Journal of Interdisciplinary Mathematics

ISSN: 0972-0502 (Print) 2169-012X (Online) Journal homepage: https://www.tandfonline.com/loi/tjim20

Soft #(*sc*)-spaces

Nadia Ali Nadhim, Rasha Naser Majeed & Haider Jebur Ali

To cite this article: Nadia Ali Nadhim, Rasha Naser Majeed & Haider Jebur Ali (2019) Soft #(*sc*)-spaces, Journal of Interdisciplinary Mathematics, 22:8, 1463-1470, DOI: 10.1080/09720502.2019.1706843

To link to this article: https://doi.org/10.1080/09720502.2019.1706843

Published online: 20 Jan 2020.

Submit your article to this journal

View related articles

View Crossmark data

Full Terms & Conditions of access and use can be found at
https://www.tandfonline.com/action/journalInformation?journalCode=tjim20

Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 22 (2019), No. 8, pp. 1463–1470
DOI : 10.1080/09720502.2019.1706843

Soft $\mathcal{K}(sc)$-spaces

Nadia Ali Nadhim *
Department of Mathematics
Faculty of Education for Pure Science
University of Al-Anbar
Al-Anbar
Iraq

Rasha Naser Majeed †
Department of Mathematics
Faculty of Education for Pure Science (Ibn al-Haytham)
University of Baghdad
Baghdad
Iraq

Haider Jebur Ali §
Department of Mathematics
College of Science
University of Al-Mustansiriyah
Baghdad
Iraq

Abstract

The aim of this work is to present and investigate the concept of soft $\mathcal{K}(sc)$-spaces. Every soft T_2-space is soft $\mathcal{K}(sc)$-space was obtaind. While a suitable condition on a set of parameters to get every soft $\mathcal{K}(sc)$-space is soft semi T_1-space. Detailed study of soft $\mathcal{K}(sc)$-spaces with examples is carried out.

Subject Classification: *06D72, 54A40*

Keywords: *Soft topological space, soft compact set, semiclosed soft set, soft semi T_1-space, soft T_1-space, soft continuous mapping*

*E-mail: nadiaalselway@gmail.com (Corresponding Author)
†E-mail: rashanm6@gmail.com
§E-mail: drhaiderjebur@uomustansiriyah.edu

© Taru Publications

1. Introduction

The theory of softs was introduce by Molodtsov [9], as a mathematical tool for solve doubts. In 2003, Maji et al. [7] provide most basic processes of the soft set theory. To improve this concept, in 2011 Shabir and Naz [12] provided and studied the concept of soft topological space (S-Top-sp, for short). In 2012, Zorlutuna et al. [14] introduced the notion of compactness in S-Top-sp's. Also, in 2014 Peyghan et al. [11] studied some properties related to the soft compact sp's. Recently, independently Mahanta and Das [6] and Chen [4] were introduced and studied the notion of semiopen soft sets in S-Top-sp's, as a weak forms of an open soft sets. Also, Chen [4] introduced a new types of soft separation axioms by using the concept of semiopen soft sets.

The notion of $\mathcal{K}c$-spaces ($\mathcal{K}c$-sp's, for short) was first introduced in general topology in 1967 by Wilansky [13]. This notion studied by several researchers (e.g. [2, 3, 5, 8]). On the other hand in 2015, Abu-Ragheef [1] introduced the concept of $\mathcal{K}(sc)$-sp's as a weak forms of $\mathcal{K}c$-sp's.

Very recently, in 2019 Nadhim et al. [10] introduced the notion of $\mathcal{K}c$-sp's in the S-Top-sp's. In the current work we introduced the concept of $\mathcal{K}(sc)$-sp's in the STop-sp's and studied several importents properties of this concept. We show that the hereditary property is not satisfies. In addition, the properties of soft $\mathcal{K}(sc)$-sp's under soft mappings, such as the topological property and some other properties are studied.

