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**Ministry of Higher Education**

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**Some Classes of Analytic Function Involving Linear Operators**

A Thesis Submitted to the Council of the College of Education for Pure Sciences,

University Of Anbar in Partial Fulfillment of the Requirements for the Degree of Master in Mathematics

**By**

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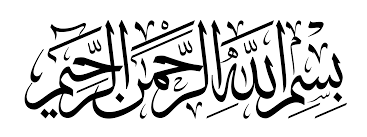
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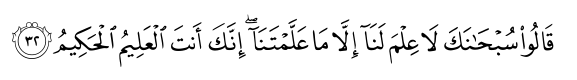
**Supervised By**

**Prof. Dr. Abdul Rahman Salman Juma**

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**سورة البقرة**

**الإهــداء**

**أحمد الله عز وجل على منه و عونه لإتمام هذا البحث.**

**إلى أبي وأمي (حفظهما الله)**

**اللذان أحاطاني بعطفهما ورعايتهما وغمراني بدعواتهما الصادقة**

**إلى زوجتي ..رفيقة حياتي**

**من أعانتني على بلوغ مأربي**

**وقاسمتني سهري ومعاناتي وكانت خير عون لي في دراستي**

**إلى ولدي .. فلذت كبدي**

كما أهدي ثمرة جهدي( لأستاذي الكريم الأستاذ الدكتور عبد الرحمن سلمان جمعه )الذي كلما تظلمت الطريق أمامي لجأت إليه فأنارها لي و كلما دب اليأس في نفسي زرع فيا الأمل لأسير قدما و كلما سألت عن معرفة زودني بها و كلما طلبت كمية من و قته الثمين وفره لي بالرغم من مسؤولياته المتعددة؛

إلى من هم قناديل حياتي وبهم تكتمل فرحتي .. وعزوتي بالدنيا .. والبلسم الشافي ... (إخواني وأخواتي )

أليكم جميعا اهدي ثمرة جهدي المتواضع

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*Osamah 2020*

**Dedication**

**I would like to dedicate this work To my**

**candles that lighten my life, My Father**

**and My Mother My Wife My Son and**

**My Brothers With love and gratitude.**

*Osamah 2020*

**Supervisor Certification**

*I certify this thesis entitled "* ***Some Classes of Analytic Function Involving Linear Operators"*** *submitted by the student "****Osamah Nadhim Kassar****", has prepared under my supervision at the University of Anbar, College of Education for Pure Sciences / Department of Mathematics, as a partial fulfillment of the requirements for the degree of master in Mathematics.*

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**List of Publications**

1. SomeProperties on aClass of Analytic Functions Involving Gener-alized linear operator**, Journal of Al-Qadisiyah for Computer S-cience and Mathematics, Vol. 25, No. 1, 2020, ppMath. 16 –22 .**



1. Certain Properties for Analytic Functions Associated with q-Ruscheweyh differential operator**, Iraqi Journal of Science, Vol. 61, No.9, 2020 (to appear).**



1. Univalent Harmonic Functions Starlike of The Complex Order De-fined By Using Catas Οperator,  **at the IOP publishing, Journal of Physics: Conference Series, June, 2020 (accepted).**
2. Certain Subclasses of Meromorphic Functions with Positive Coe-fficients associated with Linear Operator, [**Kragujevac Journal of Mathematics**](https://imi.pmf.kg.ac.rs/kjm/en/index.php)**.( accepted ).**



1. Subclass of Harmonic Univalent Functions Defined By Sriv-astava -Attiya Operator, **Annals of Oradea University - Math-e-matics Fascicola. (submitted).**

|  |  |
| --- | --- |
| **Symbol**  ***List of Symbols*** | **Meaning** |
|  | Cmplex plane. |
|  | \{0}. |
|  | Set f all real numbers. |
|  | The Integer numbers. |
|  | Set f natural numbers. |
|  | . |
|  | Open unit disk . |
|  | Punctured pen unit disk . |
|  | Bundary f unit disk }. |
|  | . |
|  | The - neighborhod of a function |
|  | Class f nrmalized analytic univalent functins in the pen unit disk . |
|  | The class of all harmonic functions |
|  | Gaussian hypergemetric functins. |
|  | Subclass of consisting of univalent functions in |
| **Symbol**  ***List of Symbols*** | **Meaning** |
|  | Class f normalized starlike functions in . |
|  | Class f normalized starlike functions f rder . |
|  | Class of normalized convex functions in . |
|  | Class of normalized convex functions of order . |
|  | Class of close-to-convex univalent functions in . |
| 𝛴 | Class of meromorphic univalent functions in . |
|  | Subclass of 𝛴 |
|  | Real partof a complex number. |
|  | The pochhammer symbol |
|  | Hadamard product (or convolutin) of funct-ions and . |
|  | q-hypergeometric function. |

**Abstract**

Abstract

The purpose of this thesis is to explore some concepts onto a survey on some analytical and geometrical properties of classes included univalent and multivalent functions in the open unit disk , and meromorphic univalent functions in the punctured open unit disk . By making use of linear operator in the class and shading light on some geometric properties, such as coefficient inequality distortion and growth theorems closure theorems and integral operators, radii of close to convexity, convexity and starlikeness for functions in the class . The study also, has shed the light on meromorphically univalent functions by using a linear operator in the punctured open unit disk . In addition to that, how to make use of Mittag-Lefflerr function certain subclass of analytic univalent functions to investigate inclusion properties associated with the concept of differential subordination . Moreover, it discusses the applications of -Ruscheweyh differen-tial operator on some certain subclass of analytic univalent functions have been obtained a necessary condition for the function to be in the , like coefficient estimates, radii of starlikeness, distortion theorem, close-to-convexity, convexity, extreme points, neighborhoods, and the integral mean inequalities of functions affiliation to these classes. Also, we have installed some interesting results by studying a certain subclass of univalent harmonic functions of the fromdefined by the Catas operator .



