Improved scatter search for 4-colour mapping problem

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Abstract: The paper presents an improvement to the Scatter Search algorithm. The improvement is achieved by adding some of Bees Algorithm concepts to the Scatter Search algorithm. These concepts provided Scatter Search algorithm with more of exploration for problem search space and intensification for promising solutions. The original and improved Scatter Search algorithms have been tested on local 4-Colour Mapping problem instances and the results have been reported. The computational results illustrate that the improved Scatter Search algorithm is better than the original Scatter Search algorithm.

Keywords: metaheuristic; scatter search; bees algorithm; 4-colour mapping problem.

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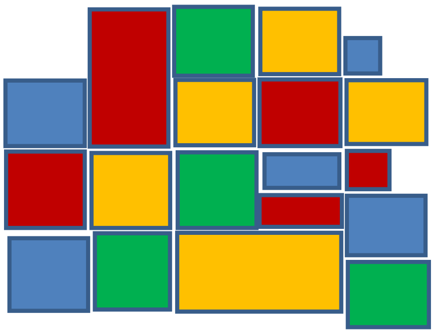
1 Introduction

Four colour mapping is one of the combinatorial optimisation problems that was first conjectured in 1852 by Francis Guthrie, and after over a century of work by many famous mathematicians (Wilson, 2002; Saaty et al., 1977). The 4-colour mapping can be defined as: The regions of any simple planar map can be coloured with only four colours, in such a way that any two adjacent regions have different colours (Gonthier, 2005). Figure 1 illustrates a vector of 20 regions that have been coloured with 4 colour and no two adjacent regions have the same colour.

Figure 1 Graph coloured with 4-colour (see online version for colours)



The scatter search algorithm, which is one of population-based metaheuristic, will be improved by using concepts that are taken from bees algorithm, which is one of several bio-inspired methods that were proposed to solve combinatorial optimisation problems. The improvement incorporates the most important concepts of bees algorithm with scatter search algorithm. 4-Colour Mapping problem will be used as a test ground for the Scatter Search and its improvement.

The rest of the paper is organised as follows. Scatter Search algorithm is described in Section 2. Section 3 presents brief description for Bees Algorithm. Section 4 presents the improved Scatter Search. Section 5 describes Improved Scatter Search for 4-Colour Mapping Problem. In Section 6, experimental results are presented. Finally, some concluding remarks are presented in Section 7.

2 Scatter search algorithm

Scatter Search (SS) is an evolutionary and population-based metaheuristic that has been applied to some of hard optimisation problems. The fundamental concepts and principles of this metaheuristic were first proposed in the 1970s and were based on formulations, dating back to the 1960s, for combining decision rules and problem constraints. The method uses strategies for search diversification and intensification that have proved effective in a variety of optimisation problems (Laguna, 2003).

Figure 2 illustrates the basic SS algorithm. SS works on a population of solutions of the problem to be solved, which are stored in a set of solutions called the Reference Set. The solutions in this set are combined in order to obtain new ones, trying to generate each time better solutions, according to high quality and diversity criteria (Laguna, 2003; Glove et al., 2000).

Figure 2 Basic scatter search algorithm

Algorithm 1: Scatter Search
Initialize the population (*P*) using a Diversification Generation Method;
Apply the Improvement Method to the *P*;
Reference Set Update Method(Good solutions for $RefSet_1$ and Diversity solutions for $RefSet_2$);
While (*itr* < *Maxitr*) do
  Subset Generation Method;
  While (subset-counter < > 0) do
    Solution Combination Method;
    Improvement Method;
    Reference Set Update Method;
  End while
End while

The design of a basic SS algorithm is generally based on the following five steps (Glove, 2000; Sagheer, 2012):