2. Preliminaries

We expect that the reader is knows about the usual notions and most basic ideas of soft set theory, soft mappings and soft topology Throughout our paper, denote by $\mathcal{V}$ an initial universe, by $\mathcal{L}$ a set of parameters and by $P(\mathcal{V})$ the power set of $\mathcal{V}$. A soft set $G_{\mathcal{A}}$ on the universe set $\mathcal{V}$ is a mapping defined from $\mathcal{A}$ into $\mathcal{A} \subseteq \mathcal{L}$, where $\mathcal{SS}(\mathcal{V})_{\mathcal{L}}$. Denote by $\mathcal{V}$ to the set of all soft sets over $\mathcal{V}$ (see [9]). For a point $x \tilde{\notin} \mathcal{V}$ and $G_{\mathcal{L}} \in \mathcal{SS}(\mathcal{V})_{\mathcal{L}}$, we say that x belongs to the soft set $G_{\mathcal{L}}$, denoted by $x \tilde{\in} G_{\mathcal{L}}$ whenever $x \in G(\ell)$ for all $\ell \in \mathcal{V}$. For any $x \in \mathcal{V}$, $x \tilde{\in} G_{\mathcal{L}}$ if $x \notin G_{\mathcal{L}}$ for some $\ell \in \mathcal{L}$. For a point $x \in \mathcal{V}$, $x_{\mathcal{L}}$ denotes to the soft set over $\mathcal{V}$ such that $x(\ell) = \{x\}$, for every $\ell \in \mathcal{V}$ (see [12]). A family $T \subseteq \mathcal{SS}(\mathcal{V})_{\mathcal{L}}$ is said to be a soft topology on $\mathcal{V}$ if the following axioms are satisfied: $\Phi_{\mathcal{L}}$, $\tilde{\mathcal{V}} \in T$ (where $\Phi_{\mathcal{L}}$, $\tilde{\mathcal{V}}$ denotes to the null (absolute) soft set) and closed under finite intersection and arbitrary union. The triple $(\mathcal{V}, T, \mathcal{L})$ is called a S-Top-sp over $\mathcal{V}$. The elements of T are called an open soft set in $\mathcal{V}$. The complement of any open soft set is called a closed soft set.(see [12]). Let W be anon-empty subset of $\mathcal{V}$ and $\mathcal{F}_{\mathcal{L}}$ be a soft set over $\mathcal{V}$. Then, the subsoft set of $\mathcal{F}_{\mathcal{L}}$ over W denoted by $\mathcal{F}_{\mathcal{L}}^{W}$ is defined as follows $\mathcal{F}^{W}(\ell) = W \cap \mathcal{F}(\ell)$ for all $\ell \in \mathcal{V}$. The soft relative topology on a non-empty subset W of $\mathcal{V}$ is defined as $T_W = \{\mathcal{F}_{\mathcal{L}}^{W} : \mathcal{F}_{\mathcal{L}} \in T\}$, and $(W, T_W, \mathcal{L})$ is called a S-subspace (S-subsp, for short) of $(\mathcal{V}, T, \mathcal{L})$. (see [12]). A soft set $G_{\mathcal{L}}$ is said to be semiopen(closed) soft set if there exists an open(closed) soft set $\mathcal{F}_{\mathcal{L}}$ such that $\mathcal{F}_{\mathcal{L}} \sqsubseteq G_{\mathcal{L}} \sqsubseteq \overline{\mathcal{F}_{\mathcal{L}}}$ $(\mathcal{F}_{\mathcal{L}}^{o} \sqsubseteq G_{\mathcal{L}} \sqsubseteq \mathcal{F}_{\mathcal{L}})$. Clearly, every open(closed) soft set is a semiopen (semiclosed) soft set (see [6]).

Definition 2.1 : [6] Let $(\mathcal{V}, T, \mathcal{L})$ be a S-Top-sp over $\mathcal{V}$ and $\mathcal{F}_{\mathcal{L}}$ be a soft set over $\mathcal{V}$. Then, the semiclosure soft set of $\mathcal{F}_{\mathcal{L}}$ is a soft set denoted by $sCl(\mathcal{F}_{\mathcal{L}})$ where $sCl(\mathcal{F}_{\mathcal{L}}) = \sqcap \{G_{\mathcal{L}} : G_{\mathcal{L}}$ is semiclosed soft set and $\mathcal{F}_{\mathcal{L}} \sqsubseteq G_{\mathcal{L}}\}$.