Finally, by using the Srivastava-Attiya operator, we give some of the results which have tackled the class of that consisting the family of harmonic functions on the open unit disk.

Introduction

A complex analysis is an important concept of mathematics which studies a function and its variables that complex numbers has an important contribution and multiple applications in other domains of science and technique. Geometric Function Theory is an important concept of complex analysis, which deals and studies the geometric properties of the analytic functions. Its origin started from the 19th century but it continued and still applicable untill now. Through attracting a large number of mathematicians and research works because it only shear beauty of its geometrical aspects and abundant avenue for research works . Many distinguished researchers in mathematicians like St.Ruscheweyh, H. M. Srivastava, H. Silverman, S. S. Miller, P. T. Mocanu,S. Owa, P. L. Duren, J. M. Jahangiri et. al., have opened new avenues in the field of complex analysis, particularly in geometric function theory.

The corner stone of the geometric function theory is the theory of univalent and multivalent function which is considered as one of the active fields of the current research. It is an old branch of mathematics. Koebe started the theory of the univalent function in 1907. One of the major problems in this field had been the Bieberbach conjecture dating from the year 1916, which asserts that the modulus of the nth Taylor coefficient of each normalized analytic univalent function is bounded by n: The conjecture was not completely solved until 1984 by French- American mathematician Louis de Branges .This theory is one of the most interesting subjects in geometric function theory that made most of researchers interested in its study. An analytic univalent and multivalent function are a holomorphic or meromorphic function. This thesis is entirely devoted to the study of meromorphic function. The theory of univalent and multivalent functions is one of the important areas of study in this field that links geometry and analysis.

This thesis is divided in to four chapters .These chapters are arranged as follows;

***Chapter One,*** gives preliminaries of geometric function theorem which are used in the following.

***Chapter Two,*** is dedicated to study some properties of certain subclasses of univalent functions, meromorphic univalent functions that defined by some operators. This chapter consists of two sections . Section one, introduces a subclass of analytic functions defined by generalized linear operator. This operator studied and introduced by Srivastava and Gaboury [76]. We obtain some geometric characterization like Coefficient estimates, distortion and growth theorems, closure theorems and integral operators, radii of closetoconvexity, convexity and starlikeness for functions in this class. Section Two, studies certain subclass of meromorphic univalent functions by using a linear operator associated with by means of a Hadamard product involving some suitably normalized meromorphically q-hypergeometric functions in the punctured open unit disk some properties has been studied like, coefficients inequalities, growth and distortion theorems, closure theorems, extreme points and radii of meromorphic starlikeness and meromorphic convexity are obtained.



***Chapter Three***, is fully devoted to the study of differential subordination prop-erties of classes of univalent functions, it is divided into two sections, sections one, intro-duces and generally studies the differential subordination operator associated with Mittag-Leffler function of certain subclass of analytic functions to investigate the inclusion of the properties of these classes. In Section two, deals with the study of certain properties for functionsdefined by q-Ruscheweyh differential operator

and the notion of the Janowski function, we obtain some subordination results for univalent functions in the open unit disk .

***Chapter Four***, deals with to study the certain subclasses of harmonic univalent functions which consists two sections, in section one, we have known the class by working applicatin of the Catas operator. Section two, introduces and studies certain subclasses , of harmonic univalent functions defined by the Srivastava-Attiya operator, we obtian essential properties, Coefficient bounds, extreme points, distortion inequality, convex combination and integral operator.

**Chapter One**

**Basic Definitions and Some Preliminaries**

**CHAPTER ONE**

**Basic Definitions and Some Preliminaries**

**1.1 Introduction**

The main subject in this chapter is the basic facts about geometric function theory, it is obtained from mixing geometric and analysis. This chapter consists of two sections. The first section reviews the basic definitions that can be found in the standard text books by Duren, Goodman and Pommerenke. Also, we introduce some classes of analytic functions, like the class of starlike functions, -starlike functions, convex functions, -convex functions, close to convex functions, -valent starlike, -valent convex, meromorphic starlike and meromorphic convex functions, which are seen to be necessary for several places and situations in the later chapters. In the second section, several fundamental lemmas and theorems have been mentioned which are essential and needed, to prove our principal results.

**1.2 Basic Definitions**

**Definition 1.2.1 [29]:**

A function of the complex variable is analytic at a point if its derivative exists not only at but each point z in some neighborhoods of . It is analytic in region if it is analytic at every point in .

**Definition 1.2.2 [29]:**

An analytic function in a domain is said to be univalent there if it does not take the same value twice, that is for all pairs of distinct points and in . In other words is one to one mapping of onto another domain *.* The theory of univalent functions is so deep. So, we need certain simplifying assumption. The most obvious one in our study is to replace the arbitrary domain by one that is convenient, and is the open unit disk

The class of all univalent functions is denoted by

**Definition 1.2.3 [29]:**

A function is said to be locally univalent at a point if it is univalent in some neighborhood of .

For analytic functions , the condition is equivalent to local univalence at . A function univalent in a domain is locally univalent at each of the points in ; but the converse is not true in general.

**Example 1.2.4 [35]:**

Consider the function in the domain Since for , it follows that is locally univalent in But so this function is not univalent in the whole domain However, is univalent on (where, denote the real part of )

**Definition 1.2.5 [56]:**

A set denote the class of all analytic functions in and be the subclass f consisting of functions of the form

with and .