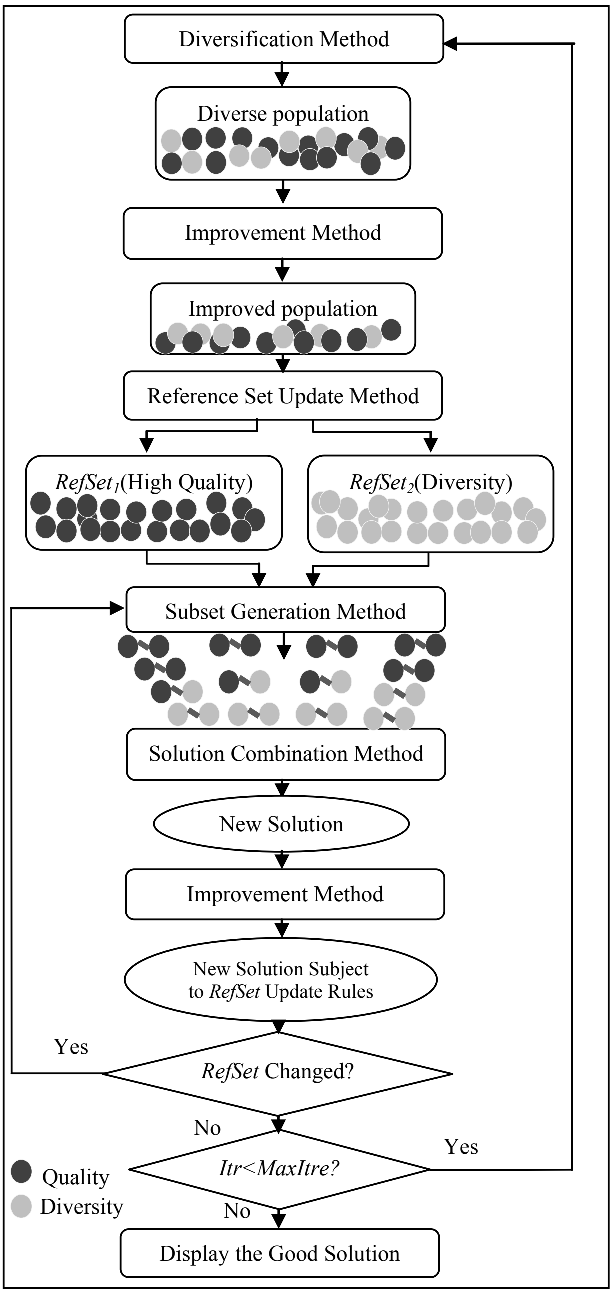
1. *A Diversification Generation Method* to generate a *P* of diverse trial solutions within the search space.
2. *An Improvement Method* to transform a trial solution into one or more enhanced trial solutions.
3. *A Reference Set Update Method* to build and maintain a Reference Set (*RefSet*). The objective is to ensure diversity while keeping high-quality solutions. For instance, one can select $RefSet_1$ solutions with the best objective function and then adding $RefSet_2$ solutions with the best diversity solutions ($RefSet = RefSet_1 + RefSet_2$).
4. *A Subset Generation Method* to operate on the reference set, to produce several subsets of its solutions as a basis for creating combined solutions.
5. *A Solution Combination Method* to transform a given subset of solutions produced by the Subset Generation Method into one or more combined solution vectors. Figure 3 illustrates the five SS steps.

The combined solutions will produce new solutions and the fitness of these new solutions will be evaluated. The new solutions will subject to the Reference Set Update Method which are (Laguna, 2003):

1. The new solution has a better objective function value than the solution with the worst objective value in $RefSet_1$.
2. The new solution has a better diversity value than the solution with the worst diversity value in $RefSet_2$.

The search continues while *RefSet* is changed. If no change in *RefSet*, the algorithm will check if number of iterations (*itr*) reaches the max iteration (*MaxItre*) that is detected by user then the algorithm will display the good solution(s) reached, else, the new *P* will be generated and $RefSet_1$ will be added to the start of new *P* (Sagheer, 2012).

Figure 3 Scatter search main steps



3 Bees algorithm

The Bees Algorithm (BA) is an optimisation algorithm proposed by Pham et al (Eberhart et al., 2001) based on the food foraging behaviour of honeybees in nature. The BA has become one of the most successful optimisation algorithms due to its successful implementation in various applications for optimisation problems such as neural network training (Pham et al., 2006), identifying homogenous data clustering (Pham et al., 2007) and a preliminary design problem (Pham et al., 2007).

