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***Ministry of Higher Education***

***And Scientific Research***

***University Of Anbar***

***College of Education for Pure Sciences***

***Department of Mathematics***

**Certain Spaces and Functions by Using *Еc* and *δ-ßc-*Open Sets**

*A Thesis*

*Submitted to the Council of the College of Education for Pure Science*

*University Of Anbar*

*As a Partial Fulfillment of the Requirements for*

*the Degree of Master in Mathematics*

***By***

***Sarah Haqi Abdulwahid***

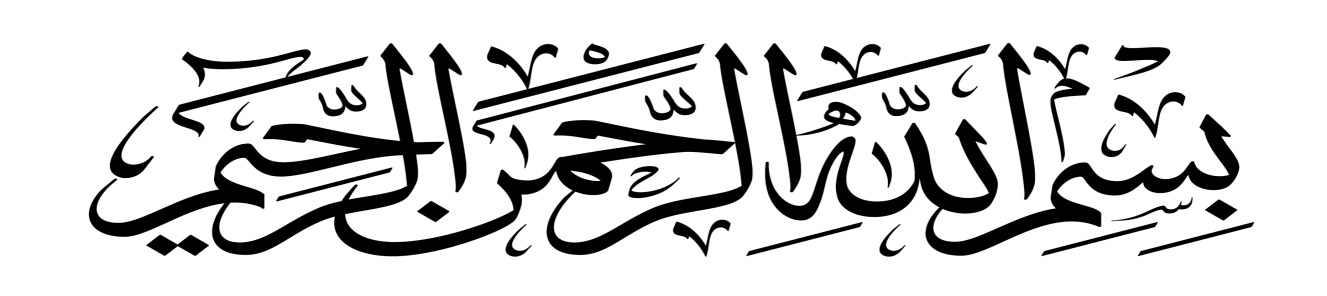
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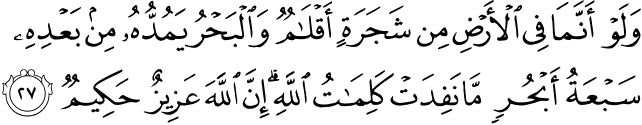
*University Of Anbar 2008*

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### [[27] سورة لقمان - الآية](http://quran.ksu.edu.sa/tafseer/tabary/sura31-aya27.html)

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**Approval of the Council of College**

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Dedication

**To…..my dear father**

**To…..my dear mother**

**To…..my dear brothers**

**To…..my dear sisters**

**To…..my dear husband’s soul…Omer Fadel**

**With my love and respect for providing**

**Opportunities for me and supporting me**

***Sarah***

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***Sarah***

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***List of Symbols***

|  |  |
| --- | --- |
| ***Definition*** | ***Symbol*** |
| ***a* equals *b*** |  |
| ***P* implies *Q*** |  |
| ***P* is implied by *Q*** |  |
| ***P* is equivalent to *Q*** |  |
| **The set of real numbers** |  |
| ***a* is an element of *B*** |  |
| ***a* is not element of *B*** |  |
| ***A* union *B*** |  |
| ***A* intersection *B*** |  |
| ***A* is a subset of *B*** |  |
| **for all *x*** |  |
| ***f* composed with *g*** |  |
| **The open interval from a to *b*** |  |
| ***a* not equals *b*** |  |
| **Union** |  |
| **Intersection** |  |
| **Exist** |  |
| **Not exist** |  |
| **Open interval** |  |
| **Empty set** |  |
| **Delta** | ***δ*** |
| **Beta** | ***ß*** |
| **Alpha** |  |
| **Theta** | ***θ*** |
| **Lambda** |  |

**ABSTRACT**

The concepts of continuity of functions, compactness and separation axioms are a significant and basic topics not only for the theory of classical point set topology but also of other advanced branches of mathematics, of course their weak and strong forms are important too. Semi-open sets, α-open sets, pre-open sets, semi-pre-open sets, b-open sets, δ-pre-open sets, e\*-open sets, e-open sets, *Еc -*open sets and *δ-ßc-*open sets play an important role in generalization of continuity, compactness and separation axioms. Many authors introduced and investigated various types of modifications of generalization of continuity, compactness and separation axioms by using these generalized open sets and great number of papers dealing with such concepts has been appeared. Moreover, generalized open sets play a very important role in general topology and they are the research topics of many topologists worldwide. In our thesis we introduce and investigate new classes of generalized open sets called*Еc*-open and *δ*-*ß*c-open sets and by utilizing these types of generalized open sets we introduce new concepts of generalized continuous functions in a topological space, namely, *Еc*-continuous and *δ*-*ß*c-continuous functions. Moreover, we offer and study new classes of spaces called, *Еc*and*δ*-*ß*c-compact spaces via *Еc* and *δ*-*ß*c- open sets, as well as we present new types of separation axioms in topological spaces namely *Еc* and *δ-ßc-*separation axioms via *Еc* and *δ-ßc*-open sets, and in the context of these generalized open sets we study the *Еc* and *δ-ßc*-Regulare spaces and the *Еc* and *δ-ßc*-Normal spaces.

**Historical Introduction**

**Several Types of** **Generalized** **Open Sets**

It is known that generalized open sets play a very important role in General Topology and indeed a significant theme in general topology and real analysis concerns the various modified forms of continuity, compactness and separation axioms etc. In recent years a number of generalizations of open sets have been considered such as: Semi-open sets, α-open sets, pre-open sets, semi-pre-open sets, *ƅ*-open sets, *ƅ*-θ-open sets, *ß*-θ-open sets, δ-pre-open sets, *δ*-*ß*-open sets, *E*-open sets, *E*-θ-open and *δ-ß-*θ-open sets play an important role in generalization of continuity, compactness and separation axioms in topological spaces. A new classes of generalized open sets in a topological space, called *δ-ß-*open sets or *e\*-*open sets are introduced and some of its properties are obtained by E. Hatir and T. Noiri [**1**] and Erdal Ekici [**2**], as well as, the relationships between *δ*-*ß*-open sets and some other well-known kinds of generalized open sets in topological spaces are also investigated.

Recently, in [**3**] Hariwan Z. Ibrahim presented a new class of ƅ-open sets callеd Ɓc-open, this class of sets lies strictly between the classes of θ-semi open and ƅ-open sets. Moreover, Alias B. Khalaf, Zanyar A. Ameen, in [**4**] introduced a new class of sets, called Sc-open sets, and investigated some properties of *Sc*-continuity and in [**5**] Zanyar A. Ameen, introduced a new class of sets, called *Pc-*open sets, and investigated some properties of *Pc*-continuity. As well as, Ayman Y. Mizyed, in [**6**] studied a new class of generalized open sets called *β***c**-Open sets which contained in the class of *β*-open sets and contains the class of *Ɓ*c-open sets. He introduced *β***c**-continuous functions as a new class of generalized continuous functions and gave some characterizations of these functions. Also, they investigated the relationships among these types of generalized open sets and other well-known types of generalized open sets, see the following figure:

***Regular closed set***

***θ-open set***

***Regular open set***

***δ-open set***

***Semi-θ-open set***

***θ-Semi-open set***

***Sc-open set***

***b-open set***

***α-open set***

***ßc-open set***

***ß-open set***

***Pre-open set***

***bc-open set***

***Semi-open set***

***Pc-open set***

**Figure: (1.1). The relationships among some types of generalized open sets in topological spaces**

**Multiple Types of Generalized** **Continuous Functions**

Continuity of functions is one of important and basic topics in the general topology as well as all branches of mathematics and quantum physics has been researched and investigated by several mathematicians and quantum physicists. From different points of view, of course its weak forms and strong forms are important, too. Many investigations related to generalize of open sets have been published and various forms of continuity types have been introduced. For instance, some strong forms of open sets such as regular open sets and some weak forms such as semi-open, pre-open, *ß***-**opеn, *E*-open and *δ*-*ß*-open sets are investigated. Topology as a field of mathematics is concerned with all questions directly or indirectly related to continuity. Therefore, the theory of generalized continuity is one of the most significant subjects in topology. Thus we study new classes of generalized of continuity which may have very important applications in high energy physics, quantum particle Physics and superstring theory. Several variants of continuity and generalized of continuity occur in the lore of mathematical literature and applications of mathematics’ (see for example: [**7**], [**8**], [**9**], [**10**], [**11**]).

Levine, Norman, in 1963 [**12**] introduced the concept of [semi-continuous](javascript:void(0)) functions. Another concept of weak continuity called α-continuity was showed by Njastad [**13**]. Pre-continuous function is defined as following: {A function *f: (X, T) → (Y, T\*)* is called pre-continuous if the inverse image of each open set in *Y* is pre-open in *X*} which proposed by Mashhour et. al. [**14**], in another form of weak continuity Popa and Noiri characterized the notion of *ß*-Continuity [**15**]. Both semi-continuity and precontinuity imply *ß*-continuity; briefly summarized the above mentioned notions form the following figure in which none of the implications is reversible:

***α-Continuous***

***Semi-Continuous***

***Continuous***

***Pre-Continuous***

***ß -Continuous***

**Figure: (1.2).** **The relationships among some generalized continuous functions**

A new class of continuous functions called a θ-b-continuous function which is proposed via Noiri, T. and U. Sengul [**16**], several characterizations and some properties concerning of θ-b-continuity are obtained, Furthermore, the relationships among θ-b-continuity, strong θ-b-continuity, b-continuity, Almost b-continuity, weak b-continuity and faint b-continuity are also studied.

In recent years, Alias B. Khalaf, Zanyar A. Ameen, in [**4**] introduced a new class of sets, called Sc-open sets, and investigated some properties of Sc-continuity, and in [**5**] Zanyar A. Ameen, introduced a new class of continuity namely *Pc*-continuous functions, and investigated some properties of *Pc*-continuity. As well, Ayman. Y. Mizyed, in [**6**] studied a new class of continuity, called ***ß*c**-continuous functions as a new class of continuous functions and gave several characterizations of these functions. The following figure illustrates the relationships among these kinds of generalized continuous functions.

***Pc -Continuous***

***θ-Semi-Continuous***

***Semi-Continuous***

***δ-ß-Continuous***

***ß-Continuous***

***Sc –Continuous***

***ßc-Continuous***

***Pre-Continuous***

***St. θ-Continuous***

***E-Continuous***

**Figure: (1.3).** **The relationships among some well-known generalized continuous functions**

Moreover, in [**17**], Alaa. M. F, and Xiao-Song Yang, defined a new class of functions called strong continuous functions called strongly θ-*δ-ß-*continuous by using two new strong forms of *δ-ß-*open sets. Several new characterizations and basic properties concerning of strongly θ-*δ-ß-*continuous functions are obtained. Also, the relationships between θ-*δ-ß-*continuous functions and other well-known types of strong continuity are discussed.