**Definition 1.2.6 [56]:**

A set denote the subclass of in unit disk and it’s functions of the following form

**Definition 1.2.7 [29]:**

A function is said to be normalized if satisfies the conditions

**Example 1.2.8 [29]:**

The leading example of a function of class is the Koebe function defined by

which is an extremal function for many subclasses of the class of univalent funct-ions. It maps the disk , one to one and conformally onto the entire complex plane minus the part of the negative real axis. This is best seen by writing

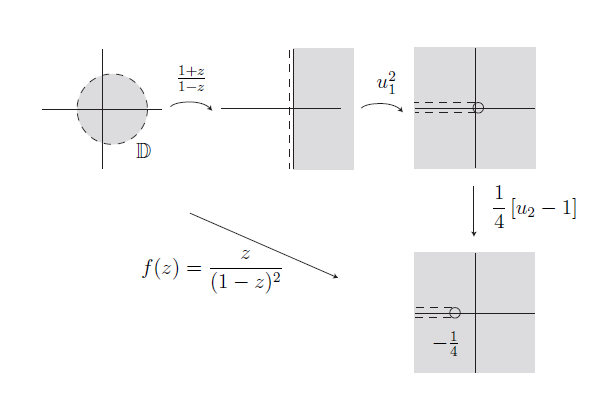


Figure 1.2.1. The image of the unit disk under the Koebe function.

**Definition 1.2.9 [29]:**

Let denote the class of function of the following form

which are meromorphic univalent function in the punctured unit disk

**Definition 1.2.10 [35]:**

Let be an analytic function in the open unit disk . If the equation has never more than -solutions in then is said to be -valent (or multivalent of order in The class of all analytic -valent functions is denoted by where is expressed of the forms

or

If is -valent with then is univalent.

**Definition 1.2.11 [29]:**

A set in the complex plane is called starlike with respect to interior point of if the line segment joining any other interior point of to lies in the inte-rior of . In a more picturesque language, the requirement is that every point of is visible from , i.e.

Furthermore, a function which maps the open unit disk onto a starlike domain is called a starlike function, the set of all starlike functions is denoted by which is analytically expressed as

It is well known that if any analytic function satisfies above equation and , then is univalent and starlike in .

**Example** **1.2.12 [29]:**

The function defined on the unit disk is a starlike functions. In Figure (1.2.2) we can see the image of this function, which is a starlike domain.

****

Figure 1.2.2

**Definition 1.2.13 [34]:**

A set in the complex plane is called convex set if it is starlike with respect to each of its points,that is, the line segment joining any two interior points of lies in the interior of , i.e.

Let and let be the functions in the open unit disk Then maps onto a convex domain, if and only if

Such function is said to be convex in The set of all convex function is denoted by . Thus .

**Example 1.2.14 [34]:**

The function is a convex function. Below is given the image of this function,



Figure 1.2.3

**Definition 1.2.15 [60]:**

The class of starlike and convex functions of order respectively which are defined by

In particular

It is clear that from the above, is convex if and only if is starlike.

**Definition 1.2.16 [29]:**

A function is said to be close to cnvex if there is a convex function such that

An equivalent formulation would involve existence of a starlike function such that

we denote by to the class of all close-to-convex functions in

**Definition 1.2.17 [29]:**

A function is said to be close-to-convex of order if there is a convex function such that

An equivalent formulation would involve existence of a starlike function such that

we denote by to the class of all close-to-convex functions of order . For we have the class of all close-to-convex function in . We note that . Every convex function is obviously close-to-convex. More generally, every starlike function is close-to-convex. Indeed, each has the form for some , and

Then from above, we conclude that

and this means that, every close-to-convex function is univalent.

**Definition 1.2.18 [35]:**

Let we denote by , and the subclasses of that are meromorphic univalent, meromorphically convex functionsorder of and meromorphically starlike functions of order,respectively. Analytically,



afunction if and only if



similarly, a function if and only if



**Definition 1.2.19 [29]:**

Radius of starlikeness of a function is the largest for which it is starlike in

**Definition 1.2.20 [29]:**

Radius of convexity of a function is the largest for which it is convex in

**Definition 1.2.21 [29]:**

If the functions belonging to the class , given by

and

then the Hadamard product (or the convolution) of and denoted by is defined by

For functions belonging to the class , where

and

The convolution or the Hadamard product of and is given by

**Definition 1.2.22 [56]:**

Let be a topological vector space over the field and let be a subset of . A point is called an extreme point of if it has no representation of the form as proper convex combination of two distinct points and in

**Definition 1.2.23 [35]:**

If the functions are analytic functions in . The function is said to be subordinate to or is said to be superordinate to if there exists a function where

the class of Schwarz functions, and

In such a case, we write

If the function is univalent in , then there will be the equivalence

**Definition 1.2.24 [76]:**

For the the generalized Srivastava - Attiya operatr is defined by

Such thatis defined by



where

Linking ( 1.20) and (1.21), we obtain

when andwhen.

**Definition 1.2.25 [21]:**

For function and let be real numbers, and and ( when

the generalization of the linear operator defined as follows:

**Definition 1.2.26 [21]:**

The q-hypergeometric function is defined by



where and ( when



**Definition 1.2.27 [82]:**

Let and The generalization Mittag-Leffler function defined as follows:

**Definition 1.2.28 [31]:**

For the differential operator is defined as follows:

where We can simply verify from that

where

And and

**Definition 1.2.29 [64]:**

For the differential q-Ruscheweyh operator of order and for given by ( 3.14) is definedas



where

and, also

**Definition 1.2.30 [20]:**

The Catas operator is the derivative operator defined as

where

**Definition 1.2.31 [72]:**

The Srivastava-Attiya operator is denoted by , and defined as

where and

**1.3 Some Basic Results**

The following lemmas and theorems will be used to prove our results in the next chapters .

**Lemma 1.3.1[55]:** Let and are real numbers. Then

if and only if . where and be any complex number.

