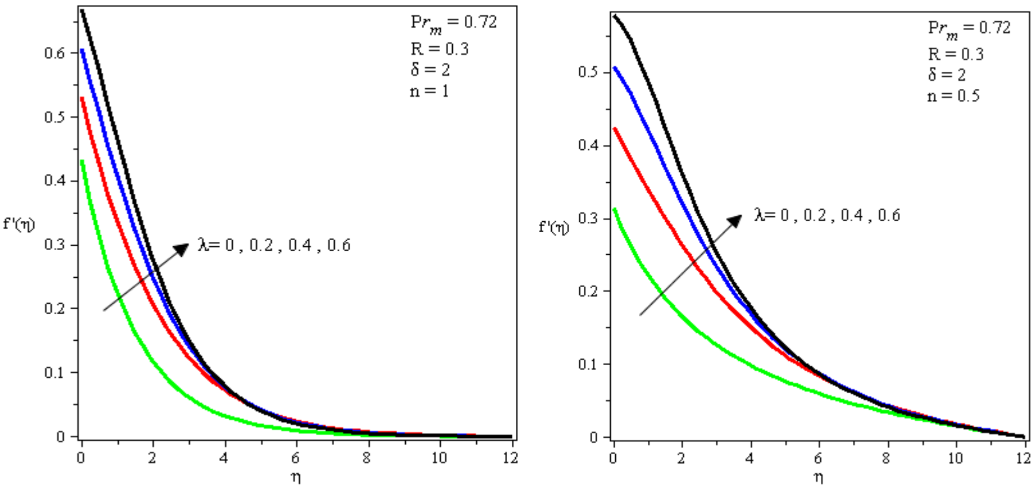
(b)

Fig. 2. Effects of thermal radiation parameter on the profile of dimensionless velocity for (a) Newtonian fluid $(n = 1)$, (b) non–Newtonian fluid $(n = 0.3)$



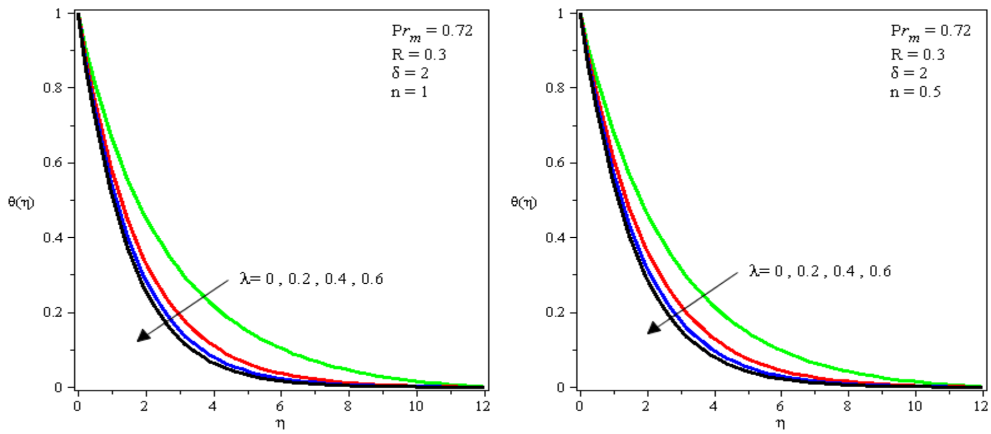
(a) (b)

Fig. 3. Effects of thermal radiation parameter on the profile of dimensionless temperature for (a) Newtonian fluid $(n = 1)$, (b) non – Newtonian fluid $(n = 0.3)$.



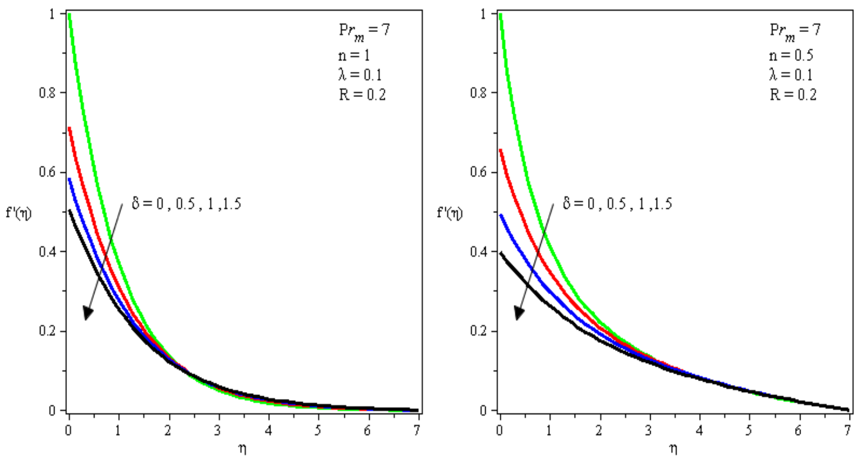
(a) (b)

Fig. 4. Effects of mixed convection parameter on the profile of dimensionless velocity for (a) Newtonian fluid $(n = 1)$, (b) non–Newtonian fluid $(n = 0.4)$.