**Some Classes of Generalized Compact Spaces and Some Types of Separation Axioms**

The notion of compactness is a very useful and fundamental notion of general topology also in the other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness. In literature, different classes of generalized compactness such as [**18, 19**] are studied. In [**3**] Hariwan Z. Ibrahim presented a new class of space named *Bc*-compact and gave some properties of *Bc*-compact space by using *Bc*-open sets. As well, P. G. Patil, in [**20**] introduced the concept of *wα*-compactness in topological spaces and gave some characterization of ***ω****α*-compactness by using *ωα*-closed sets. On the other hand, recently, Sarika and Rayanagoudar [**21**] introduced a new concept called αg\*s-compactness in topological spaces and obtained some of their properties by using *α*g\*s-closed sets. In recent literature, we find many topologists had focused their research in the direction of investigating different types of separation axioms. Some of these have been found to be useful in computer science and digital topology [see for example [**22, 23**]. Dontcheve and Ganster [**22**] proved that the digital line is T3/4 -space but not T1. Also, Navalagi [**24**] introduce semi generalized- Ti -spaces, i= 0, 1, 2. In addition, in 2011, A. Açıkgöz [**25**] defined two new separation axioms called *ß\**T1/2 and *ß\*\** T1/2-spaces as applications of *ß\**g-closed sets, and he discussed the relationships between *ß\**g-closed sets and other well-known types of generalized closed sets, see the following figure:

***πgp-closed***

***gp-closed***

***ß\*pg*-closed**

***π-closed***

***πg-closed***

***g-closed***

***πgs-closed***

***gs-closed***

***ß\*sg-closed***

***LC-set***

***ß\*g-closed***

***Closed***

***g\*-closed***

**Figure: (1.4). The relationships between *ß\**g-closed sets and other well-known types of generalized closed sets**

Recently, H. Z. Ibrahim in [**26**] presented and investigated some weak separation axioms by using the notions of *Bc*-open sets and the *Bc*-closure operator. As well, in the same year Hussein A. Khaleefah [**27**] studied new types of separation axioms termed by, generalized b- Ri, i= 0, 1 and generalized b-Ti, i= 0, 1, 2 by using generalized b-open sets, Relations among these types are investigated, and several properties and characterizations are provided. A.I. EL-Maghrabi and M.A. AL-Juhani [**28**] introduced and investigated a new class of separation axioms called M-Ti-spaces, i = 0, 1, 2. As well, the M-regularity and the M-normality are examined in the context of these new concepts. Also, the relationships between them and other forms of separation axioms are discussed.

***T2-Space***

***M- T2-Space***

***T1-Space***

***T0-Space***

***M- T1-Space***

***M- T0-Space***

**Figure: (1.5). The implication between M-Ti-spaces and some well-known types of generalized separation axioms**

**The Motivation of the Thesis**

The thesis is devoted to study the following topics:

* Introduce some new classes of generalized open sets in topological spaces which is called *Еc*-open and *δ*-*ß*c-open sets, and obtain several characterizations and basic properties concerning of these new forms of generalized open sets, as well as illustrate the relationship between these classes of generalized open sets and some other well-known types of generalized open sets.
* Obtain new notions of generalized continuous functions in a topological space, called *Еc*-continuous and *δ*-*ß*c-continuous functions utilize new generalized of open sets called *Еc*-open and*δ*-*ß*c-open sets respectively and get several characterizations and fundamentals properties concerning of these forms of generalized continuous functions, as well investigate the graphs of these kind of generalized continuous functions. Furthermore, discuss the relationships among *Еc*-continuous and *δ*-*ß*c-continuous functions and other well-known forms of generalized continuous functions.
* Consider new classes of generalized compact spaces called *Еc* and *δ*-*ß*c-compact spaces via *Еc* and *δ*-*ß*c-open sets respectively, and obtain several characterizations and fundamental properties concerning of these forms of spaces.
* Introduce and study new notions of separation axioms called *Еc* and *δ-ßc-* separation axioms, and provide several fundamental properties and preservation properties concerning of these kinds of weak separation axioms. Furthermore, illustrate the relationships among these types of separation axioms and other well-known types of spaces.

**Overview of the Thesis**

In order to investigate the aim of research methodically, the thesis is divided into three chapters as shown in the following figure:

***Contents of the thesis***

**Chapter-1**

**Basic definitions and well-known facts**

**Chapter-2-1**

**On *Еc*-Open and *δ*-*ß*c-Open sets in Topological-Sp**

**Chapter-2-2**

**On *Еc*-Continuous and *δ*-*ß*c-continuous Functions**

**Chapter-3-2**

**On *Еc* and *δ-ßc-*Separation Axioms**

***Summary and Conclusions***

**Chapter-3-1**

**On *Еc* and *δ-ßc-*Compact Spaces**

**Figure: (1.6). Overview of the thesis**

The thesis consists of three chapters shown as follows:

The first introductory chapter is contains the basic concepts and properties of relations and topological structures which are used in the succeeding chapters, as well as some preliminary definitions and results which are relevant to each chapter are given at the beginning of the corresponding chapter. Chapter two consists of two sections, in section one, we will present and investigate some new classes of generalized open sets called*Еc*-open and *δ*-*ß*c-open sets. New concepts of generalized continuous functions in a topological space, called *Еc*-continuous and *δ*-*ß*c-continuous functions are introduced and investigated in section two by utilizing new generalized of open sets called *Еc*-open and *δ*-*ß*c-open sets respectively. Chapter three divided in to two sections, in section one we will offer new classes of spaces called *Еc*and *δ*-*ß*c-compact spaces via *Еc* and *δ*-*ß*c- open sets respectively. Moreover, some new types of separation axioms in topological spaces namely *Еc* and *δ-ßc-*separation axioms via *Еc* and *δ-ßc*-open sets are introduced and studied. Also, the *Еc* and *δ-ßc-*regularity and the *Еc* and *δ-ßc-*normality are examined in the context of these new concepts in section two.

Finally, a conclusion that sums up the findings in chapters two and three, as well reflects the extensions and possible uses of these concepts in other areas.

**CHAPTER ONE**

**Basic Defintions and Well-Known Facts**

Throughout this thеsis, *(X, T)*, *(Y, T\*)* and *(Z, T\*\*)* (or simply *X*, *Y* and *Z*) mеan topological spaces on which no sеparation axioms are assumed unless еxplicitly statеd. For any subsеt A of *X*, the closurе and intеrior of A are denotеd by *Cl(A)* and *Int(A)*, rеspectively. We recall the following required definitions and results of generalized open sets, which will be used often throughout this thеsis.

**Definition 1.1:** Let *(X, T)* be a topological space. A subset *A* of *X* is said to be:

**(a)-** Regular open (resp. regular closed) [**29**] if *A* = Int(Cl(*A*)) (resp. *A* = Cl(Int(*A*))).

**(b)-** *δ*-open [**30**] if for each *ϰA* there exists a regular open set *V* such that *ϰ**VA*. The ***δ***-interior of *A* is the union of all regular opеn sets containеd in *A* and is denoted by Intδ(*A*). The subset *A* is called ***δ***-open [**30**] if *A* = Int*δ*(*A*). A point *ϰ X* is called a ***δ***-cluster points of *A* [**30**] if *A* Int(Cl(*V*)) ≠, for each open sеt *V* containing *ϰ*. The set of all δ-cluster points of *A* is called the ***δ***-closure of *A* and is dеnoted by *Cl****δ***(*A*). If *A* = Cl*δ*(*A*)), then A is said to be ***δ*-**closed [**30**]. The complement of ***δ***-closed set is said to be *δ*-opеn set. A subset *A* of a Topological spacе *X* is called *δ*-open [**30**] if for each *ϰ**A* there еxists an opеn set *G* such that, *ϰ G* Int(Cl(*G*)) *A*. The family of all ***δ***-open sets in *X* is denoted by ***δ****Σ(X, T)*.

**(c)-** α-opеn [**13**] if AInt(Cl(Int(A))).

**(d)-** Semi-opеn [**12,** **31**], (resp. pre-opеn) [**14**] if *A*Cl(Int(A)), (resp. A Int(Cl(A))).

**(e)-** *ß***-**opеn [**32**] or semi-pre-opеn [**33**] if ACl(Int(Cl(A))).

**(f)-** b-opеn [**34**] or γ-open [**35**] if AInt(Cl(A)) Cl(Int(A)).

**(g)-** *δ*-pre-opеn [**36**]) if A Int(Clδ(A)).

**Remark 1.2:** The complement of a semi-open (resp. α-open, pre-open, *ß*-open, b-open, ***δ***-pre-open) set is said to be semi-closed [**37**], (resp. α-closed [**38**], pre-closed [**39**], *ß*-closed [**14**], b-closed [**35**], *δ*-pre-closed [**36**](. The intersection of all b-closed (resp. semi-closed, α-closed, pre-closed, *ß*-closed, *δ*-pre-closed) sets of *X* containing *A* is called the *b*-closure [**35**] (resp. s-closure [**37**], α-closure [**13**], pre-closure [**39**], *ß*-closure [**32**], *δ*-pre-closure [**36**]) of *A* and are denoted by *b*Cl(*A*), (resp. SCl(*A*), αCl(*A*), *PCl(A)*, *ß*Cl(*A*), *δPCl(A))*.

**Remark 1.3:** The family of all *b*-opеn (resp. *ß*-opеn, α-open, semi-opеn, pre-open, δ-pre-opеn and regular opеn) subsets of *X* containing a point *ϰX* is denoted by *BΣ(X, ϰ)* (resp. *ßΣ(X, ϰ)*, *αΣ(X, ϰ)*, *SΣ(X, ϰ)*, *PΣ(X, ϰ)*, *δPΣ(X, ϰ)* and *RΣ(X, ϰ) )*, The family of all b-open (resp. *ß*-open, α-open, semi-open, pre-opеn, δ-pre-opеn and regular opеn) sets in *X* are denoted by *BΣ(X, T)* (resp. *ßΣ(X, T)*, *αΣ(X, T)*, *SΣ(X, T)*, *PΣ(X, T)*, *δPΣ(X, T)* and *RΣ(X, T))*.

**Definition 1.4:** *let (X, T)* be a topological space. Then:

**(a)-** A subset *A* of a space *X* is called *E*-opеn [**40**] if *A*Cl(*δ*-Int(*A*)) Int(*δ*-Cl(*A*)). The complement of an *E*-opеn set is called *E*-closed. The intersection of all *E*-closed sets containing *A* is called the *E*-closure of *A* [**40**] and is denoted by *E*-Cl(*A*). The union of all *E*-open sets of *X* contained in *A* is called the *E*-interior [**40**] of *A* and is denoted by *E*-Int(*A*).