**Lemma 1.3.2[55]** : Let Then

if and only if (1.31)

**Lemma 1.3.3[29]: (Schwarz Lemma)**

Let be analytic in the unit disk with. And in .Then

in Strict inequality holds in both estimates unless is a rotation of the disk

**Lemma1.3.4[44]:**The coefficients of a function stisfy the sharp inequality . (1.32)

**Lemma 1.3.5[24]**

Let be convex, univalent in with and

if is analytic in with then

implies

**Lemma 1.3.6** **[77]** Let be convex, univalent in U and be analytic in with If is analytic in with then

implies

**Theorem 1.3.7 [29]: (Alexander's Theorem)**

Let be an aalytc function in with . Then if and only if.

**Theorem1.3.8 [29] :( Distortion Theorem)**

For each

For each equality occurs if and only if is a suitable rotation of the Koebe function. We call the upper and lower bounds for as distortion bounds.

**Theorem1.3.9 [29] :( Growth Theorem)**

For each

For each equality occurs if and only if is a suitable rotation of the Koebe function.

**Theorem 1.3.10 [24] :( Bieberbach Conjecture)**

The coefficients of each satisfy for *n=2,3*,….. The strict inequality holds for all n unless is the Koebe function or one of its rotation.

The next theorem is about coefficient of functions in and due to Littlewood [20].

**Theorem 1.3.11[29] :( Littlewood's Theorem)**

For the constant , the coefficient of each function satisfy for n=2, 3,….. .

**Theorem 1.3.12 [29]**

Assume that is analytic and not constant in a domain of the complex -plan. For any point for which ,this mapping is conformal, that is, it preserves the angle between two differentiable arcs.

**Theorem 1.3.13 [29]: (Maximum Modulus Theorem)**

Suppose that a function is continuous on a boundary of ( any disk or region). Then the maximum value of which always reached, occurs somewhere on the boundary of and never in the interior.

**Theorem 1.3.14 [52]** **:** If and are analytic in with , then fr and

**Chapter Two**

**Subclass of Analytic and Meromorphic Univalent Functions**

**CHAPTER TWO**

**Subclass of Analytic and Meromorphic Univalent Functions**

**2.1 Introduction**

The theory of univalent functions is an age-old branch of mathematics, in particular complex analysis. Its origin can be traced to the first decade of the 19th century due to Koebe [30], Gronwall's proof of the Area Theorem [34] and to Bieberbach's estimate for the second coefficient of normalized univalent functions [18] which is readily yielding the growth and distortion bounds for univalent functions.The current work is a piece of the geometric function theory .This chapter is mainly concerned with the study of analytic univalent functions in the open unit disk, and univalent meromorphic function in the punctured open unit disk. This chapter consists of two sections. Section one is dedicated to the study some properties on a class of analytic functions involving generalized linear operator , also we give some geometric characterization like coefficient estimates, distortion and growth theorems, closure theorems and integral operators, radii of close to convexity, convexity and starlikeness for functions in the class .

Section two, deals with the study of certain subclassof meromorphic univalent functions through using the linear operator associated by means of Hadamard product which associated some suitably normalized meromorphically q-hypergeometric functions, we have studied some geometric properties like distortion theorem, coefficient bounds, and radii of starlikeness, convexity for these classes of functions. Extreme points and integral operator are also investigated.

**2.2 Some Properties on a Class of Analytic Function Involving**

**Generalized Linear Operator**

Let denotes the class of analytic functions that from   
which are analytic and normalized in the open unit disk . Next we will provide generalized linear operator drawn up and introduced by Srivastava and Gaboury [76] as follows



,

when characterized by

Such thatis defined by



where

linking ( 2.2) and (2.3), we obtain

min when;when

for more details see[79].

**Defintion 2.2.1:** A function be given by (2.1) is said to be in the class if the following condition holds:



Or, equivalently:

where

Some special cases of the above class can be found in [7] and [26].

Let denote the subclass of consisting of function of the form



Now we define theclass by:



In our present study, we obtain some interesting geometric properties in the class

We use techniques like those utilized before by Darus and Faisal [26],Al-Hawary et.al. [3], and Amourah et.al.[8,14,9].

**Theorem 2.2.1.** A function given by (2.1) is in the class

if and only if

minwhenandwhen

**Proof:** Let. Then for we have



This implies

Contrariwise, let inequality (2.10) is satisfied.Then



This Completes the proof of Theorem 2.2.1. 

**Corollary 2.2.1** If inis given by (2.1), then



Now, we give distortion and growth bounds for the functions belonging to

the class which is found in the following theorem.

**Theorem 2.2.2.** Let which is defined by (1.8). Then for we have



and

**Proof:** Since, from Theorem 2.2.1 we can write



Thus, for and making use of (2.11), we have



and

As well from Theorem 2.2.1, it follows that

Hence

And

Then, the proof of Theorem 2.2.2.is complete .



Closure theorems for the class are given by the following .

**Theorem 2.2.3**. Let the functions be defined by

be in the class,



minwhenandwhen.Thenthe



function E(z) defined by

is a member of the class in ,where

**Proof:** Since it follows from Theorem 2.2.1 that

for every Hence

which implies that E(z).



**Theorem 2.2.4** The class is closed under convex linear combi-nation,

minwhenandwhen.

**Proof:** Suppose that the funcions defined by (2.12) are in



the class, it is suffices to prove that the function

is also in the class .



Since, for

we observe that

Hence K(z).

This completes the proof of Theorem 2.2.4 .



In this part, we review integral transforms of functions.in the class .

**Theorem 2.2.5** If the function defined by ( 2.1) is in the class ,

then the function defined by   
also belongs to the class .



**Proof:** From it follows that where

Therefore,

since . Hence by Theorem 2.2.1, F(z).



**Defintion 2.2.2.**[1] A function is said to be close –to –convex of order if it satisfies

**Theorem 2.2.6.**The functionbelong to the class is close-to-convex of order  in. where

**Proof:** It is sufficient to show that

and

Observe that ( 2.17) is true if

Solving ( 2.18) for we get

is close-to-convex of order . 