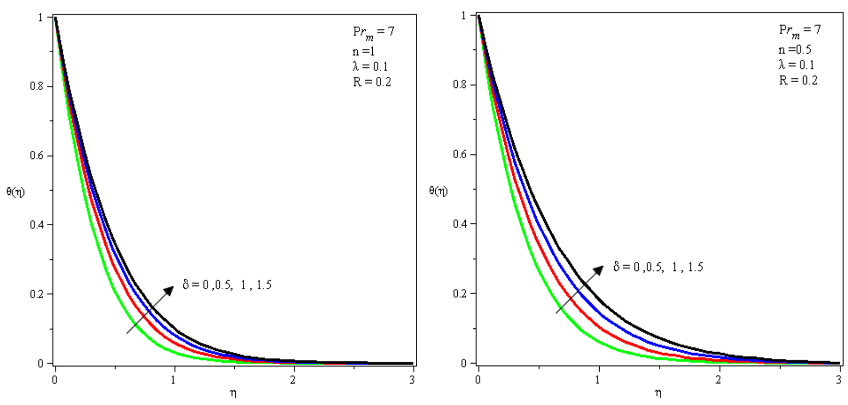
(a) (b)

Fig. 5. Effects of mixed convection parameter on the profile of dimensionless temperature for (a) Newtonian fluid $(n = 1)$, (b) non–Newtonian fluid $(n = 0.4)$



(a) (b)

Fig. 6. Effects of slip parameter on the profile of dimensionless velocity for (a) Newtonian fluid $(n=1)$, (b) non – Newtonian fluid $(n=0.2)$.



(a) (b)

Fig. 7. Effects of slip parameter on the profile of dimensionless temperature for (a) Newtonian fluid $(n=1)$, (b) non – Newtonian fluid $(n=0.2)$.

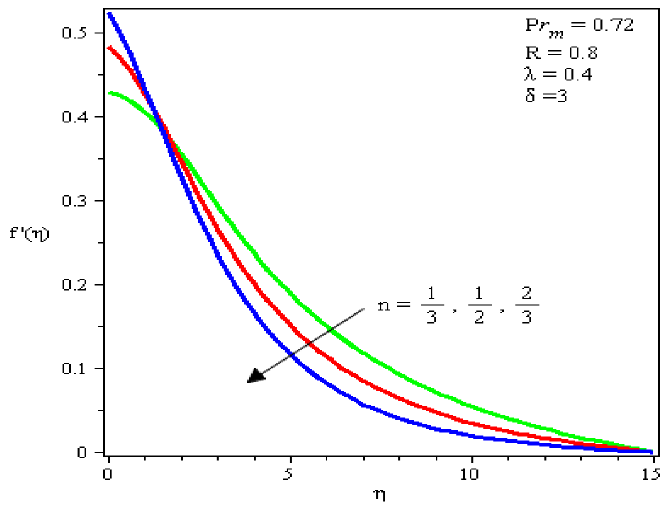


Fig. 8. Effects of power low index parameter on the profile of dimensionless velocity.

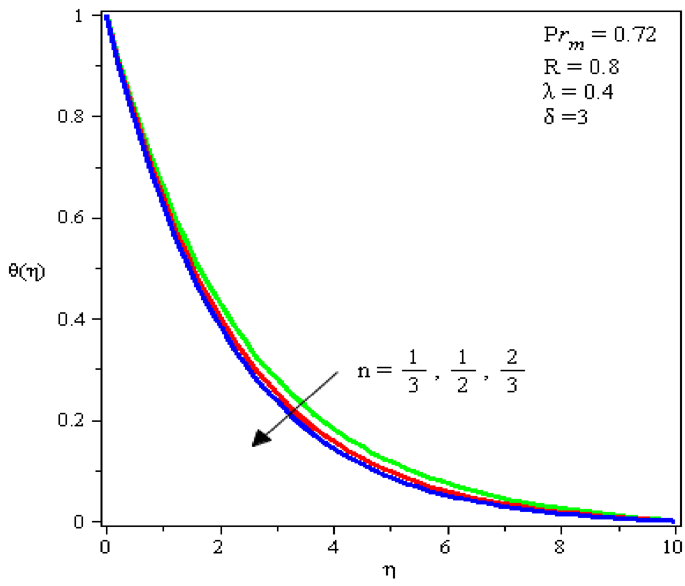


Fig. 9. Effects of power low index parameter on the profile of dimensionless temperature.

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