The basic steps of the BA are explained in Figure 4. The Algorithm requires a number of parameters to be set. These are; number of scout bees (*n*), number of sites selected for neighbourhood search (*m*), number of best "elite" sites out of m selected sites (*e*), number of bees recruited for the best *e* sites (*nep*), number of bees recruited for the other (*m*) selected sites (*nsp*) and a stopping criterion (Pham et al., 2008).

Figure 4 Bees algorithm

Algorithm 2: Bees Algorithm
Initialize population with random solutions (*n*).
Evaluate fitness of the population.
While (stopping criterion not met)
  Select sites for neighborhood search (*m*).
  Recruit *nsp* of bees for selected (*m*) sites (elite *e* from *m* then recruits more *nep* of bees for *e* sites) and evaluate fitnesses.
  Select the fittest bee from each patch.
  Assign remaining bees to search randomly and evaluate their fitnesses.
End While.

In step 1 the algorithm generates *n* of scout bees which will return *n* of food sources (sites). In step 2 each site will evaluate its fitness. After evaluating the fitness of each site, it will be ordered depending on their objective function (fitness) where the fitness that has high quality will be in the beginning.

In step 4 and after ordering operation, *m* good sites will be taken. Then *e* elite the best sites in m. Then in step 5, the colony recruits some of bees *nsp* to *m* sites and neighbourhood search generates new solutions and the best solution will replace with worst one in *m*. At the same time the colony will recruit more of bees *nep* to the *e* sites to make neighbourhood search on each site in *e* to generate new site and if the new site is better than the previous it will be replaced. In step 6, select the fittest bee from each site where the best sites in *m* and *e* will be selected. Step 7, the remaining bees (*n*-*m*) will assign to random function to create new solutions and evaluate their fitness. After evaluating their fitness, the algorithm will redo the step 4, 5, 6 and 7 and the process of search continues until stopping criteria is met.

4 Improved scatter search algorithm

The improvement to SS algorithm was accomplished by using algorithm inspired from biological nature. BA is inspired from the behaviour of bees in forage of nectar. This algorithm has proved its ability in solve most combinatorial problems and finding the nearest global optimum solution in reasonable time.

Not all concepts or steps of Bees has been used in improve SS algorithm. The selected steps from Reference Set Update Method are the *m* steps which are representing the sites (solutions) and the other *e steps* which elite best sites in *m*. Figure 5 illustrates the improved SS algorithm. The new proposed changes in the original SS algorithm are illustrated in the bold font in Figure 5.

Figure 5 Improved scatter search algorithm

Algorithm 3: Improved Scatter Search
Initialize the population Pop using a Diversification Generation Method;
Apply the Improvement Method to the population;
Reference Set Update Method (Good solutions for $RefSet_1$ and Diversity solutions for $RefSet_2$);
While (*itr* < *Maxitr*) do
    While (Reference set is changed) do
        Select *m* sites (from Reference Set) for neighborhood search.
        Recruit *nsp* of bees for selected (*m*) sites (elite *e* from *m* then recruits more *nep* of bees for *e* sites) and evaluate fitnesses.
        Reference Set Update Method
        Subset Generation Method;
        While (subset-counter < > 0) do
            Solution Combination Method;
            Improvement Method;
            Reference Set Update Method;
        End while
    End while
End while

In *Reference Set Update Method* the best and diversity solutions are determined and the number of best solutions b_1 and diversity solutions b_2 will be chosen. The input to BA concepts from SS algorithm is b_1 best solutions and the output is b_1 best solution after updating (generating new best solutions from *m* and *e*) and these new solutions are input to other SS algorithm steps to complete the search.

5 Improved scatter search for 4-colour mapping

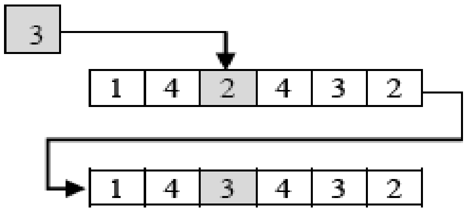
The original and improved SS will be tested on 4-Colour Mapping problem which is one of combinatorial optimisation problem. We will set the 4-Colour Mapping problem depending on the original and improved SS steps.