**Theorem 2.2.7.** Ifbelong to the class ,then is starlike of order  inwhere

**Proof:** We must show that for

since

Observe that (2.19) is true if

Hence

The result is desired . 

**Theorem 2.2.8.** Ifbelongs to the class ,then is convex of order in. where

**Proof**. We need only show that

Since

observe that (2.19) is true if

The result is desired . 

**2.3** **Certain Subclasses of Meromorphic Functions with Positive**

**Coefficients associated with Linear Operator**

Letdenote the class of meromorphic functions in the punctured open unit disk of the form

There are many other classes of meromorphically univalent functions has been extensively studied by (for instance) Altintas[6], Aouf[10],AbdulRahman S.Juma[45] and others (see[27,13, 67]).

Now, we define the Quasi-Hadamard product of the functions for

is deﬁned by

Next, by using the Quasi-Hadamard for we will present a generalization to the linear operator as follows:

,

which are defined by

such that

where

and is the q-hypergeometric function is defined by

where and ( and

also

Cho et al. [23] and Ghanim and Darus [33] studied the above function. Now, by combining (2.21), (2.23)and(2.25), we get

For convenience, we will henceforth denote

Notice,the above linear operator(2.26) was introduced and studied by K. A. Challab, M. Darus, and F. Ghanim[21].

For convenience, we let

**Definition 2.3.1**: For and we let be the subclass of consisting of functions of the form (2.21) and satisfying the analytic criterion

where is given by (2.28)

Remarking that, for , the class provides atransition, a from meromorphically starlike functions to meromorphically convex functions. Also, by suitably specializing the parameters involved in the operator, the class reduces to various new subclasses .These considerations can fruitfully be worked out and we skip the details in this regard.

Some special cases of the above class are studied by[55].

**Theorem2.3.1** Let in be given by (2.18). Then if and only if

where is given by (2.28).

**Proof.** Let . Then by definition and using Lemma 1.3.1, we get

For easiness, we let

Hence, the equation ( 2.31) is equivalent to

From Lemma 1.3.2, we only need to prove that

Therefore,

and

it is now easy to show that

by the provided condition (2.30).

On the other hand ,let .Then by Lemma 1.3.1, we get (2.31).

Selection the values of on the positive real axis the inequality (2.31) shorter to

since the above inequality shorter to

Letting and by the mean value theorem we get the desired inequality (2.30). 

**Corollary 2.3.1 .** If ,then



**Corollary 2.3.2.** If be given by , then if and only if

where in Theorem 2.3.1. 

**Theorem 2.3.2.** Let given by (2.21) .Then for we get

And

The result is sharp for

**Proof.** Since and then

and

Next,

And

This completes the proof of Theorem 2.3.2 

Letbe the function given by

**Theorem 2.3.3.** Let the function defined by (2.33) be in the class for every . Then the function defined by

Belongs to the class where

**Proof.** Since it follows from Theorem 2.3.1 that

for every Hence,

which implies that 

**Theorem 2.3.4.** Let

and  
Then if and only if it can be represented in the form

**Proof.** From and ,we have

Since

it follows from Theorem 2.3.1 that the function

Conversely,suppose that since

Setting

it follows that   
 This completes the proof of the theorem. 

**Theorem 2.3.5.** Let Then is meromorphically starlike of order , in the unit disk ,where

The result is sharp for the extremal function given by (2..32) .

**Proof**. We must show that

Since

hence (2.42) holds true if

or,

Hence is starlike of order 

**Corollary 2.3.3.** LetThen is meromorphically convex of order in the unit disk , where

The result is sharp for the extremal function given by (2.32). 

**Chapter Three**

**Some Applications of Differential Subordi-nation on Univalent Functions**

**CHAPTER THREE**

**Some Applications of Differential Subordination on Univalent Functions**

**3.1 Introduction**

In this chapter, we will discuss several geometric properties of univalent analytic functions in the open unit disk . This chapter consists of two sections. The first section, deals with the studies of some results for differential subordination for analytic univalent function in open unit disk involving the generalized derivative operator associated Mittag-Lefflerr function .These results are obtained by investigating appropriate classes of adm-issible functions. Section two, introduces and studies some subclasses of analytic univalent functions in the open unit disk by making use of the q-R uscheweyh differential operator , and the notion of the Janowski function. We have obtained results numerous like coefficient estimates, radii of starlikeness ,distortion theorem, close-to-convexity, convexity, extreme points, neighborhoods, and the integral mean inequalities of functions affiliation to these classes.

**3.2 Inclusion Properties of Certain Subclasses of Analytic Func-tions Associated with Generalized Differential Operator Involv-ing Mittag-Leffler Function**

Let denote the class of univalent functions normalized by

which are analytic in the open unit disk

For we introduce the following subclasses of starlike, convex and close-to-convex functions ) and of order, are studied by several authors [46,80,59] and are respectively defined by

The next, we define the familiar Mittag-Leffer function introduced by Mittag-Leffer [59] and [58] and its generalization introduced by Wiman [82]are defined by

and

where and As a result, a lot of useful work have been made by many researchers in attempt to explain Mittag-Leffler function and its generalization, see for example [78],[16] and[70].

Now, we define the function by

Corresponding to (see [31]) defined the differential operator as follows

where We can simply verify from (3.2) that

We note for and we have Al-Oboudi operator [5],also for

we have Slgean operator [69], for we have .

Let be the class of all functions which are univalent and analytic in for

which is convex such that (0) = 1 and

Next, we provide a differential operator on the class and making use of the principle of subordination between analytic functions to investigat and study the class of starlike, convex and close-to-convex functions ) and of order respectively,for the function which are defined by

We also note that

As follows, we give some inclusion properties of the operator using the principle of subordination.