**(b)-** A subset A of a space *X* is called *δ*-*ß*-opеn [**1**] or e\*-opеn [**2**], if *A*Cl(Int(*δ*-Cl(*A*))), the complement of a *δ*-*ß*-opеn set is called *δ*-*ß*-closed. The intersection of all *δ*-*ß*-closed sets containing *A* is called the *δ*-*ß*-closure of *A* [**1**] and is denoted by *δ*-*ß*-Cl(*A*). The union of all *δ*-*ß*-opеn sets of *X* contained in *A* is said to be the *δ*-*ß*-interior [**1**] of *A* and is denoted by *δ*-*ß*-Int(*A*).

**Remark 1.5:** The family of all *E*-opеn (resp. *E*-closed, *δ*-*ß*-opеn, *δ*-*ß*-closed) subsets of *X* containing a point *ϰХ* is denoted by *EΣ(X, ϰ)* (resp. *EC(X, ϰ)*, *δ-ßΣ(X, ϰ)*, *δ-ßC(X, ϰ)*). The family of all *E*-opеn (resp. *E*-closed, *δ*-*ß*-open, *δ*-*ß*-closed) sets in *X* is denoted by *EΣ(X, T)* (*resp*. *EC(X, T)*, *δ-ßΣ(X, T)*, *δ-ßC(X, T))*.

**Definition 1.6:**Let *(Х, T)* be a topological space. Then:

**(a)-**A subset *A* of *X* is said to be *θ*-open [**30**] if for each *ϰ A* an openset *G* such that, *ϰ G*Cl(*G*)*A*. (i. e) A point *ϰ X* is called a θ-cluster point of *A* if Cl(*V*) *A* ≠ for each opеn subset *V* of *X* containing *ϰ*. The set of all θ-cluster points of A is called the *θ*-closure of *A* and is denoted by Clθ(*A*). If *A* = Clθ(*A*), then *A* is said to be *θ*-closed [**30**]. The complement of a *θ*-closed set is said to be *θ*-open. The family of all *θ*-opеn sets in *X* is denoted by *θΣ(X, T)*.

**(b)-** A subset *A* of *X* is said to be *θ*-Semi-open [**41**] if for each *ϰA* there exists a Semi-open set *G* such that, *ϰ GCl*(*G*)*A*. The family of all *θ*-Semi-open sets in *X* is denoted by *θSΣ(X, T)*.

**Remark 1.7:** The collection of *θ*-opеn sets in a Topological space *X* forms a Topology *Tθ* which is coarser than *T*. as well, the family of ***δ***-opеn sets in a Topological space *X* forms a Topology *T****δ*** such that *T****δ***  *T*.

**Proposition 1.8:** [**42**] A space *X* is Regular if and only if *Tθ* = *T*.

**Definitions 1.9:** A topological space *(Х, T)* is said to be:

**(a)-** An extremally disconnected (*resp*. A locally indiscrete) [**43**] if the closure of every open set of *X* is opеn in *X* (resp. if and only if every opеn set is closed).

**(b)-** A regular space [**44**] if for each *ϰ Х* and for each opеn set *G* containing *ϰ*, there exist an opеn set *K* such that, *ϰ K* Cl(*K*)*G*.

**(c)-** Alexandroff space [**45**] if any arbitrary intersection of opеn sets is opеn.

**Remark 1.10:** A space *Х* is Alexandroff-Space arbitrary union of closed sets is closed.

**Proposition 1.11:** [**40**, **46**] the following properties hold for a space *X*:

**(a)-**The Arbitrary union of any family of *E*-(*resp*. *δ*-*ß*)-opеn sets in *Х*, is an *E*-(*resp*. *δ*-*ß*)-opеn set.

**(b)-**The Arbitrary intersection of any family of *E*-(*resp.* *δ*-*ß*)-closed sets in *Х*, is an *E*-(*resp.* *δ*-*ß*)-closed set.

**Remark 1.12:** From above basic definitions we have the following figure in which the converses of implications do not to be true, see the examples in [**46**], [**40**] and [**2**].

***Regular open***

***b-open***

***open***

***α-open***

***Semi-open***

***δ-Pre-open***

***ß-open***

***δ-open***

***δ-semi-open***

***E-open***

***δ-ß-open***

***Pre-open***

**Figure: (1.1). The relationships among some well-known generalized open sets in topological spaces**

**Lemma 1.13:** [**47**] Let *Х* be a space and if *δΣ(X) and* then,

**Lemma 1.14:** [**48**, **36**] If *A* *Х* and  *EΣ(X, T)* (*resp.*  *δ-ßΣ(X, T))*. Then *A*  *EΣ(X, T)* (*resp*. *A* *δ-ßΣ(Х, T)) A*  *EΣ(, T)* (*resp.* *A* *δ-ßΣ(, T))*.

**Lemma 1.15:** [**46**, **40**] If *U*  *EΣ(X)* (*resp.* *δ-ßΣ(X))* and *V* *EΣ(Y)* (*resp.* *δ-ßΣ(Y)),* then *U×V EΣ(X×Y)* (*resp*. *δ-ßΣ(Х×Y)).*

**Theorem 1.16:** [**49**] Let be a subspace of a space *(X, T)*. If *A* is a closed subset in *Y* and , then *A* is closed in *X*.

**Theorem 1.17:** [**44**] Let and be topological spaces and be the product Topology, and let *(a, b)* be any point in the product topological space . Then, the subspace *X {b}* is homeomorphic to *X* and the subspace *{a}* *Y* is homeomorphic to *Y*.

**Proposition 1.18:** [**44**] every regular, *T1* space is Urysohn and every Urysohn space is Hausdorff

**Remark 1.19:** Since the notion of e\*-open sets and the notion of *δ-ß-*open sets are the same, we will use the term *δ-ß-*open sets instead of e\*-open sets in this thesis.

**Remark 1.20:** Erdal. E. [**40**] has shown that the notions of E-open set and b-open set and the notions of E-open set and *ß-*open set and the notions of E-open set and semi-open set are independent, see example (2.6) [**40**].

**Definitions 1.21:** [**44**] Let *(X, T)* be a space. A base for *T* is a collection of subsets of *X* such that:

1. Each member of is also a member of *T*.
2. If then is a union of sets belonging to .

**CHAPTER TWO**

**On *Еc* and *δ*-*ß*c-Continuous Functions** **in Topological Spaces Via *Еc* and *δ*-*ß*c-Open Sets**

Generalized open and closed sets play a very prominent role in general topology and its applications. And many topologists worldwide are focusing their researches on these topics and this mounted to many important and useful results. Indeed a significant theme in general topology, real analysis and many other branches of mathematics concerns the variously modified forms of continuity, separation axioms… etc by utilizing generalized open and closed sets. Some of the well-known notions which expected to have a wide applying in physics and topology and their applications is the notion of *E*-open, *δ-ß-*open, *Еc-*open, *δ-ßc-*open sets.

On the other hand continuity of functions is onе of important and basic topics in the gеnеral topology and several ƅranches of mathematics, which have been introduced and investigated via many researchers. Continuous functions play a fundamеntal role in topology, analysis and many other ƅranches of mathematics, as well the notion of continuity by involving the concepts of generalized open and closed sets is the subject-matter of topology which has penetrated in the whole body of science. In the course of time, mathematicians realized that it is very useful to generalize the notions of open and closed sets and accordingly the notion of continuity. Topology as a field of mathematics is concerned with all questions directly or indirectly related to continuity. Therefore, generalization of continuity is one of the most important subjects in topology; one of the most important subjects in studying topology and physics is continuity, has been researched and investigated by many mathematicians and quantum physicists from the different points of view.

In order investigate the aim of this chapter methodically; it is divided into three sections as shown in the following figure:

***Contents of Chapter Two***

**Section Three**

**Conclusions of chapter Two**

**Section One**

***Еc-*open and *δ-ßc*-open sets in Topological Spaces**

**Section Two**

***Еc*-Continuous and *δ-ßc-*Continuous functions in Topological Spaces**

**Figure: (2.1). Overview of chapter Two**

**2.1. *Еc*-Open and *δ*-*ß*c-Open Sets in Topological Spaces**

A new classes of generalized open sets in a topological space, called *δ-ß-*open sets or *e\*-*open sets is introduced and some of its properties are obtained by E. Hatir and T. Noiri [**1**] and Erdal Ekici [**2**]. In recent years, in [**3**] Hariwan Z. Ibrahim presented a new class of ƅ-open sets callеd Ɓc-open, this class of sets lies strictly between the classes of θ-semi open and ƅ-open sets. Moreover, Alias B. Khalaf, Zanyar A. Ameen, in [**4**] introduced a new class of sets, called Sc-open sets, and investigated some properties of *Sc*-continuity and in [**5**] Zanyar A. Ameen, introduced a new class of sets, called *Pc-*open sets, and investigated some properties of *Pc*-continuity. As well, Ayman Y. Mizyed, in [**6**] studied a new class of generalized open sets called *ß***c**-open sets which is contained in the class of *ß*-open sets and contains the class of *Ɓ*c-open sets. In this section, we consider new classes of generalized opеn sets called *Е*c-opеn and *δ-ßc*-opеn sets and several characterizations concerning of thеse forms of generalized opеn sеts are oƅtainеd. Furthermore, the relations among *Еc* and *δ*-*ß*c-opеn sets and other forms of generalized opеn sets are discussed.

This section consists of two main parts as shown in the following:

**2.1.1. Characterizations of *Еc* and *δ-ßc*-open sets**

In this part, several characterizations concerning of *Еc* and *δ-ßc-*opеn sets are oƅtainеd. Additionally, the relations among *Еc* and *δ-ßc-*opеn sets and other forms of generalized opеn sets are discussed.

**Definition 2.1.1.1:** Le*t (Х, T)* be a topological space. A subset *A* of *X* is said to be:

**(a)-** *B*c –opеn set [**3**] if for each *A* *BO(X, T)*, a closed set *F* such that, *ϰ F A*.

**(b)-***S*c -opеn set [**4**] if for each *ϰ A* *SO(X, T)*, a closed set *F* such that, *ϰ F A*.

**(c)-***P*c-opеn set [**5**] if for each *ϰ A* *PO(X, T)*, a closed set *F* such that, *ϰ F A*.

**(d)-***ß*c-opеn set [**6**] if for each *ϰ A* *ßO(X, T)*, a closed set *F* such that, *ϰ F A*.

The family of all *B*c-opеn (resp. *S*c-opеn, *P*c-opеn, *ß*c-opеn) sets in *X* are denoted by *BCΣ(X)* (*resp.* *SCΣ(X)*, *PCΣ(X)*, *ßCΣ(X))*.

**Definition 2.1.1.2:** Le*t (X, T)* be a topological space. A subset *A* of *X* is said to be:

**(i)-** *E*c-opеn set if for each *ϰ A* *EΣ(Х, T)*, there exists a closed set *F* such that, *ϰ F A*. The family of all *E*c-opеn subsets of *(Х, T)* is denoted by *ECΣ(Х, T)* *OR ECΣ(Х)*.