In the Diversification Method, *P* of solutions of coloured region will be generated by using randomised local search procedure, which depends on seed that represents the initial solution will enter to local search to generate *P* from it. This procedure generates random solutions each solution is different from another and each solution is represented as array containing the colours of each region. The colours in each solution will be represented by the numbers (1,2,3,4) where each number represents a colour in solution such as 1=red, 2=green, 3=blue and 4=yellow.

After generating *P* by Diversification Generation Method, the fitness of each solution in *P* will be evaluated. The fitness in 4-Colour Mapping problem is the summation of neighboured regions that have the same colour.

After evaluating the fitness of each solution, all the solutions in *P* will be improved or remain without change by Improvement Method. The improvement is achieved by using local search algorithm which is heuristic replaces the current solution by a neighbour that improves the objective function. The neighbourhood is done by mutating the solution by inserting new colour instead of another. Figure 6 illustrates the mutation in the solution.

Figure 6 One mutation (6 regions)



The mutation will change the fitness of solution because the region that was with 2=blue changed to 3=green.

After local search generates a new solution, the fitness of this new solution will be evaluated. If the new solution is better than current solution then current solution = new solution else the current solution will remain without change. These operations will be applied to all solutions in *P* to obtain more of feasible solutions.

Solutions of *P* will be ordered depending on their fitness and Reference Set Update Method will chose $RefSet_1$ which will take the first b_1 solutions in *P* and delete them from P, while $RefSet_2$ will be generated by Euclidean distance to select diversity solutions. The Euclidean distance computes the dissimilarity of the solutions in $RefSet_1$ with solutions of $RefSet_1 - P$. These solutions of *P* which are dissimilar with solutions of $RefSet_1$ will be chosen as solutions of $RefSet_2$.

Solutions of $RefSet_1$ will enter to *m* which is the first concept that taken from BA, number of bees (*nsp*) will recruit for solutions of m in order to generate new solutions. After *m* generate new solutions and updated, *e* will elite the best solutions in *m* and number of bees (*nep*) will recruit in order to generate new solutions. *e* and *m* use the mutation mechanism in Figure 6 in generating the new solution. After *e* solutions are updated, solutions of *e* will be compared with solutions of $RefSet_1$ and the best solutions will be replaced with worst one.

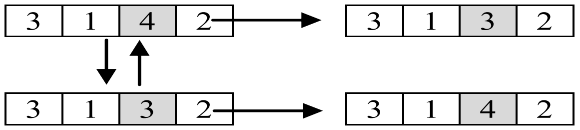
Each solution in RefSet will make subset with all other solutions in RefSet by Subset Generation Method.

Solution Combination Method uses the subsets generated with the Subset Generation Method to combine the solutions in each subset with the purpose of creating new trial solutions. Depending on the form of the solution in 4-Colour Mapping problem the new solution can be generated by two techniques:

1 Crossover the colours of solutions in the subset (see Figure 7).

2 Mutation of solutions in the subset (see Figure 6).

Figure 7 The crossover operation



The combination method can use either one of these two techniques or it can hyper the combination for solution by using these two techniques. During the experiments, the second technique is more appropriate for generating new feasible solutions in SS, so in our experiments we will use the second technique with the following conditions (Laguna, 2003):

1 If both x and x' in subset are elements of $RefSet_1$, then generate 3 solutions by applying mutation($x(i)a$,$x(i+1)b$,$x(i+2)c$) where x = first solution in subset, x' =second solution in subset i=number of solution.

2 If only one of x and x' in subset is a member of $RefSet_1$, then generate 3 solutions for member of $RefSet_1$ and generate 2 solutions for $RefSet_2$ solutions by applying mutation($x(i)a$,$x(i+1)b$) where i=number of solution.

The combination method will search for regions in solution that have neighbours and neighbours have same colour, if they found the combination will mutate these regions to create new solution.

After generating the new solution by applying Solution Combination Method to each subset, it will be subjected to reference set update rules where the new generated solution may become a member of the reference set if either one of the following conditions is satisfied:

- The new solution has a better objective function value than the solution with the worst objective value in $RefSet_1$.
- The new solution has a better dissimilarity value than the solution with the worst dissimilarity value in $RefSet_2$.