Theorem 2.2 : [6] *Let $(\mathcal{V}, T, \mathcal{L})$ be a S-Top-sp over $\mathcal{V}$, and $\mathcal{F}_{\mathcal{L}}$ is a soft sets over $\mathcal{V}$. Then*

1. $\mathcal{F}_{\mathcal{L}}$ is a semiclosed soft set if and only if $\mathcal{F}_{\mathcal{L}} = sCl(\mathcal{F}_{\mathcal{L}})$.
2. $sCl(\mathcal{F}_{\mathcal{L}}) = sCl(sCl(\mathcal{F}_{\mathcal{L}}))$
3. $sCl(\mathcal{F}_{\mathcal{L}})$ is the smallest semiclosed soft set over $\mathcal{V}$ contains $\mathcal{F}_{\mathcal{L}}$.

Definition 2.3 : [12, 4] A S-Top-sp $(\mathcal{V}, T, \mathcal{L})$ over $\mathcal{V}$ is called

1. soft T_1 (semi T_2)-sp, if for every $x, y \in \mathcal{V}$ such that $x \neq y$, $\exists\, \mathcal{F}_{\mathcal{L}}, G_{\mathcal{L}} \in T$ ($\exists$ semiopen soft sets $\mathcal{F}_{\mathcal{L}}, G_{\mathcal{L}}$.) such that $x \tilde{\in} \mathcal{F}_{\mathcal{L}}$ and $y \tilde{\notin} \mathcal{F}_{\mathcal{L}}$ and $y \tilde{\in} G_{\mathcal{L}}$ and $x \tilde{\notin} G_{\mathcal{L}}$.
2. soft T_2(semi T_2)-sp, if for every $x, y \in \mathcal{V}$ such that $x \neq y$, $\exists\, \mathcal{F}_{\mathcal{L}}, G_{\mathcal{L}} \in T$ ($\exists$ semiopen soft sets $\mathcal{F}_{\mathcal{L}}, G_{\mathcal{L}}$.) such that $x \tilde{\in} \mathcal{F}_{\mathcal{L}}$, $y \tilde{\in} G_{\mathcal{L}}$ and $\mathcal{F}_{\mathcal{L}} \sqcap G_{\mathcal{L}} = \Phi_{\mathcal{L}}$.

Theorem 2.4 : [4] *Let $(\mathcal{V}, T, \mathcal{L})$ be a S-Top-sp over $\mathcal{V}$. If $x_{\mathcal{L}}$ is a semiclosed soft set in $(\mathcal{V}, T, \mathcal{L})$ for each $x \in \mathcal{V}$, then $(\mathcal{V}, T, \mathcal{L})$ is a soft semi T_1-sp.*

Definition 2.5 : [14] A S-Top-sp $(\mathcal{V}, T, \mathcal{L})$ is said to be a soft compact, if each soft open cover of $\tilde{\mathcal{V}}$ has a finite subcover.

Theorem 2.6 : [11] *Let $(W, T_W, \mathcal{L})$ be a S-subspace of a S-Top-sp $(\mathcal{V}, T, \mathcal{L})$. Then, $(W, T_W, \mathcal{L})$ is a soft compact if and only if every soft open cover of $\tilde{W}$ by open soft sets in $\mathcal{V}$ contains a finite subcover.*

Definition 2.7 : [15] A soft mapping f_{qv} from a S-Top-sp $(\mathcal{V}, T, \mathcal{L})$ into a S-Top-sp $(\mathcal{V}^{*}, T^{*}, \mathcal{D})$ is called

1. soft continuous, if for each $G_{\mathcal{D}} \in T^{*}$, $f_{qv}^{-1}(G_{\mathcal{D}}) \in T$.

2. soft open, if for each $\mathcal{F}_{\mathcal{L}} \in T$, $f_{qv}(\mathcal{F}_{\mathcal{L}}) \in T^{*}$..