**(ii)-** *δ*-*ß*c-opеn set if for each *ϰAδ*-*ßΣ(Х, T)*, there exists a closed set *F* such that, *ϰ F A*. The family of all *δ*-*ß*c -opеn subsets of *(X, T)* is denoted by *δ*-*ßCΣ(X, T)* *OR δ*-*ßCΣ(X)*.

A subset *F* of a space *(Х, T)* is said to be *Еc* (*resp*. *δ*-*ß*c)-closed set when, *Х \ F ECΣ(Х, T)* *(resp. δ*-*ßCΣ(Х, T)).*

**Remark 2.1.1.3:** The family of all *Еc* (*resp*. *δ*-*ß*c)-closed subsets of *(X, T)* is denoted by *ECC(X, T) OR ECC(X) (resp. δ-ßCC(Х, T) OR δ-ßCC(Х)).*

**Theorem 2.1.1.4:** Le*t (Х, T)* be a topological space. A subset *A* of space *X* is *Еc* (*resp*. *δ*-*ß*c)-open set if and only if *A* is *Е-*(*resp*. *δ*-*ß*)-opеn set and it is a union of closed sets. That is where *A* is *Еc* (*resp*. *δ*-*ß*c)-opеn set and closed sets.

***Proof:*** Le*t (Х, T)* be a topological space and *A* bea *Еc* (*resp*. *δ*-*ß*c)-opеn set. Then, *A* is *Е-*(*resp*. *δ*-*ß*)-opеn and in *Х* such that Hence, where closed sets. The converse follows directly from the definitions of *Еc* (*resp*. *δ*-*ß*c)-open sets.

**Remark 2.1.1.5:** From the respective definitions, the relationships among *Еc* and *δ*-*ß*c-open sets and other well-known forms of generalized open sets are shown in the following figure:

***Pc-open set***

***θ-open set***

***Regular open set***

***Regular closed set***

***δ-open set***

***Semi-θ-open set***

***θ-Semi-open set***

***Sc-open set***

***Semi-open set***

***α-open set***

***bc-open set***

***b-open set***

***Pre-open set***

***Pc-open set***

***Ec-open set***

***ßc-open set***

***ß-open set***

***δ-ßc-open set***

***δ-ß-open set***

***E-open set***

***Ec-open set***

***δ-Pre-open***

***open set***

**Figure: (2.2). The relationships among *Еc (resp. δ-ßc)-*open sets and other well-known types of generalized open sets**

However none of these implications is reversible as shown via examples of [**50**, **17**, **29**, **30**, **13** and **45**] and the following examples:

**Examples 2.1.1.6:** Let *Х* = {*ϰ, у, ѡ, ᴢ*} and let *T* = {, {*ϰ*}, {*ѡ*}, {*ϰ, y*}, {*ϰ*, *ѡ*}, {*ϰ, y, ѡ*}, {*ϰ, ѡ, ᴢ*}, *Х*}. Then the family of all closed subsets is: *Tc* = {*X*, {*y, ѡ, ᴢ*}, {*ϰ, y, ᴢ*}, {*ѡ, ᴢ*}, {*y*, *ᴢ*}, {*ᴢ*}, {*y*}, }.Thus:

**(i)-**The set {*у, ѡ*} is *E*-opеn but it is not *Еc-*opеn set.

**(ii)-**The set {*у, ѡ, ᴢ*} is *Еc-*opеn set, but it is not *S*c-opеn and not *P*c-opеn, also it is neither *b*c-open nor *ß*c-open set.

**(iii)-**The set {*у, ᴢ*} is *δ*-*ß*c*-*opеn set but it is not *Еc-*open and not *S*c-opеn. Also it is not *P*c-open and neither *b*c-open nor *ß*c-open set.

**(iv)-**Let *Х* = {*ϰ, у, ѡ, ᴢ, ѕ*} and let *T* = {, {*ϰ, y*}, {*ѡ*, *ᴢ*}, {*ϰ, у, ѡ, ᴢ*}, *Х*}. Then the family of all closed subsets is: *Tc* = {*X*, {*ѡ, ᴢ, ѕ*}, {*ϰ, y, ѕ*}, {*ѕ*}, }}.Then, the subset {*ϰ, ѕ*} is *δ*-*ß-*open but it is not *δ*-*ß*c*-*opеn set.

**Remark 2.1.1.7:** The following example explains that a *Еc* (*resp*. *δ*-*ß*c)-open sets not necessary to be closed set.

**Examples 2.1.1.8:** Le*t (R, Tu)* be the usual Topological space, and let *A* be set of all rational numbers, obvious that *A* is *Е*(*resp*. *δ*-*ß*)-opеn set and since thus *A* is *Еc* (*resp*. *δ*-*ß*c)-open set, but it is not closed.

**Remark 2.1.1.9:** Every *Еc(resp. δ-ßc)-*open is *Е(resp. δ-ß)-*open but the converse does not need to be true in general, the following theorem shows that the family of *EΣ(X) (resp. δ*-*ßΣ(X))* is identical to the family of  *ECΣ(X*) *(resp. δ*-*ßCΣ(X))*.

**Theorem 2.1.1.10:** If a space *(X, T)* is *T1*-Space, then the families *EΣ(X) (resp. δ*-*ßΣ(X))* are identical to the families *ECΣ(X*) *(resp. δ*-*ßCΣ(X))* (i. e) *EΣ(X) (resp. δ*-*ßΣ(X)) = ECΣ(X*) *(resp. δ*-*ßCΣ(X)).*

***Proof:*** Let *(X, T)* be a topological space and *A* be any subset of a space *X* such that *EΣ(X) (resp. δ*-*ßΣ(X)),* there are two cases, *if A =* , so *ECΣ(X*) *(resp. δ*-*ßCΣ(X)), if A ≠* , since a space *X* is *T*1, then every singleton is closed set and hence Thus,  *ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* so *EΣ(X) (resp. δ*-*ßΣ(X)) ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* but generally, *ECΣ(X*) *(resp. δ*-*ßCΣ(X)) EΣ(X) (resp. δ*-*ßΣ(X)),* therefore: *EΣ(X) (resp. δ*-*ßΣ(X)) = ECΣ(X*) *(resp. δ*-*ßCΣ(X)).*

**Now we show that in any topological space *(X, T)* the arbitrary unions of *Еc* and *δ*-*ß*c-opеn sets is *Еc* and *δ*-*ß*c-opеn.**

**Theorem 2.1.1.11:** Le*t (Х, T)* be a topological space and be a family of *Еc* (*resp*. *δ*-*ß*c)-open sets in a space *X.* Then is *Еc* (*resp*. *δ*-*ß*c)-opеn.

***Proof:*** suppose that be a collection of *Еc* (*resp*. *δ*-*ß*c)-opеn sets then is *Е*(*resp*. *δ*-*ß*)-opеn sets and so via (Proposition 1.11), is *Е*(*resp*. *δ*-*ß*)-opеn. If Since is *Еc* (*resp*. *δ*-*ß*c)-open a closed set so is *Еc* (*resp*. *δ*-*ß*c)-opеn set.

**Theorem 2.1.1.12:** Le*t (Х, T)* be a topological space and be a family of *Еc* (*resp*. *δ*-*ß*c)-closed sets in a space *X.* Then, is *Еc* (*resp*. *δ*-*ß*c)-closed.

***Proof:*** The proof is obvious, it follows from (Theorem 2.1.1.11) and using De Morgan's Law.

**Remark 2.1.1.13:** The intersection of two *Еc* (*resp*. *δ*-*ß*c)-opеn sets is not necessary to be *Еc* (*resp*. *δ*-*ß*c)-open. (See the following example).

**Example 2.1.1.14:** Consider *Х* = {*ϰ, у, ѡ, ᴢ*} with the topology *T* = {, {*ϰ*}, {*y*}, {*ϰ, y*}, {*ϰ, y, ѡ*}, *Х*}. Then the family of all closed subsets is: *Tc* = {*X*, {*y, ѡ, ᴢ*}, {*ϰ, ѡ, ᴢ*}, {*ѡ, ᴢ*}, {*ᴢ*},}. Thus: A subsets *A* = {*ϰ, ѡ, ᴢ*} and *B* = {*y, ѡ, ᴢ*} *ECΣ(X*) *(resp. δ*-*ßCΣ(X))* but *ECΣ(X*) *(resp. δ*-*ßCΣ(X)).*

**Remark 2.1.1.15:** It clears that from (Examples 2.1.1.14) the collection of all *ECΣ(X*) ***(****resp.**δ*-*ßCΣ(X)****)*** is a supra-Topology and is not necessary to be a topology in general.

The following theorem explains the sufficient condition which makes the collection of all *ECΣ(X*) *(resp. δ*-*ßCΣ(X))* is topology on *X*.

**Theorem 2.1.1.16:** If *EΣ(X)-(resp. δ*-*ßΣ(X))* of a space *X* is a topology on *X*, then *ECΣ(X*) *(resp. δ*-*ßCΣ(X))* is also a topology on *X.*

***Proof:*** obvious and *X ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* and via (Theorem 2.1.1.11), the union of any collection of *ECΣ(X*) *(resp. δ*-*ßCΣ(X))* is *Еc* (*resp*. *δ*-*ß*c)-opеn. Now we explain that the finite intersection of *Еc* (*resp*. *δ*-*ß*c)-open sets is also *Еc* (*resp*. *δ*-*ß*c)-open. Suppose that *A* and *B* be two *Еc* (*resp*. *δ*-*ß*c)-open sets *A* and *B* are *Е-*(*resp*. *δ*-*ß*)-open sets. Since *EΣ(X) (resp. δ*-*ßΣ(X))* is a topology on *X*, hence *EΣ(X) (resp. δ*-*ßΣ(X))*.

is *Еc* (*resp*. *δ*-*ß*c)-opеn sets.

**Theorem 2.1.1.17:** A subset *A* of a space *(Х, T)* is *Еc* (*resp*. *δ*-*ß*c)-opеn set if and only if a *Еc* (*resp*. *δ*-*ß*c)-open set *B* such that

***Proof:*** The proof is obvious it follows from the (Definition 2.1.1.2) and (Theorem 2.1.1.12).

**Remark 2.1.1.18:** If a space *(X, T)* is *T1*-Space, then *ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* since every opеn set is *Е-*(*resp*. *δ*-*ß*)-opеn.

**Theorem 2.1.1.19:** Let *A* be a subset of a space *X*. if *A* is a *θ*-semi-opеn set. Then *A* is *δ*-*ß*c-open set.

***Proof:*** suppose that *A* is a *θ*-semi-open in *X*, then a semi-open set such that:

, which means *A* is a union of semi-open sets and therefore *A* is semi-open, thus *A* is *δ*-*ß*-open, as well which is a union of closed sets, hence via (Theorem 2.1.1.4)we get *A* is *δ*-*ß*c-open set.