**Theorem 3.2.1.** Let be analytic function of the form (3.1) and let with Then,

**Proof.** Let belongs to the class and let

Applying ( 3.3) in (3.4) we get

we get

Now from (3.5), we obtain

Otherwise

From (3. 6) and( 3.7) we have

Applying Lemma 1.3.5 ,to (3.8) shows that

i.e.

Thus,



**Theorem 3.2.2.** Let be analytic function of the form (3.1) and let with

**Proof.** From definition 1.2.15, we get

Now, by Theorem 3.2.1we obtain

Thus, 

The function is analytic and satisfies Thus, we obtain the following corollaries

**Corollary 3.2.1.** Let and in Theorem3.2.1. Then

**Corollary 3.2.2.** Let and in Theorem 3.2.1 . Then

**Theorem 3.2.3.** Let and let with Then

**Proof.** Let in then there exist a function in such that

That is, we get

Let

From (3.3), we have

From (3.9), we get

This implies that

Also, by Theorem 3.2.1, , let

By using (3.3) in (3.11) we get

and further, from ( 3.10) and(3.12) we get

Algebraic manipulation in (3.13) gives

Thus, making

And apply Lemma 1.3.6, we get that which implies that

. 

**3.3 Certain Properties for Analytic Functions Associated with q-Ruscheweyh Differential Operator**

Let represent the class of functions which are analytic functions in the unit disk and of the form

By application the notion of subordination, Janowski provided the class . A given analytic function with is called in the class , if and only if the following condition satisfies

geometrically, the function maps the unit disk onto the domain defined by

This domain symbolizes an open circular disk centered on real axis with diameter end points and with

Now ,we define the Ruscheweyh derivative operator as follows

Hence

For . (see [43],[15])

The use of the q-calculus was introduced by Jackson[37] (see also [12]) which plays a vital role in the theory of hypergeometric series, quantum physics and operator theory regard the applications of q-calculus in geometric structure theory, see the papers by Mohamad and Darus[61], Mohamad and Sokol[53].

Next, we provide some fundamental definitions and results of-calculus which we will apply in our results. For more information, see [68,65,48,39].The application of q-calculus was initiated by Jackson [39] (also see [38]). For the applications of q-calculus in geometric function theory.

Now if is fixed, then Jackson explained q-derivative and the q-integral of as the next step

and

if that series converges.

It can simply be show that for and

where

For every non-negative integer n the q-number shift factorial is defined by

In addition, the q-generalized Pochhamer symbol for is defined as

Let be the function given as

For absolutely approximate in this series. The differential q-Ruscheweyh operator of order and for given by ( 3.14) is defined as

where Formoredetails see [64]

and, also

Equation (3.19) can be expressed as

Since  
it follows that

**Definition 3.3.1.** Let indicate the subclass of consisting of functions of the form (3.14)  and satisfy the following subordination condition,

where

We note the following:

**(i)** For the class reduces to the class discussed by Agrawal and Sahoo[1].

**(ii)** For and the class reduces to the class discussed by Ismail et.al. [ 36].

**(iiii)** For and the class reduces to the class discussed by Libera [51].

**(iv)** For the class reduces to class discussed by Janowski [41].

**(v)** For and the class reduces to the class discussed by Padmanabhan and Ganesan [66] .

**(vi)** For and the class reduces to class discussed by Eker and Owa [49].

**(vii)** For and the class

reduces to the class discussed by Shams et. al.[71].

**Definition 3.3.2.** Let represents the subclass of functions of of the form

Further, we define the class

for more details, see[63].

In this part, we will prove our main results.

**Theorem 3.3.1.** A function of the form (3.14) belongs to the class if

where .

**Proof.** It is sufficient to prove that

where

We obtain

This final statement is bounded above by one if

Hence, the proof is complete. 

**Theorem 3.3.2.** Consider .Then if and only if

**Proof.** Since for functions

we can put

where

Then

Since , we get

Now taking z to be real and letting , we have

Or equivalently



**Corollary 3.3.1.** A function be in the class .Then

The result is sharp for the function

That is, the function defined in (3.23) can be achieve the equality.

**Theorem 3.3.3.** Consider the function defined by (3.20) in the class .Then

and

The result is sharp.

**Proof.** From Theorem 3.3.2, let the function

Then, it is obvious that it is an increasing the function of, then

That is

Thus, we get

Likewise, we get

Lastly, can be get the equality for the function

At and This ended the result. 

**Theorem3.3.4.** Let the function be defined by ( 3.20) in the class Then

and

The result is sharp.

**Proof.** Since is anincreasing function for from Theorem 3.3.2, we get

that is

Thus, we obtain

Likewise, we obtain

Lastly , we can view that the affirmation of theorem are sharp for function

 definedby (3.24).



**Theorem 3.3.5.** A function of the from (3.20) belongs theclass Then

**(i)** is starlike of order in ,where

**(ii)** is convex of order in in where

**(iii)** is close to convex of order in where

The function provide by (3.23), all of these results are sharp

**Proof.** We need to show

where

where is specified by (3.25) . Indeed, we get from (3.20) that

Hence, we obtain

if and only if

By Theorem3.3.2, is true the relation (3.28) if

That is, if

for

Implies

This completes the proof (3.25).

For proving (3.26) we need only show that

since

Then,we have

If and only if

By Theorem3.3.2, is true the relation (3.29) if

That is, if

for

Implies

This completes the proof (3.26).

For proving (3.27), It is sufficient to show that

where

and

Observe that ( 3.30) is true if

Solving ( 3.31) for we get

for

Implies

This completes the proof (3.27). 

**Theorem 3.3.6.** Consider

and

Then in if and only if it can be written in the following form

where

**Proof.** assume that

Then, by Theorem 3.3.2, we get

Thus, in view of Theorem 3.3.2, we obtain

Contrariwise, let us assume that, in then

By setting

for

we obtain



**Corollary 3.3.2.** The extreme points of the class are given by

and

Integral mean inequalities.