Theorem 2 : 8 [14] *If* $f_{qv} : (\mathcal{V}, T, \mathcal{L}) \to (\mathcal{V}^{*}, T^{*}, \mathcal{D})$ *is a soft continuous and onto mapping and* $(\mathcal{V}, T, \mathcal{L})$ *is a soft compact Top-sp, then* $(\mathcal{V}^{*}, T^{*}, \mathcal{L})$ *is a soft compact Top-sp.*

Theorem 2.9 : [11] *Every soft compact subsp* $(W, T_W, \mathcal{L})$ *of a soft* T_2*-sp* $(\mathcal{V}, T, \mathcal{L})$ *is a closed soft set.*

Theorem 2.10 : [11] *Every closed soft subset of soft compact sp is a soft compact.*

Proposition 2.12 : [10] If $f_{qv} : (\mathcal{V}, T, \mathcal{L}) \to (\mathcal{V}^{*}, T^{*}, \mathcal{D})$ is a bijective soft open mapping, then f_{qv}^{-1} is a soft continuous mapping.

3. Soft $\mathcal{K}(sc)$-spsces

In this section we introduce the definition of soft $\mathcal{K}(sc)$-sp's and give some properties about it, examples and it is relationship with soft separation axioms.

Definition 3.1 : A S-Top-sp $(\mathcal{V}, T, \mathcal{L})$ is said to be a soft $\mathcal{K}(sc)$-sp, if every soft compact subsp $(W, T_W, \mathcal{L})$ is semiclosed soft set in $\mathcal{V}$.

Example 3.2 : Let $\mathcal{V} = \mathbb{R}$, $\mathcal{L} = \mathbb{R}$ where $\mathbb{R}$ and $\mathbb{N}$ are the set of real and natural numbers respectively, let $(\mathcal{F}_{\mathcal{L}})_{ab} = \{(n,(a,b)) : n \in \mathcal{L}\}$ and T = $\{\Phi_{\mathcal{L}}, \tilde{\mathbb{R}}\} \cup \{(\mathcal{F}_{\mathcal{L}})_{ab} : a, b \in \mathbb{R}\}$. Then, it is clear that $(\mathbb{R}, T, \mathbb{N})$ is a soft $\mathcal{K}(sc)$-sp.

Proposition 3.3 : Every soft $\mathcal{K}c$-sp is a soft $\mathcal{K}(sc)$-sp.

Proof *:* Let $(\mathcal{V}, T, \mathcal{L})$ be soft $\mathcal{K}c$-sp and W be a non empty subset of $\mathcal{V}$, such that $(W, T_W, \mathcal{L})$ be a soft compact subsp of $(\mathcal{V}, T, \mathcal{L})$ which is soft $\mathcal{K}c$-sp, then $\tilde{W}$ is a closed soft set in $\mathcal{V}$ which implies, $\tilde{W}$ is a semiclosed soft set in $\mathcal{V}$.Therefore, $(\mathcal{V}, T, \mathcal{L})$ is a soft $\mathcal{K}(sc)$-sp. □

In the next the relationship between the soft $\mathcal{K}(sc)$-sp's with soft separation axioms are discussed.

Proposition 3.4 : Every soft T_2-sp is a soft $\mathcal{K}(sc)$-sp.

Proof : Let $\tilde{W}$ be a soft compact subsp of $(\mathcal{V}, T, \mathcal{L})$. From hypothesis and Theorem 2.9, we get $\tilde{W}$ is a closed soft set in $\mathcal{V}$ and by every closed soft set is semiclosed soft set, we get $\tilde{W}$ is a semiclosed soft set in $\mathcal{V}$ Therefore, $(\mathcal{V}, T, \mathcal{L})$ is a soft $\mathcal{K}(sc)$-sp. □

Remark 3.5 : The opposite of Proposition 3.4 is not necesarily true as the next example shown.

Example 3.6 : Consider the S-Top-sp ($\mathbb{R}$, T_*, $\mathcal{L}$), where $\mathcal{L}$ is a countable set and $T_* = \{\mathcal{F}_{\mathcal{L}} : \cup_{\ell\in\mathcal{L}} \mathbb{R} - \mathcal{F}(\ell)$ is countable set$\} \cup \{\Phi_{\mathcal{L}}\}$. Then, ($\mathbb{R}$, T_*, $\mathcal{L}$) is soft $\mathcal{K}c$-sp but not soft T_2-sp (see Example 3.8 [16]). Thus, it follows ($\mathbb{R}$, T_*, $\mathcal{L}$) , is soft $\mathcal{K}(sc)$-sp but not soft T_2-sp.