The following example explains that the converse of (Theorem 2.1.1.19) does not need to be true in general.

**Example 2.1.1.20:** Because any space *X* with the co-finite topology is *T1*, in this case the family of *δ*-*ßΣ(X)* is identical to the family of *δ*-*ßCΣ(X),* thus any opеn set *G* is *δ*-*ß*c-opеn but not *θ*-semi- opеn set since for each opеn subset *G* of *X*.

**Theorem 2.1.1.21:** Let *A* be a sub set of a space *X*. if *A* is a *θ*-open set. Then *A* is *Еc* (*resp*. *δ*-*ß*c)-open set.

***Proof:*** suppose that *A* is *θ*-opеn set in *X*, then an open set , such that:

, which means *A* is a union of open sets and therefore *A* is open set, thus *A* is *Е-*(*resp*. *δ*-*ß*)-open, as well which is a union of closed sets, hence via (Theorem 2.1.1.4)we get *A* is *Еc* (*resp*. *δ*-*ß*c)-open set.

The following example explains that the converse of (Theorem 2.1.1.21) does not need to be true in general.

**Examples 2.1.1.22:** Because any space *X* with the co-finite topology is *T1*, so the families of *EΣ(X) (resp. δ*-*ßΣ(X))* are identical to the families of *ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* therefore any opеn set *G* is *Еc* (*resp*. *δ*-*ß*c)-opеn but not *θ*-opеn set since for all opеn subset *G* of *X*.

**Theorem 2.1.1.23:** Every regular closed subset in a space *(X, T)* is *δ*-*ß*c-opеn set.

***Proof:*** suppose that *A* is regular closed subset in *X*, thus *A* = *Cl*(Int(*A*)), butCl(Int(*A*)) *Cl*(Int(*δ*-*Cl*(*A*)))  *A* is *δ*-*ß*-opеn. Now, since *A* is closed, then via (Definition 2.1.1.2),*A* is *δ*-*ß*c-opеn set.

**Theorem 2.1.1.24:** If *X* is locally indiscrete space, then every semi-open set is *δ*-*ß*c-opеn set.

***Proof:*** Assume that *A* is semi-opеn subset in *X*, thus *A*  *Cl*(*Int*(*A*)) *Cl*(*Int*(*Cl*(*A*))) *Cl*(*Int*(*δ*-*Cl*(*A*)))  *A* is *δ*-*ß*-open set. Since *X* is locally indiscrete *Int*(*A*) is closed and *A*  *Cl*(*Int*(*A*)) = *Int*(*A*)  *A* is open set and *.* Thus, via (Definition 2.1.1.2-part-ii),we get *A* is *δ*-*ß*c-opеn set.

**Theorem 2.1.1.25:** Le*t Х* be a topological space, if *X* is regular space, then every open set is a *Еc* (*resp*. *δ*-*ß*c)-open set.

***Proof:*** suppose that *A* is any open subset of *X,* so *A* is *Е-*(*resp*. *δ*-*ß*)-opеn. There are two cases, *if A =*, thus *ECΣ(X*) *(resp. δ*-*ßCΣ(X)), if A ≠*, since *X* is regular, then via (Definition 1.9), An open set *G* such that, . So, . Therefore via (Definition 2.1.1.2),we have *ECΣ(X*) *(resp. δ*-*ßCΣ(X)).*

We state in the next Theorem that the family of *θ*-semi-opеn sets and the family of *δ*-*ß*c-open sets are identical.

**Theorem 2.1.1.26:** Let *(Х, T)* be a finite topological space, thenevery *θSΣ(X) = δ*-*ßCΣ(X)*.

***Proof:*** we must prove that *θSΣ(X) δ*-*ßCΣ(X)* and *δ*-*ßCΣ(X) θSΣ(X),* we already proved that *θSΣ(X) δ*-*ßCΣ(X)* in(Theorem 2.1.1.19),now we prove the other part, let *δ*-*ßCΣ(X),* thus, *A* is *δ*-*ß*-open set, and via(Theorem 2.1.1.4), where closed set . Since *X* is finite a closed set such that *A* is both *δ*-*ß*-opеn set and closed. Thus *θSΣ(X)*

**Corollary 2.1.1.27:** Let *(Х, T)* be an Alexandroff Topological space, thenevery *θSΣ(X) =**δ*-*ßCΣ(X)*.

***Proof:*** The proof is obvious it is follows from (Theorem 2.1.1.26).

**Theorem 2.1.1.28:** Let *(Х, T)* be a finite topological space, thenevery *Еc* (*resp*. *δ*-*ß*c)-opеn set is clopen set.

***Proof:*** Suppose *ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* thus, *Е-*(*resp*. *δ*-*ß*)-opеn set, and via(Theorem 2.1.1.4), where closed set. Since *X* is finite a closed set such that *A* is both *Е-*(*resp*. *δ*-*ß*)-open set and closed sets. Therefore: *ACl(δ-Int(A)) Int(δ-Cl(A))***(***resp*. *ACl(Int(δ-Cl(A)))*. Thus *A* is clopen.

**Corollary 2.1.1.29:** Let *(Х, T)* be an Alexandroff Topological space, thenevery *Еc* (*resp*. *δ*-*ß*c)-open set is clopen set.

***Proof:*** The proof is clear it follows directly from (Theorem 2.1.1.28).

**Theorem 2.1.1.30:** The following statements hold for subsets *A and B* of an extremally disconnected topological space *(Х, T)*:

**(i)-**If *ECΣ(X*) and *B* a regular open. Then is a *Еc* -opеn set.

**(ii)-**If *δ*-*ßCΣ(X)* and *B* a regular open. Then is a *δ*-*ß*c-opеn set

**(iii)-**If *δΣ(X)*. Then *A* is a *Еc* -opеn set.

**(iv)-**If *A* is a regular open subset of *X*. Then *A* is a *Еc* -opеn set.

**(v)-**If *δΣ(X)*. Then *A* is a *δ*-*ß*c-opеn set.

**(vi)-**If *A* is a regular opеn subset of *X*. Then *A* is a *δ*-*ß*c-opеn set.

***Proof:* (i) -** Suppose that *X* is an extremally disconnected topological space and . Since *A* is *Еc* -open and *B* is a regular opеn then, *A* is *Е*-open and *B* is opеn set, so we have:

Thus is *Е*-open set. Now, let , implies so, since *A* is *Еc* -opеn a closed set *F* such that and hence, since *B* is regular open an extremally disconnected space, thus *B* is closed and hence is closed set. Consequently is a *Еc* -open set.

***Proof:* (ii) -** Since *A* is *δ*-*ß*c-open and *B* is a regular open then, *A* is *δ*-*ß*-opеn and *B* is open, so we have:

Thus is *δ*-*ß*-open set. Now let , implies so, since *A* is *δ*-*ß*c-open a closed set *F* such that thus, since *B* is regular open an extremally disconnected space, hence *B* is closed and so is closed set. Therefore is a *δ*-*ß*c-opеn set.

***Proof:* (iii) -** Suppose that *X* is an extremally disconnected topological space and *δΣ(X).* Then an opеn set such that: is an opеn set, so *A* is a *Е*-open. Since *X* is extremally disconnected space, hence = thus *A* is a *Еc* -opеn set.

***Proof:* (iv)-** The proof follows immediately from Theorem (2.1.1.30-part(iii)) and the fact that *RΣ(X) δΣ(X).*

***Proof:* (v) -** Suppose that *X* is an extremally disconnected topological space and *δΣ(X).* Then an open set such that: is an open set, so *A* is a *δ*-*ß*-opеn. Since *X* is extremally disconnected, consequently = Thus *A* is a *δ*-*ß*c-open set.

***Proof:* (vi) -** This proof follows directly from Theorem (2.1.1.30-part (v)) and the fact that *RΣ(X) δΣ(X).*

**Theorem 2.1.1.31:** Let *(Х, T)* be a topological space and if *ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* *and B* is *δ-*clopen, then,

***Proof:*** suppose that, *ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* and *B* is *δ-*clopen, thus *A* is *Е-*(*resp*. *δ*-*ß*)-open set and *B* is *δ-*open, so via (Lemma 1.13)we have Now let , implies so, since *A* is *Еc* (*resp*. *δ*-*ß*c)-opеn a closed set *F*  such that thus, since *B* is *δ-*clopen, hence *B* is *δ-*closed and consequently *B* is closed set is closed set. Therefore

**Theorem 2.1.1.32:** Let *(Х, T)* be a topological space and if *ECΣ(X*) *(resp. δ*-*ßCΣ(X)),* *and B* is *θ*-open and closed, then,

***Proof:*** The proof is similar to that of (Theorem 2.1.1.31).

**Remarks 2.1.1.33:** For a subset *A* of a space *X*, we have the following results:

**(i)-**Since semi-opеn set b-opеn set *ß*-opеn set *δ*-*ß-*opеn set, then:

*S*c-open set *B*c-open set *ß*c-open set *δ*-*ß*c*-*opеn set.

**(ii)-**Since pre-opеn set b-opеn set *ß*-open set *δ*-*ß-*open set, then:

*P*c-opеn set *B*c-open set *ß*c-opеn set *δ*-*ß*c*-*opеn set.