**Theorem 3.3.7.** Suppose that

and is defined by

then for we get

**Proof.** For

the relation (3.29)is equivalent to proving that

From using theorem 1.3.14, it suffices to show that

By setting

and using (3.21) we get

This completes the proof of the Theorem 3.3.7. 

Now**,** we defined the -neighborhood of a function in by

In particular, for

On the other hand, a function defined by is said to be in the class if there exists a function such that

**Theorem 3.3.8.**If

and

then

**Proof.** Let .Then from Theorem 3.3.2 and the condition

We get,

which implies

By using Theorem 3.3.2 with (3.32), we obtain

by (3.30) we get .

This completes the proof of the Theorem3.3.8 . 

**Theorem 3.3.9.**If and

Then

**Proof.** Let in . we find by (3.30) that

which mean the coefficient of inequality

Following, since ,then from (3.32)we obtain

using (3.34) and (3.35), we get

provided that is given by (3.33),

hence by condition (3.31) in . 

**Chapter Four**

**Some Results of Sub-classes on Univalent Harmonic Functions**

**CHAPTER FOUR**

**Some Results of Subclasses on Univalent**

**Harmonic Functions**

**4.1 Introduction**

Chapter four is fully devoted to the study of some results of certain classes of harmonic univalen functions of the form   
where h and 𝑔 are analytic functions in the unit disk . This chapter consists of two sections. The first section, deals and studies the working application of the Catas operator on the class , and provides sufficient and necessary condit-ions for the functions to be in this class. In Section two, a lot of interesting properties leading to coefficient bounds, extreme points,distortion inequality and convex combination for the functions in this class are obtained and also for a class preserving integral operator by making use of the Srivastava-Attiya operator on subclass of harmonic univalent functions of the form in the unit disk .

**4.2 Univalent Harmonic Functions Starlike of The Complex Order Defined By Using Catas Operator**

Consider that symbolizes the class of the functions that are harmonic univalent and sense-preserving in the open unit disk with. Hence, we can express analytic and co-analytic parts of the function as

We can show that decrease to , the class of normalized univalent analytic functions whenever the co-analytic part .

Let represent the family of functions that are harmonic belong to with the normalization

For and and , let symbolize the class of all functions in for which

Some special cases f the above class can be found in[42].

Here, the Catas operator is the derivative operator provided and studied through Catas et. al. [20]. As following   
where

Note that, the following are the special cases of operator

1- when,include the Slgean operator(see[69]).

2- when include the Uralegaddi-Somanatha operator (see [81]).

3- when include the Cho-Kim operator (see [22]).

4-when,include the Al-oboudi operator (see [5]).

Now if we can define the Catas operator as follows

(4.4)

where

Further, let symbolize the subclass at comprised of func-tions that satisfy the condition

Also let the subclass of comprised of functions that satisfy the following

(4.6)

The starlike harmonic functions discussed by Avci and Zlotkiewicz [17], Jahangiri [40], Silverman [73], and Silverman and Silvia [74].

The coefficient condition with is enough for to be harmonic starlike fixed by Avci and Zlotkiewicz [17], while Silverman[73] fixed that this coefficient condition is also important and necessary if and if and in (4.1) are negative. Jahangiri [40] proved that where is specific by (4.1) and if   
hence is harmonic and univalent, also starlike of order in. This proviso is proved to be also important if and are of the form (4.2). the instance when is specified in [25], and for (see [73]).

**Theorem 4.2.1.** .

**Proof.** Assume that .We need to show that the condition (4.3) satisfies, hence

where

Now, by taking the fact that if and only if, hence we get and by (4.2)

For sharpness let the function

where are non-negative and and for all functions of the form (4.8 ) are in, since



**Theorem 4.2.2.** .

**Proof.** consider, hence by (4.3) we obtain

By application (4.2), we get

therefore, we get

Taking on the real axis, we get

thus,

hence we obtain

Then by (4.6) we get. 

**Theorem 4.2.3.** ,

where

**Proof.** If , then by (4.5) and (4.6) are the equivalent and hither By making use the previous two theorems, we get the result and this complete the proof. 

**Theorem** **4.2.4.**  , if and or

**Proof.** Assume that the function

since

,

when and

Also, if and let be negative real number such that

If we tookthen and by we obtain

hence

By the same way, consider then if we get

since

Now, if be a negative real number such that

choose then and by definition of we find



**Theorem 4.2.5.**  whenever and

**Proof.** Assume that the function and

then since

We have, also

because

then . 

**4.3 Subclass of Harmonic Univalent Functions Defined by Srivastava-Attiya Operator**

Consider the complex valued be a harmonic function in the complex domain that is and both are the real harmonic enter . Consider   
where h and are analytic in and is denoted any simply connected domain.

Let symbolized being a class of the functionsthat is univalent harmonic and sense-preserving enter the open unit disk such that and define as follows

In [25] Clunie and Shell-Small tested the class in addition to its geometric subclasses and gotted some coefficient bounds. Since then, there have been many connected papers on and the subclasses.

Consider represent the subclasses of consisting of functions such that and provided as next  
Where .Here, the Srivastava-Attiya operator provided and studied through Srivastava-Attiya [75] . Defined as nexit

where

For more details see[54].

See that, the following are the special Srivastava-Attiya of operator

1. for we obtain the well-known operator studied by Alexander [2].
2. for we have the operator defined by Libera [70].
3. for such that this operator defined by Bernardi [19].
4. for such that this operator studied by Jung–Kim–Srivastava [47].
5. for and () was studied by Ram N. Mohapatra and Trailokya Panigrahi [57]
6. for and was studied by Slgean [69] (also see [32]).