To study the relationship between soft $\mathcal{K}(sc)$-sp and soft semi T_1-sp, we need first to recall the following lemma that introduced in [10].

Lemma 3.7 : *Let ($\mathcal{V}$, T, $\mathcal{L}$) be a S-Top-sp with finite set of parameters $\mathcal{L}$. Then, for all $x \in \mathcal{V}$, $x_{\mathcal{L}}$ is a soft compact subset in $\mathcal{V}$.*

Remark 3.8 : In Lemma 3.7, it is clear that if we put $W = \{x\}$, then (W, T_W, $\mathcal{L}$) is soft compact subsp of ($\mathcal{V}$, T, $\mathcal{L}$).

Theorem 3.9 : *Every soft $\mathcal{K}(sc)$-sp with a finite set of parameters is a soft semi T_1-sp.*

Proof *:* Let $x \in \mathcal{V}$. Then, from Lemma 3.7, it follows $x_{\mathcal{L}}$ is a soft compact subset in $\mathcal{V}$, and by Theorem 2.6, it follows ($\{x\}$, $T_{\{x\}}$, $\mathcal{L}$) is soft compact subsp of ($\mathcal{V}$, T, $\mathcal{L}$). But ($\mathcal{V}$, T, $\mathcal{L}$) is a soft $\mathcal{K}(sc)$-sp, which implies $x_{\mathcal{L}}$ is a semiclosed soft set in $\mathcal{V}$. By Theorem 2.4, we get ($\mathcal{V}$, T, $\mathcal{L}$) is a soft semi T_1-sp. □

Remark 3.10 : The convers of Theorem 3.9 is not satisfied in general, we shows that by the next example.

Example 3.11 : Let $\mathcal{V} = \{a, c\}$, $\mathcal{L} = \{\ell_1, \ell_2\}$ and $T = \{\Phi_{\mathcal{L}}, \tilde{\mathcal{V}}, (\mathcal{F}_{\mathcal{L}})_1, (\mathcal{F}_{\mathcal{L}})_2, (\mathcal{F}_{\mathcal{L}})_3\}$, where $(\mathcal{F}_{\mathcal{L}})_1 = \{(\ell_1, \mathcal{V}), (\ell_2, \{c\})\}$, $(\mathcal{F}_{\mathcal{L}})_2 = \{(\ell_1, \{a\}), (\ell_2, \mathcal{V})\}$, and $(\mathcal{F}_{\mathcal{L}})_3 = \{(\ell_1, \{a\}), (\ell_2, \{c\})\}$. Then, ($\mathcal{V}$, T, $\mathcal{L}$) is a soft T_1-sp and from Proposition 2.30 part(1), ($\mathcal{V}$, T, $\mathcal{L}$) is a soft semi T_1-sp, but not soft $\mathcal{K}(sc)$-sp because there exists $W = \{a\}$ such that $\tilde{W} = \{(\ell_1, \{a\}), (\ell_2, \{a\})\}$ is soft compact subsp of $\mathcal{V}$. But $\tilde{W}$ is not semiclosed soft set in $\mathcal{V}$, since $\tilde{W}^c$ is not semiopen soft set, since the only open soft set which is contained in $\tilde{W}^c$ is $\Phi_{\mathcal{L}}$ but $\Phi_{\mathcal{L}} \sqsubseteq \tilde{W}^c$ $\not\sqsubseteq \Phi_{\mathcal{L}} = \overline{\Phi_{\mathcal{L}}}$. Therefore, ($\mathcal{V}$, T, $\mathcal{L}$) is not soft $\mathcal{K}(sc)$-sp.

The next example shown that the hereditarily property of soft $\mathcal{K}(sc)$-sp's does not true in general.