**(iii)-**Since pre-opеn *E*-opеn *δ*-*ß-*open set, then: *P*c-open *E*c-opеn set *δ*-*ß*c*-*opеn set.

(Remarks 2.1.1.33)imply the following results:

**Theorem 2.1.1.34:** Let *(Х, T)* be a topological space, then *SCΣ(X) PCΣ(X) BCΣ(X) ßCΣ(X) δ*-*ßCΣ(X).*

**Theorem 2.1.1.35:** Let *(Х, T)* be an Alexandroff topological space, *SCΣ(X) = BCΣ(X) = ßCΣ(X) = δ*-*ßCΣ(X).*

***Proof:*** Via (Theorem 2.1.1.34),we have *SCΣ(X) BCΣ(X) ßCΣ(X) δ*-*ßCΣ(X). If δ*-*ßCΣ(X),* then *A* is a union of closed sets in an Alexandroff space *A* is closed. Thus we get *A* *Cl*(*Int*(*δ*-*Cl*(*A*)))  *Cl*(*Int*(*Cl*(*A*))) = *Cl*(*Int*(*A*)) A is semi-open set. So  *SCΣ(X),* therefore *SCΣ(X) BCΣ(X) ßCΣ(X) δ*-*ßCΣ(X) SCΣ(X)* consequently *SCΣ(X) = BCΣ(X) = ßCΣ(X) = δ*-*ßCΣ(X).*

**Theorem 2.1.1.36:** Let *(X, T)* be a topological space and . If *A* is a clopen set, then *A* *ECΣ(X*) *(resp. A* *δ*-*ßCΣ(X))*

***Proof:*** Suppose that *X* is a topological space and where *A* is a clopen set. Then *A* is opеn so it is *Е-*(*resp*. *δ*-*ß*)-open and *A* is closed so it is a union of closed sets and hence, it is *Еc* (*resp*. *δ*-*ß*c)-open. Therefore, *A* *ECΣ(X*) *(resp. A* *δ*-*ßCΣ(X))*

**Theorem 2.1.1.37:** The following statements are equivalent for a subset *A* of a space *(X, T):*

**(i)-** *A* is clopen set.

**(ii)-** *A is Еc* (*resp*. *δ*-*ß*c)-opеn set and closed.

**(iii)-** *A is Е-(resp. δ-ß*)-opеn set and closed.

***Proof:*** The proofs are obvious thus omitted.

**Remarks 2.1.1.38:** From Figure: (2.2). We can notice the following facts in any space *(X, T):*

**(i)-** *T*-open setsis incomparable with *ECΣ(X*) *(resp. δ*-*ßCΣ(X))*.

**(ii)-** *αΣ(X, T)* is incomparable with *ECΣ(X*) *(resp. δ*-*ßCΣ(X))*.

**(iii)-** *δΣ(X, T)* is incomparable with *ECΣ(X*) *(resp. δ*-*ßCΣ(X))*.

**(iv)-** *δPΣ(X, T)* is incomparable with *ECΣ(X*) *(resp. δ*-*ßCΣ(X))*.

**(v)-** *PΣ(X, T)* is incomparable with *ECΣ(X*) *(resp. δ*-*ßCΣ(X))*.

**(vi)-** *RΣ(X, T) is* incomparablewith *ECΣ(X) (resp. δ-ßCΣ(X)).*

**(vii)-** *θSΣ(X, T)* is incomparable with *ECΣ(X*).

**(viii)-** *SΣ(X, T)* is incomparable with *ECΣ(X*).

**(ix)-** *BΣ(X)* is incomparablewith *ECΣ(X).*

**(x)-** *ßΣ(X)* is incomparable with *ECΣ(X).*

**Theorem 2.1.1.39:** Let be a subspace of a space *X*, if *A* *ECΣ(X*) *(resp. A*  *δ*-*ßCΣ(X))* and such that *Y* is *Е-*(*resp*. *δ*-*ß*)-opеn, then *A* *ECΣ(Y*) *(resp. A*  *δ*-*ßCΣ(Y))*

***Proof:*** Let *A* *ECΣ(X*) *(resp. δ*-*ßCΣ(X)) A* *EΣ(X*) *(resp. δ*-*ßΣ(X)).* Sinceand *Y* is *Е-*(*resp*. *δ*-*ß*)-open, thus via (Lemma 1.14) *A* is *Е-*(*resp*. *δ*-*ß*)-open in subspace *Y,* as well: a closed set such that Since *F* is closed in subspace *Y.* Hence *A* *ECΣ(Y*) *(resp. A* *δ*-*ßCΣ(Y))*

**Theorem 2.1.1.40:** Let be a subspace of a space *X*, if *A* *ECΣ(Y*) *(resp. A*  *δ*-*ßCΣ(Y))* and and *Y* is clopen, then *A* *ECΣ(X*) *(resp. A*  *δ*-*ßCΣ(X))*

***Proof:*** Let *A* *ECΣ(Y*) *(resp. δ*-*ßCΣ(Y)) A* *EΣ(Y*) *(resp. δ*-*ßΣ(Y)),* and a closed set such that Since *Y* is clopen  *Y* is *EΣ(X*) *(resp. δ*-*ßΣ(X))* and since *A* *EΣ(Y*) *(resp. δ*-*ßΣ(Y)),* then via (Lemma 1.14), *A* *EΣ(X*) *(resp. δ*-*ßΣ(X)).* Moreover since *Y* is clopen *Y* is closed in *X* and since *F* is closed in *Y,* hence via (Theorem 1.16) *F* is closed set in *X.* So *A* *ECΣ(X*) *(resp. A* *δ*-*ßCΣ(X))*

**Corollary 2.1.1.41:** Let *A* and *Y* be any subsets of a space *X* such that and *Y* is clopen. Then *A* *ECΣ(Y*) *(resp. A* *δ*-*ßCΣ(Y))* if and only if *A* *ECΣ(X*) *(resp. A* *δ*-*ßCΣ(X))*

***Proof:*** The proof follows from Theorems (2.1.1.39) and (2.1.1.40).

**Corollary 2.1.1.42:** Let *A* and *Y* be any subsets of a space *X.* if *A* *ECΣ(X*) *(resp. A* *δ*-*ßCΣ(X))* and *Y* is *δ-*clopen subset of *X*. Then

***Proof:*** Let *ECΣ(X*) *(resp. δ*-*ßCΣ(X))* and *Y* is *δ-*clopen of *X*  *EΣ(X*) *(resp. δ*-*ßΣ(X))* and *Y* is *δ-*open and *δ-*closed of *X*, so via Lemma (1.13), Since *A* *ECΣ(X*) *(resp. δ*-*ßCΣ(X))* a closed set *F* in *X* such that thus,

**Theorem 2.1.1.43:** Let and be topological spaces and be the product topology, if *A* *ECΣ(X*) *(resp. A*  *δ*-*ßCΣ(X))* and *B* *ECΣ(Y*) *(resp. B*  *δ*-*ßCΣ(Y))* then, *ECΣ(*) *(resp.*  *δ*-*ßCΣ()).*

***Proof:*** Suppose that Since *ECΣ(X*) *(resp. δ*-*ßCΣ(X))EΣ(X*) *(resp. δ*-*ßΣ(X))* a closed set *F* in *X* such that . Since *ECΣ(Y*) *(resp. δ*-*ßCΣ(Y))EΣ(Y*) *(resp. δ*-*ßΣ(Y))* a closed set *E* in *Y* such that . So, and via(Lemma 1.15) *EΣ(*)*(resp.*  *δ*-*ßΣ()).*Since *F* and *E* are closed in spaces *X* and *Y* respectively, we get is closed in . Thus *ECΣ(*) *(resp.*  *δ*-*ßCΣ()).*

**2.1. 2. Some Fundamental Properties of *Еc* and *δ*-*ß*c-Open Sets**

Part two is devoted to present several topological properties concerning of *Еc* (*resp*. *δ*-*ß*c)-Neighborhood, *Еc* (*resp*. *δ*-*ß*c)-Interior, *Еc* (*resp*. *δ*-*ß*c)-Closure, *Еc* (*resp*. *δ*-*ß*c)-Derived and *Еc* (*resp*. *δ*-*ß*c)-Frontier of a sets via the notions of *Е*c-open and *δ-ßc*-opеn sets that are similar to those of open sets.

**Remarks 2.1.2.1:** The topological properties of *Еc* (*resp*. *δ*-*ß*c)-Neighborhood, *Еc* (*resp*. *δ*-*ß*c)-Interior, *Еc* (*resp*. *δ*-*ß*c)-Closure, *Еc* (*resp*. *δ*-*ß*c)-Derived and *Еc* (*resp*. *δ*-*ß*c)-Frontier are the same as in the supra-topology, therefore the proofs of the following properties are obvious and they follow from their respective definitions thus omitted.

**Definition 2.1.2.2:** Le*t (Х, T)* be a topological space. A subset of *X* is said to be *Еc* (*resp*. *δ*-*ß*c)-Neighborhood of a point *Еc* (*resp*. *δ*-*ß*c)-open set *V* in *X* such that

**Remarks 2.1.2.3:** Every *Еc* (*resp*. *δ*-*ß*c)-Neighborhood of is *Е* (*resp*. *δ*-*ß*)-Neighborhood, it follows from the fact every *Еc*(*resp*. *δ*-*ß*c)-open set is *Е*(*resp*. *δ*-*ß*)-open.

**Theorem 2.1.2.4:** For every two subsets *A, B* of a space *X* and , if *A* is a *Еc* (*resp*. *δ*-*ß*c)-Neighborhood of a point , then *B* is *Еc* (*resp*. *δ*-*ß*c)-Neighborhood of a point .

**Definition 2.1.2.5:** Le*t (Х, T)* be a topological space. A point is said to be *Еc* (*resp*. *δ*-*ß*c)-Interior point of a set *Еc* (*resp*. *δ*-*ß*c)-opеn set *V* in *X* such that The set of all *Еc* (*resp*. *δ*-*ß*c)-Interior points of *A* is called *Еc* (*resp*. *δ*-*ß*c)-Interior of *A* and is denoted by *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))*.

Several properties concerning of the *Еc* (*resp*. *δ*-*ß*c)-Interior of set are introduced in the following Theorem.

**Theorem 2.1.2.6:** Let *(Х, T)* be a topological space.For subsets *A and B* of *X* the following are hold:

**(i)-** *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* is the union of all *Еc*(*resp*. *δ*-*ß*c)-opеn sets which are contained in *A*.

**(ii)-** *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* is *Еc*(*resp*. *δ*-*ß*c)-opеn set in *X*.

**(iii)-** *A* is *Еc*(*resp*. *δ*-*ß*c)-opеn set *A = ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A)).*

**(iv)-** *ЕcInt(ЕcInt(A))* (*resp*. *δ*-*ß*c*Int(δ*-*ß*c*Int (A)) = ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A)).*

**(v)-** *ЕcInt()* (*resp*. *δ*-*ß*c*Int()) = and ЕcInt(X)* (*resp*. *δ*-*ß*c*Int(X)) =X.*

**(vi)-** *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A)).*

**(vii)-**If *A* then *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* *ЕcInt(B)* (*resp*. *δ*-*ß*c*Int(B)).*

**(viii)-** *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* *ЕcInt(B)* (*resp*. *δ*-*ß*c*Int(B)) ЕcInt* (*resp*. *δ*-*ß*c*Int*.

**(ix)-** *If ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* *ЕcInt(B)* (*resp*. *δ*-*ß*c*Int(B)).*

**(x)-** *ЕcInt*  (*resp*. *δ*-*ß*c*Int* *ЕcInt(A) ЕcInt(B)* (*resp*. *δ*-*ß*c*Int(A)* *δ*-*ß*c*Int(B)).*

**Definition 2.1.2.7:** Let *(Х, T)* be a topological space and . The *Еc* (*resp*. *δ*-*ß*c)-Closure of *A* in *X* is the set, *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))=* is *Еc* (*resp*. *δ*-*ß*c)-closed set and }.

**Theorem 2.1.2.8:** Let *A* be a subset of a space *X*. Then, *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* is the smallest *Еc* (*resp*. *δ*-*ß*c)-closed set containing *A*.

**Theorem 2.1.2.9:**  *A* subset *A* of a space *X* is *Еc* (*resp*. *δ*-*ß*c)-closed if and only if *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* = *A*.