In both cases, the new solution replaces the worst and the ranking which is updated to identify the new worst solution in terms of either quality or diversity. This loop terminates when the reference set does not change and all the subsets have already been subjected to the Solution Combination Method. At this point, the regeneration by Diversification Generation Method is used to construct a new diversity solutions and the search continues. The regeneration consists of keeping $RefSet_1$ in first of new population and using the Diversification Generation Method to construct new diverse solutions

6 Computational experiments

SS and its improvement algorithms were implemented in Microsoft Visual C# 2005 Express Edition and run on a computer whose processor is AMD Turion™ 64 2 Mobile Technology TL60 (2 CPU) 2.0 GHz, with 3 GB main memory, 200 GB hard disk. The algorithms were applied to local instances with small sizes ranging from 5 to 30 regions. The stop criteria are chosen as follow:

1 If no change in Reference Set.

2 To reach a maximum number of iterations = 10.

The following parameters are chosen:

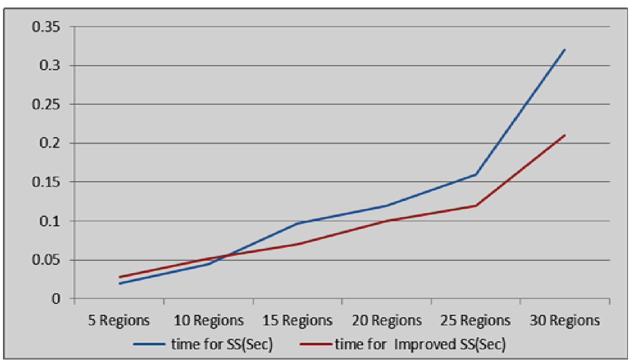
- Initial population P =100,
- The size of $|RefSet_1|$ $=b_1$=10, the size of $|RefSet_2|$ $=b_2$=10 and the size of reference set $|RefSet|$ = $|RefSet_1|+|RefSet_2|$=20.
- Size of selected sites m=10.
- Number of bees recruit for selected site nsp=25.
- Size of the best sites in m, e=5.
- Number of bees recruit for elite sites nep =30.

As the 4-Colour Mapping can colour small instance and find the optimal solution (which is represented by the vector that contains regions adjacent and have different colours) rapidly, the comparison of the original and improved SS algorithms will depend on the time required to find the optimal solutions for each vector of regions. Fifteen run for each instances have been achieved and the average time required to reach the optimal solutions for each vector of regions have been reported in Table 1. It shows how the original SS is faster than the improved SS with small instance such as with 5 and 10 regions but when the instance extended the improved SS is faster than the original SS algorithm. Figure 8 illustrates the difference in reaching to the optimal solution in faster time for original and improved SS algorithms.

Table 1 Average of elapsed time for SS and improved SS for 4-colour mapping problem

Instances	*Average of elapsed time for SS(Sec)*	*Average elapsed time for improved SS(Sec)*
5 Regions	0.02	0.028
10 Regions	0.044	0.052
15 Regions	0.097	0.07
20 Regions	0.12	0.1
25 Regions	0.16	0.12
30 Regions	0.32	0.21

Figure 8 Optimal solution time (see online version for colours)



7 Conclusion and future work

The improvement for Scatter Search algorithm has been achieved by adding some of Bees Algorithm concepts to the Scatter Search algorithm. The original and improved Scatter Search algorithms have been tested on 4-Colour Mapping problem which is one of the combinatorial optimisation problems. The computational results illustrate that the improved Scatter Search algorithm present better performance than original Scatter Search algorithm where the improved Scatter Search reached to the optimal solution faster than the original Scatter Search.

The algorithms presented are still being developed; the next step would be testing them on a greater variety of problems with different parameters, stopping criterion, and problem-specific combination operators. The other SS and SBS contain several parameters that can operate on it to improve the performance such as using Population-based Metaheuristic in step 2 which is improvement method. Also can be use ant colony or partial swarm concepts on reference set which represents the most effective step in SS and its improvements.

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