Example 3.12 : In Example 3.2, ($\mathbb{R}$, T, $\mathbb{N}$) is a soft $\mathcal{K}(sc)$-sp. Let $W = [0,1] \subseteq \mathbb{R}$. Then, $T_W = \{\Phi_{\mathcal{L}}, \tilde{W}, (G_{\mathcal{L}})_1, (G_{\mathcal{L}})_2, (G_{\mathcal{L}})_3\}$, where $(G_{\mathcal{L}})_1 = \{(n, (a,b)), n \in \mathbb{N}\}$, $(G_{\mathcal{L}})_2 = \{(n, [0,b)), n \in \mathbb{N}\}$, $(G_{\mathcal{L}})_3 = \{(n, (a,1]), n \in \mathbb{N}\}$. To show ($W$, T_W, $\mathcal{L}$) is not soft $\mathcal{K}(sc)$-sp. Let $\mathcal{V} = [1/2, 3/4] \subseteq W$. Then $\tilde{\mathcal{V}} = \{(n, [1/2, 3/4]) : n \in \mathbb{N}\}$ is a soft compact subsp but it is not semiclosed soft set in $\tilde{W}$, since for any closed soft set $\mathcal{H}_{\mathbb{N}}$ in W such that $\tilde{\mathcal{V}} \sqsubseteq \mathcal{H}_{\mathbb{N}}$ we have $\mathcal{H}^{o}_{\mathbb{N}} \not\sqsubseteq \tilde{\mathcal{V}} \sqsubseteq \mathcal{H}_{\mathbb{N}}$. Hence, $(W, T_W, \mathcal{L})$ is not soft $\mathcal{K}(sc)$-subsp of $(\mathbb{R}, T, \mathbb{N})$.

4. Some properties of soft $\mathcal{K}(sc)$-spaces related with soft mappings

In this section we introduce some results about soft $\mathcal{K}(sc)$-sp's under soft mappings. In order to discuss the topological property of a soft $\mathcal{K}(sc)$-sp's, we need first to introduce the following lemma.

Lemma 4.1 : *Let $f_{qv} : (\mathcal{V}, T, \mathcal{L}) \to (\mathcal{V}^*, T^*, \mathcal{L})$ be a soft homeomorphism mapping. Then:*

1. *$f_{qv}(\mathcal{F}_{\mathcal{L}})$ is semiclosed soft set in $\mathcal{V}^*$, for each $\mathcal{F}_{\mathcal{L}}$ semiclosed soft in $\mathcal{V}$.*
2. *$f_{qv}^{-1}(G_{\mathcal{L}})$ is semiclosed soft set in $\mathcal{V}$, for each $G_{\mathcal{L}}$ semiclosed soft in $\mathcal{V}^*$.*

Proof. The proof follows from the definition of soft semiopen(closed) and the definition of image(inverse image) of soft set. □

Theorem 4.4 L *The property of a soft $\mathcal{K}(sc)$-sp's, is a topological property.*

Proof : Let f_{qv} be a soft homeomorphism mapping from a S-Top-sp $(\mathcal{V}, T, \mathcal{L})$ into a S-Top-sp $(\mathcal{V}^*, T^*, \mathcal{L})$. Suppose $(\mathcal{V}, T, \mathcal{L})$ is a soft $\mathcal{K}(sc)$-sp, to show $(\mathcal{V}^*, T^*, \mathcal{L})$ is soft $\mathcal{K}(sc)$-sp. Let $(S, T_S^*, \mathcal{D})$ be a soft compact subsp of $(\mathcal{V}^*, T^*, \mathcal{D})$. Then, we have $f_{qv}^{-1}(\tilde{S}) = \tilde{W}$ whenever $v^{-1}(S) = W$ and $W \subseteq \mathcal{V}$. From hypothesis and Theorem 2.8, it follows $(W, T_W, \mathcal{L})$ is soft compact subsp of $(\mathcal{V}, T, \mathcal{L})$ which is a soft $\mathcal{K}(sc)$-sp, this implies $\tilde{W}$ is a semiclosed soft in $\mathcal{V}$, by Lemma 4.1 part (1), $f_{qv}(\tilde{W}) = f_{qv}(f_{qv}^{-1}(\tilde{S})\ \ ) = \tilde{S}$ is a semiclosed soft in $\mathcal{V}^*$. Thus, $(\mathcal{V}^*, T^*, \mathcal{D})$ is a soft $\mathcal{K}(sc)$-sp. By the same way, we can prove that if $(\mathcal{V}^*, T^*, \mathcal{D})$ is soft $\mathcal{K}(sc)$-sp, then $(\mathcal{V}, T, \mathcal{L})$ is a soft $\mathcal{K}(sc)$-sp. *□