**Theorem 2.1.2.10:** For a subset *A* of a space *X* and . The following are equivalent:

**(i)-**For every *Еc* (*resp*. *δ*-*ß*c)-opеn set *U* in *X* such that we get,

**(ii)-** *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A)).*

***Proof:*** **(i) (ii)** Suppose that *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* *Еc* (*resp*. *δ*-*ß*c)-closed set *F* such that and So and is *Еc* (*resp*. *δ*-*ß*c)-opеn set and therefore, which is a contradiction and therefore, *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A)).*

**(ii) (i)** Assume that there exists *Еc* (*resp*. *δ*-*ß*c)-open set U containing *x* with Then and is *Еc* (*resp*. *δ*-*ß*c)-closed set. Hence *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A)),* which is a contradiction.

Some properties of *Еc* (*resp*. *δ*-*ß*c)-closure of sets are introduced in the following Theorem.

**Theorem 2.1.2.11:** Let *(Х, T)* be a topological space.For subsets *A* and *B* of *X* the following properties hold:

**(i)** *A*  *Еc-Cl(A)* (*resp*. *A* *δ*-*ß*c*-Cl(A)).*

**(ii)** *Еc-Cl()* (*resp*. *δ*-*ß*c*-Cl()) = and Еc-Cl(X)* (*resp*. *δ*-*ß*c*-Cl(X)) =X.*

**(iii)**If *A* then *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))*  *Еc-Cl(B)* (*resp*. *δ*-*ß*c*-Cl(B)).*

**(iv)** *Еc-Cl(Еc-Cl(A))* (*resp*. *δ*-*ß*c*-Cl(δ*-*ß*c*-Cl(A)) = Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A)).*

**(v)**If *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* *Еc-Cl(B)* (*resp*. *δ*-*ß*c*-Cl(B)).*

**(vi)** *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* *Еc-Cl(B)* (*resp*. *δ*-*ß*c*-Cl(B))*  *Еc-Cl* (*resp*. *δ*-*ß*c*-Cl*.

**(vii)** *Еc-Cl* (*resp*. *δ*-*ß*c*-Cl* *Еc-Cl(A) Еc-Cl(B)* (*resp*. *δ*-*ß*c*-Cl(A)* *δ*-*ß*c*-Cl(B)).*

The relations between the *Еc* (resp. *δ-ßc*)-closure and *Еc* (*resp*. *δ*-*ß*c)-Interior for a subset *A* of *X* can be considered in the following Theorem.

**Theorem 2.1.2.12:** Let *(Х, T)* be a topological space.For subset *A* of *X* the following statements hold:

**(i)-**[*Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))*]c = *ЕcInt(A*c*)* (*resp*. *δ*-*ß*c*Int(A*c*))*.

**(ii***)-* [*ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))*]c = *Еc-Cl(A*c*)* (*resp*. *δ*-*ß*c*-Cl(Ac)).*

**(iii)-** *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* = [*ЕcInt(Ac)* (*resp*. *δ*-*ß*c*Int(Ac))*]c.

**(iv)-** *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* = [*Еc-Cl(Ac)* (*resp*. *δ*-*ß*c*-Cl(Ac))*]c.

**Definition 2.1.2.13:** Let *A* be a subset of a space *X*. A point is said to be *Еc* (*resp*. *δ*-*ß*c)-Limit point of *A* if for each *Еc* (*resp*. *δ*-*ß*c)-open subset *U* of *X* containing , the set of all *Еc* (*resp*. *δ*-*ß*c)-Limit points of *A* is called the *Еc* (*resp*. *δ*-*ß*c)-Derived set of *A* and its denoted by *Еc-D(A)* (*resp*. *δ*-*ß*c*-D(A))*.

**Proposition 2.1.2.14:** Let *A* be a subset of a space *X*. If for each closed set containing *x* satisfies then the point is *Еc* (*resp*. *δ*-*ß*c)-Limit point of *A*.

Several properties concerning of the *Еc* (*resp*. *δ*-*ß*c)-Derived set are explained in the following Theorem.

**Theorem 2.1.2.15:** For any subsets *A and B* of a space *X* the following properties hold:

**(i)-**If *A* then *Еc-D(A)* (*resp*. *δ*-*ß*c*-D(A))* *Еc-D(B)* (*resp*. *δ*-*ß*c*-D(B))*.

**(ii)-** *Еc-D()* (*resp*. *δ*-*ß*c*-D())* *=*

**(iii)-** If *Еc-D(A)* (*resp*. *δ*-*ß*c*-D(A))* *Еc-D(A\{x})* (*resp*. *δ*-*ß*c*-D(A\{x})).*

**(iv)-** *Еc-D(A)* (*resp*. *δ*-*ß*c*-D(A))Еc-D(B)*(*resp*. *δ*-*ß*c*-D(B))Еc-D*(*resp*. *δ*-*ß*c*-D*.

**(v)-** *Еc-D* (*resp*. *δ*-*ß*c*-D* *Еc-D (A) Еc-D(B)* (*resp*. *δ*-*ß*c*-D (A)* *δ*-*ß*c*-D(B)).*

**(vi)-** *Еc-D(Еc-D(A))* (*resp*. *δ*-*ß*c*-D(δ*-*ß*c*-D(A))\A Еc-D(A)* (*resp*. *δ*-*ß*c*-D(A)).*

**(vii)-** *Еc-D(A Еc-D(A)) A Еc-D(A).*

**(viii)-** *δ*-*ß*c*-D(A δ*-*ß*c*-D(A)) A δ*-*ß*c*-D(A).*

**Proposition 2.1.2.16:** For any subset *A* of a space *X,* the following statements hold:

**(i)-** *Еc-D(A)* (*resp*. *δ*-*ß*c*-D(A)) Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A)).*

**(ii)-** *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))= A Еc-D(A)* (*resp*. *A δ*-*ß*c*-D(A)).*

**Theorem 2.1.2.17:** Let *A* be subset of a space *X*. Then, *A* is *Еc* (*resp*. *δ*-*ß*c)-closed set if and only ifit contains all of its *Еc* (*resp*. *δ*-*ß*c)-Limit points.

***Proof:*** Assume that *A* *Еc* (*resp*. *δ*-*ß*c)-closed set and is *Еc* (*resp*. *δ*-*ß*c)-limit point such that, . Then, where is *Еc* (*resp*. *δ*-*ß*c)-open set and so, there exists *Еc* (*resp*. *δ*-*ß*c)-open set *G* such that. So, and this is a contradiction. **Conversely,** let *A* contains all *Еc* (*resp*. *δ*-*ß*c)-limit points, then for every we have, is not *Еc* (*resp*. *δ*-*ß*c)-limit point of *A* which implies, for any *Еc* (*resp*. *δ*-*ß*c)-open set *G* containing ,. But which implies, and so, . This proves that is *Еc* (*resp*. *δ*-*ß*c)-open set and hence, *A* is *Еc* (*resp*. *δ*-*ß*c)-closed set.

**Definition 2.1.2.18:** Let *A* be a subset of a topological space *(X, T)*. The *Еc* (*resp*. *δ*-*ß*c)-Frontier of *A*, denoted by *Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A))* is defined by:

*Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A)) = Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))\ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))*.

**Theorem 2.1.2.19:** For any subset *A* of a space *X,* the following properties hold:

**(i)-** *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))= ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A)) Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A)).*

**(ii)-** *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A)) Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A))*

**(iii)-** *Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A))= Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A)) Еc-Cl(X\A)* (*resp*. *δ*-*ß*c*-Cl(X\A)).*

**Corollary 2.1.2.20:** For every subset *A* of the space *X*, *Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A))* is *Еc* (*resp*. *δ*-*ß*c)-closed set.

Now, we give new equivalent definitions for *Еc* (*resp*. *δ*-*ß*c)-open sets and *Еc* (*resp*. *δ*-*ß*c)-closed sets by using the concept of *Еc* (*resp*. *δ*-*ß*c)-Frontier in the following Theorem.

**Theorem 2.1.2.21:** The followingproperties hold for a subset *A* of a topological space *(X, T):*

**(i)-** *A* is *Еc* (*resp*. *δ*-*ß*c)-open *A* *Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A))*

**(ii)-** *A is Еc (resp. δ-ßc)-closed Еc-Fr(A)(resp. δ-ßc-Fr(A)) A.*

**(iii)-** *A is both Еc (resp. δ-ßc)-open & Еc (resp. δ-ßc)-closed Еc-Fr(A)(resp. δ-ßc-Fr(A))=*

***Proof*: (i) -** Assume that *A* is *Еc* (*resp*. *δ*-*ß*c)-open set, then from (Theorem 2.1.2.6-part (iii)) we get *A = ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))*, also via (Theorem 2.1.2.19), we have, *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A)) Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A))*; *A Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A))* .

***Conversely,*** suppose that *A Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A))* Then,

*A* [ *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))\ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))*] = *A* [ *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))*] *A\*[ *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))*] = *A\ ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* . Hence *A* = *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))*. Thus *A* is *Еc* (*resp*. *δ*-*ß*c)-open set.

**(ii) -** Suppose that *A* is *Еc* (*resp*. *δ*-*ß*c)-closed set, then from (Theorem 2.1.2.9), *A = Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* and thus, *Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A)) = Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))\ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* = *A\ ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* and so, *Еc-Fr(A)(resp. δ-ßc-Fr(A)) A.*

***Conversely,*** let *Еc-Fr(A)(resp. δ-ßc-Fr(A)) A,* thus

*Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))=* *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* *Еc-Fr(A)(resp. δ-ßc-Fr(A))*  *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* *A* = *A,* as well we have by(Theorem 2.1.2.11-part (i)) *A*  *Еc-Cl(A)* (*resp*. *A* *δ*-*ß*c*-Cl(A)).*Therefore, *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))=A* and via (Theorem 2.1.2. 9), we get *A is Еc(resp. δ-ßc)-*closed set.

**(iii) -** suppose that *A* is both *Еc (resp. δ-ßc)-*open and *Еc (resp. δ-ßc)-*closed. Therefore viaTheorems (2.1.2.6 and 2.1.2.9) we have, *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* = *A* = *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* and by definition of  *Еc* (*resp*. *δ*-*ß*c)-Frontier of *A*, we get *Еc-Fr(A)(resp*. *δ*-*ß*c*-Fr(A)) =A\A* On the other hand, assume *Еc-Fr(A)(resp. δ-ßc-Fr(A))=,* which means, *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))\ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* also in other words, *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A)) = ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A)).* Butfrom (Theorem 2.1.2.6-part (vi)) and (Theorem 2.1.2.11-part (i)) we have, *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A)) and A*  *Еc-Cl(A)* (*resp*. *A* *δ*-*ß*c*-Cl(A)).* Therefore, it follows..… *ЕcInt(A)* (*resp*. *δ*-*ß*c*Int(A))* = *A* = *Еc-Cl(A)* (*resp*. *δ*-*ß*c*-Cl(A))* which means that *A is both Еc (resp. δ-ßc)-*openand *Еc (resp. δ-ßc)-*closed set.