For ease we assume that

Now, where given by (4.10),and ,

we can define the Srivastava-Attiya operator as follows

. (see[72]) (4.13)

Such that

For and for all let denote the family of harmonic functions where and given by (4.10) and satisfying the analytic criterion

Let be the subclass of consisting of functions such that and given by (4.11).Some special cases of the above class studied by ,[62],[28] and [4] .

Unless in the other ways mentioned, we shall suppose in the notice of this paper that,the parameters

**Theorem 4.3.1.** Consider that where and as in equation (4.10) are defined. Furthermore , let

Then is sense-preserving, harmonic and univalent in and

**Proof.** If then

which proves univalence. See that the function in is sense-preserving. Because of

Now we make prove that in .We need only to see that if (4.15) satisfy then the condition ( 4.14 ) is holds.

taking the fact that if and only if it's enough to prove that

thus,

Which is bounded above by using (4.15). This completes the proof of the Theorem 4.3.1. 

**Theorem 4.3.2.** Let the function be form (4.9) is in the class if and only if

**Proof.** Since we need only to show the "only if" section of this theorem. To this end, for the functions of the form (4.11), we observe that the condition

equivalent the following

Taking  along of real axis, we have the inequality (4.15).This completes the proof the Theorem 4.3.2. 

**Theorem 4.3.3.** Let be the function defined by ( 4.9) in class Then for we get

where The results are sharp with equality for functions defined by

and

**Proof.** We only prove the right-hand inequality. The left-hand inequality is similar in proof and will be deleted. Consider Taking absolute value of we obtain

Likewise, we can show

This completes the proof of the Theorem 4.3.3.



**Theorem 4.3.4.** Let be defined by (4.9) .Then in if and only if

for

and

In special, the class of extreme points are and respectively.

**Proof.** Assume that

Hence

And so in .

Contrariwise, if in , then

and

Setting

since and

hence, we can show that  in the form (4.19) can be expressed. This completes the proof of the Theorem 4.3.4. 

For our following theorem, we make to define the convolution of two harmonic the functions. For harmonic functions of the form

and

now the convolution of and is defined as the following

Applying this definition, the next theorem shows that the class is closed under convolution .

**Theorem 4.3.5.** For , let for is defined by (4.22 ) and where is given by (4.23). Then

**Proof.** We want to prove that the coefficients of holds the required condition defined in Theorem 4.31. For we show that and

Now, for the convolution function we get

since and .

Next, we prove that the class is closed under convex combinations of

its members.

**Theorem 4.3.6.** The class is closed under convex combination.

**Proof.** For let for

By Theorem 4.3.1,we obtain

Where the convex combination of may be written as

Then by (4.25), we obtain

This is the condition required by ( 4.15 ) and so . 

**Theorem 4.3.7.** Consider the function of the form ( 4.9) belongs to the class and be a real number such that .Then is the function given by

also in the class

**Proof.** Consider the function of the from (4.9) Then from by representation

(4.27) of ,it follows that

for

and

Therefore, we obtain

since .

Then, by Theorem 4.3.1, .

This completes the proof of Theorem 4.3.7. 

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**المستخلص**

**الغرض من هذه الرسالة هو عرض بعض المفاهيم الجديدة في دراسة استقصائية عن بعض الخصائص التحليلية والهندسية التي تضمنت الدوال أحادية التكافؤ ومتعددة التكافؤ في قرص الوحدة المفتوحة ، والدوال الميرومورفية أحادية التكافؤ في قرص الوحدة المثقوب .**

بالاستفادة من استخدام المؤثر الخطي في الصنف , تم ألقاء الضوء على بعض الخواص الهندسية مثل مبرهنات التشويه وعدم المساواة بالمعامل ونظريات النمو ونظريات الإغلاق والعوامل المتكاملة ،انصاف الاقطار المحدبة ,المحدبة ,النجمية للدوال في الصنف . ألقت هذه الدراسة أيضًا الضوء على الدوال الميرومورفية أحادية التكافؤ بشكل متماثل بواسطة استخدام مؤثر خطي في قرص الوحدة المفتوحة المثقبة. بالإضافة إلى ذلك ، كيفية الاستفادة من دالة ميتاك لفلر على صنف فرعي معين من الوظائف التحليلية المتكافئة للتحقيق في خصائص التضمين المرتبطة بمفهوم التبعية التفاضلية. علاوة على ذلك، تناقش تطبيقات المؤثر التفاضل Ruscheweyh - على بعض الفئات الفرعية من الدوال التحليلية الاحادية

تم الحصول على شرط ضروري وكاف للدالة التحليلية لتكون في مثل تقديرات المعامل، نصف الاقطار النجمية ، نظرية التشويه ، الاقتراب من التقارب ، التحدب، النقطة القصوى، والجوارات، متباينات قيم التكامل للدوال المنتمية إلى هذه الاصناف . أيضًا، تم أثبات بعض النتائج المثيرة للاهتمام من خلال دراسة صنف فرعي معين من الدوال الاحادية المتجانسة من الشكل معرفة بواسطة المؤثر كاتاس . وأخيرا، باستخدام مؤثر سريفاستافا - عطية، فإننا نعطي بعض النتائج التي تم تناولها في فئة التي تتألف من عائلة الدوال التوافقية على قرص الوحدة المفتوحة.

 **جمهورية العراق**

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**قسم الرياضيات**

**بعض اصناف الدالة التحليلية باستخدام المؤثرات الخطية**

رسالة مقدمة

إلى مجلس كلية التربية للعلوم الصرفة - جامعة الانبار

وهي جزء من متطلبات نيل شهادة ماجستير في الرياضيات

من قبل

**اسامة ناظم كسار**

بكالوريوس رياضيات- كلية التربية للعلوم الصرفة - جامعة الأنبار- 2011

**بإشراف**

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