Theorem 4.3 : *Let $f_{qv} : (\mathcal{V}, T, \mathcal{L}) \to (\mathcal{V}^*, T^*, \mathcal{D})$ be a soft open bijective mapping, and $(\mathcal{V}, T, \mathcal{L})$ is a soft T_2-sp. Then, $(\mathcal{V}^*, T^*, \mathcal{L})$ is a soft $\mathcal{K}(sc)$-sp.*

Proof : Let $(S, T_S^*, \mathcal{D})$ be a soft compact subsp of $(\mathcal{V}^*, T^*, \mathcal{L})$, to prove $\tilde{S}$ is a semiclosed soft set in $\mathcal{V}^*$, we must prove $\tilde{S}$ is a semiclosed soft set in $\mathcal{V}^*$. By the definition of inverse image of soft set we get, $f_{qv}^{-1}(\tilde{S}) = \tilde{W}$ whenever $v^{-1}(S) = W$ and $W \subseteq \mathcal{V}$, and by Proposition 2.11 and Theorem 2.8, it follows $(W, T_W, \mathcal{L})$ is soft compact subsp of $(\mathcal{V}, T, \mathcal{L})$, but $\mathcal{V}$ is a soft T_2-sp, so by Theorem 2.9, $\tilde{W}$ is a closed soft set in $\mathcal{V}$ implies $\tilde{W}^c$ is an open soft set in $\mathcal{V}$. From hypothesis, we get $f_{qv}(\tilde{W}^c) = f_{qv}((f_{qv}^{-1}(\tilde{S}))^c) = f_{qv}(f_{qv}^{-1}(\tilde{S}^c)) = \tilde{S}^c$

is an open soft set in $\mathcal{V}^*$.which implies, $\tilde{S}$ is a semiclosed soft set in $\mathcal{V}^*$. Thus, we prove $(\mathcal{V}^*, T^*, \mathcal{D})$ is a soft $\mathcal{K}(sc)$-sp. □

Definition 4.4 : A mapping $f_{qv} : (\mathcal{V}, T, \mathcal{L}) \to (\mathcal{V}^*, T^*, \mathcal{L})$ is said to be a soft $\mathcal{S}$-semi closed mapping, if for each closed S-subsp $\tilde{W}$ of $\mathcal{V}$, the image $f_{qv}(\tilde{W})$ is semiclosed soft set of $\mathcal{V}^*$.

Theorem 4.5 : *If $f_{qv} : (\mathcal{V}, T, \mathcal{L}) \to (\mathcal{V}^*, T^*, \mathcal{D})$ is a soft continuous mapping, $(\mathcal{V}, T, \mathcal{L})$ is a soft compact sp and $(\mathcal{V}^*, T^*, \mathcal{D})$ is a soft $\mathcal{K}(sc)$-sp, then f_{qv} is a soft $\mathcal{S}$-semi closed mapping.*

Proof : Let $\tilde{W}$ is closed soft set in $\mathcal{V}$ which is a soft compact sp, then by Theorem 2.10, it follows $(W, T_W, \mathcal{L})$ is a soft compact subsp of $(\mathcal{V}, T, \mathcal{L})$, since f_{qv} is a soft continuous mapping and Theorem 2.8, $f_{qv}(\tilde{W})$ is a soft compact subset in $(\mathcal{V}^*, T^*, \mathcal{D})$, but $(\mathcal{V}^*, T^*, \mathcal{D})$ is a soft $\mathcal{K}(sc)$-sp, then $f_{qv}(\tilde{W})$ is a semiclosed soft subset in $\mathcal{V}^*$, that is f_{qv} is a soft $\mathcal{S}$-semi closed mapping. □

Definition 4.6 : A S-Top-sp $(\mathcal{V}, T, \mathcal{L})$ is said to be soft semi $\mathcal{K}_2$-sp, if $sCl(\tilde{W})$ is a soft compact subset in $\mathcal{V}$, whenever $(W, T_W, \mathcal{L})$ is a soft compact subsp of $\mathcal{V}$.

Example 4.7 : Let $(\mathcal{V}, T, \mathcal{L})$ be the soft discrete. Then, $(\mathcal{V}, T, \mathcal{L})$ is a soft semi $\mathcal{K}_2$-sp.

Proposition 4. 8 : Every soft $\mathcal{K}(sc)$-sp is a soft semi $\mathcal{K}_2$-sp.

Proof : Let $(W, T_W, \mathcal{L})$ is a soft compact subsp of $\mathcal{V}$ which is soft $\mathcal{K}(sc)$-sp, then $\tilde{W}$ is a semiclosed soft set in $\tilde{W}$ =, by Theorem 2.2 part (1), $sCl(\tilde{W})$, then $\mathcal{V}$ is a soft semi $\mathcal{K}_2$-sp. □

References

[1] H. A. Abu-Ragheef, Some type of Kc-spaces and Lc-spaces, M.Sc. Thesis, Al mustansiriyah University, (2015).

[2] Haider Jabur Ali and A. H. Al-Sheikhly, New forms of KC- and LC-Spaces, *Al-Mustansiriyah J. Sci* 22.5, pp. 231–327, (2011).

[3] Chiara Baldovino and Camillo Costantini, On some questions about *KC* and related spaces, *Topology and its Applications* 156, pp. 2692–2703, (2009).

[4] Bin Chen, Soft semi-open sets and related properties in soft topological spaces, *Appl. Math. Inf. Sci.* 7.1, pp. 287–294, (2013).

[5] N. Kalaivani, A. I. El-Maghrabi, D. Saravanakumar and G. Sai Sundara Krishnan, Operation-compact spaces, regular spaces and normal spaces with α-γ-open sets in topological spaces, *Journal of Interdisciplinary Mathematics*, 20.2, pp., 427-441, (2017).

[6] Juthika Mahanta and Pramod Kumar Das, On soft topological space via semi open and semi closed soft sets, arXiv:1203.4133vl, (2012).

[7] Pradip Kumar Maji, Ranjit Biswas, Ankhi Ranjan Roy, Soft set theory, *Computers and Mathematics with Applications* 45, pp. 555–562, (2003).

[8] Sonal Mittal and Rashmi Singh, Fuzzy G-semi open sets and fuzzy G-semi continuous functions, *Journal of Information and Optimization Sciences* 38.6, pp. 1039-1046, (2017).

[9] Dmitri Molodtsov, Soft set theory first results, *Computers and Mathematics with Applications* 37, 19–31, (1999).

[10] Nadia Ali Nadhim, Rasha Naser Majeed and Haider Jebur Ali, Soft $\mathcal{K}c$-spaces, *Journal of the India Mathematical Society*, to appear.

[11] Esmaeel Peyghan, Babak Samadi and Abedlhamid Tayebi, Some results related to soft topological spaces, *arXiv : 1401. 6933v1 [math. GM]* 21 Jan., (2014).

[12] Muhammad Shabir and Munazza Naz, On soft topological spaces, *Computers and Mathematics with Applications* 61, pp. 1786–1799, (2011).

[13] Albert Wilansky, Between T_1 and T_2, *A mer, Math Monthly* 74, pp. 261–266, (1967).

[14] İdris Zorlutuna, muhammed Akdag, W. K. Min and Serkan Atmaca, Remarks on soft topological spaces, *Annals of Fuzzy Mathematics and Informatics* 3.2, pp. 171–185, (2012).

[15] İdris Zorlutuna and Hatice Cakir, On continuity of soft mappings, *Appl. Math. Inf. Sci.*9.1, pp. 403–409, (2015).

[16] Morteza Norouzi (2019) Normal subfuzzy (m, n)-hypermodules, *Journal of Discrete Mathematical Sciences and Cryptography,* 22:3, 433-451, DOI: 10.1080/09720529.2019.1614336

[17] Chung-Ho Chen (2019) Optimal manufacturing target setting by considering process adjustment cost and quality loss, *Journal of Information and Optimization Sciences,* 40:1, 23-27, DOI: 10.1080/02522667.2017.1411016

Received July, 2019