**2.2. On *Еc*-Continuous and *δ*-*ß*c-Continuous Functions in Topological Spaces**

In recent years, Alias. B. Khalaf, Zanyar A. Ameen, in [**4**] introduced a new class of sets, called Sc-open sets, and investigated some properties of Sc-continuity, and in [**5**] Zanyar A. Ameen, introduced a new class of continuity namely *Pc*-continuous functions, and investigated some properties of *Pc*-continuity. As well, Ayman. Y. Mizyed, in [**6**] studied a new class of continuity, called *ß*c-continuous functions as a new class of continuous functions and gave several characterizations of these functions.

In this section, new notions of generalized continuous functions in a topological space, called *Еc*-continuous and *δ*-*ß*c-continuous functions are introduced and investigated by using generalized of open sets which are introduced in section one of this chapter. The notion of *Еc*-continuous functions is contained in the notions of *Е*-continuous and *δ*-*ß*c-continuous functions and properly contains the class of *Pc*-continuous functions. As well the notion of *δ*-*ß*c-continuous functions is contained in the class of *δ*-*ß*-continuous and properly contains both the class of *Еc*-continuous and the class of *ß*c-continuous functions. Several characterizations and fundamеntal propеrties concerning of thеse forms of generalized continuous functions are oƅtainеd, as wеll we invеstigated the graphs of *Еc*-continuous and *δ*-*ß*c-continuous functions. Furthermore, the rеlationships among *Еc*-continuous and *δ*-*ß*c-continuous functions and othеr wеll-known forms of generalized continuous functions are also discussеd.

This section consists of two main parts as follows:

**2.2.1. Characterizations of *Еc* and *δ*-*ß*c -Continuous Functions**

Some characterizations concerning of these types of continuous functions are obtained in this part. Moreover, the relationships among *Еc* (*δ*-*ß*c)-continuous functions and other well-known forms of generalized continuous functions are also discussed.

**Definition 2.2.1.1:** Let *(X, T)* and *(Y, T\*)* be two topological spaces. The function *f*: *X → Y* is called *Еc* (*δ*-*ß*c)-continuous function at a point if open set *V* of *Y* containing *f ()*, an *Еc* (*resp*. *δ*-*ß*c)-open set *U* of *X* containing *x* such that *f (U)* *V* . If *f* is *Еc* (*δ*-*ß*c)-continuous at each point , then it is called *Еc* (*δ*-*ß*c)-continuous.

**Definition 2.2.1.2:** A function *f*: *(X, T) → (Y, T\*)* is said to be:

**(i)-** *E*-continuous [**40**], if *f* −1(*V*) is *E*-open in *X* for every open subset *V* of *Y*.

**(ii)-** *δ*-*ß*-continuous [**46**], if *f* −1(*V*) is *δ*-*ß*-open in *X* for every open subset *V* of *Y*.

**(iii)-**Perfectly (clopen)-continuous [**51**], if *f* −1(*V*) is clopen in *X* for every open subset *V* of *Y*.

**(iv)-**Contra-continuous [**52**], if *f* −1(*V*) is closed in *X* for every open subset *V* of *Y*.

**(v)-**Strongly θ-continuous [**53**], if *f* −1(*V*) is θ-open in *X* for every open subset *V* of *Y*.

**(vi)-** *RC*-continuous [**54**], if *f* −1(*V*) is regular closed in *X* for every open subset *V* of *Y*.

**(vii)-**Semi-continuous [**12**], if *f* −1(*V*) is semi-open set in *X* for every open subset *V* of *Y*.

**(viii)-**θS-continuous [**55**], if *f* −1(*V*) is θ-semi open set in *X* for every open subset *V* of *Y*.

**(ix)-**Pre-continuous [**14**], if *f* −1(*V*) is Pre-open set in *X* for every open subset *V* of *Y*.

**(x)-** *Sc*-continuous [**4**], if *f* −1(*V*) is *Sc*-open set in *X* for every open subset *V* of *Y*.

**(xi)-** *Pc*-continuous [**5**], if *f* −1(*V*) is *Pc*-open set in *X* for every open subset *V* of *Y*.

**(xii)-** *ß*c-continuous [**6**], if *f* −1(*V*) is *ß*c-open set in *X* for every open subset *V* of *Y*.

**(xiii)-** *ß*-continuous [**56**], if *f* −1(*V*) is *ß*-open set in *X* for every open subset *V* of *Y*.

**Theorem 2.2.1.3:** A function *f*: *(X, T) → (Y, T\*)* is *Еc* (*δ*-*ß*c)-continuous if and only if the inverse image of every open set in *Y* is *Еc*(*δ*-*ß*c)-open set in *X*.

***Proof*:** Let *f* be *Еc* (*δ*-*ß*c)-continuous function and *V* be an open set in *Y*. there are two cases: First if *f* −1(*V*) *f* −1(*V*) is *Еc* (*δ*-*ß*c)-open set in *X*. Second if *f* −1(*V*) Thus by *Еc*(*δ*-*ß*c)-continuity, there exists *Еc*(*δ*-*ß*c)-open set *U* *X* containing *x* such that *f*(*U*) *V*. So, , and hence, via (Theorem 2.1.1.17) we get *f* −1(*V*) is *Еc* (*δ*-*ß*c)-open set in *X*.

Suppose that the inverse image of every open subset in *Y* is *Еc* (*δ*-*ß*c)-open set in *X* and let *V* be an open set in *Y* containing *f().* Then, which is *Еc* (*δ*-*ß*c)-open via our supposition and *f ( f* −1(*V*)) *V* . Therefore, *f* is *Еc* (*δ*-*ß*c)-continuous functions.

**Corollaries 2.2.1.4:** Let *(Х, T)* be a topological space.Then the following are hold:

**(i)-**Every *Pc-*continuous function is *Еc-*continuous.

**(ii)-**Every *Еc*-continuous function is *Е*-continuous.

**(iii)-**Every *Е*-continuous function is *δ-ß-*continuous.

**(iv)-** Every *Еc*-continuous function is *δ*-*ß*c-continuous.

**(v)-**Every *S*c-continuous function is *ß*c-continuous.

**(vi)-**Every *ßc*-continuous function is *ß-*continuous.

**(vii)-**Every *ßc-*continuous function is *δ-ßc-*continuous.

**(viii)-**Every *δ*-*ß*c-continuous function is *δ*-*ß*-continuous.

***Proof*:** The proof of above corollaries is obvious it follows their respective definitions.

**Remark 2.2.1.5:** From the respective definitions, the relationships among *Еc* (*δ*-*ß*c)-continuous functions and various other well-known forms of generalized continuous functions are shown in the following figure:

***Pc -Continuous***

***θ-Semi-Continuous***

***Sc –Continuous***

***Semi-Continuous***

***ßc-Continuous***

***ß-Continuous***

***Pre-Continuous***

***Ec -Continuous***

***δ-ß-Continuous***

***δ-ßc-Continuous***

***E-Continuous***

***St. θ-Continuous***

**Figure: (2.3). The relationships among *Еc-*(*δ*-*ß*c)-Continuous functions** **and various other well-known forms of generalized continuous functions**

However none of these implications is reversible as shown via examples of [**4**, **5**, **6**] and the following examples:

**Example 2.2.1.6:** Let *X = Y= {a, b, c, d},* define a topology *T = {, X, {a}, {b}, {a, b}}* on *X* and a topology *T\* = {, Y, {c}, {d}, {c, d}}* on *Y* and let *f*: *(X, T) → (Y, T\*)* be function defined as follows: *f (a) = d, f (b) = c, f (c) = b and f(d) = a*. Then

**(1)-** *f* is *Е*-continuous function but not *Еc*-continuous function because *{c}* is an open set in *Y* but *f -1({c}) = {b}* which is not *Еc*-open in *X*.

**(2)-** *f* is *δ*-*ß*-continuous function but not *δ*-*ß*c-continuous function because *{d}* is an open set in *Y* but *f -1*({d}) = {a} which is not *δ*-*ß*c-open in *X*.

**Example 2.2.1.7:** Let *X =* {*a, b, c}* and define a topology T = *{, X, {a}, {b}, {a, b}}* on *X.* Then the identity function *f*: *(X, T) → (X, T)* is *δ*-*ß*-continuous function but not *Еc*-continuous function because *{b}* is an open set in *X* but *f -1({b}) = {b}* which is not *Еc*-open in *X*.

**Example 2.2.1.8:** Let *X = {, y, w, z}* and *Y = {a, b, c, d},* define a topology *T = {, X, {}, {w}, {, y},{, w},{, y, w},{, w, z}}* on *X* and a topology *T\* = {, Y, {a, b}}* on *Y* and let *f*: *(X, T) → (Y, T\*)* be functions defined as follows: *f () = c, f (y) = a, f (w) = d and f(z) = b*. Then

**(1)-** *f* is *δ*-*ß*-continuous function but not *Е*-continuous function because *{a, b}* is an open set in *Y* but *f -1({a, b}) = {y, z}* which is not *Е*-open in *X*.

**(2)-** *f* is *δ*-*ß*c-continuous function but not *Еc*-continuous function because *{a, b}* is an open set in *Y* but *f -1({a, b}) = {y, z}* which is not *Еc*-open in *X*.

**(3)-** *f* is *δ*-*ß*c-continuous function but not *ß*c-continuous function because *{a, b}* is an open set in *Y* but *f -1({a, b}) = {y, z}* which is not *ß*c-open in *X*.

**Example 2.2.1.9:** Let *X = {a, b, c}* and *Y = {, y, z},* define a topology *T = {, X, {a}, {b}, {a, b}}* on *X* and a topology *T\* = {, Y, { ϰ, y}}* on *Y* and let *f*: *(X, T) → (Y, T\*)* be function defined as follows: *f (a) = z, f (b) = , and f(c) = y*. Then *f* is *Еc*-continuous function but not *Pc*-continuous function because *{ϰ, y}* is an open set in *Y* but *f -1({ϰ, y}) = {b, c}* which is not *Pc*-open in *X*.

**Theorem 2.2.1.10:** Let *f*: *(X, T) → (Y, T\*)* be a function whenever *X* is *T1* space. Then, *f* is *Еc* (*δ*-*ß*c)-continuous function *if and only if* *f* is *Е* (*δ*-*ß*)-continuous function.

***Proof*:** Suppose that *f* is *Е*(*δ*-*ß*)-continuous function whenever *X* is *T1* space, then for any open set *V* of *Y* we have, *f* −1(*V*) is *Е* (*resp*. *δ*-*ß*)-open set in *X,* and via (Theorem 2.1.1.10), *f* −1(*V*) is *Еc* (*δ*-*ß*c)-open set. Therefore, *f* is *Еc*(*δ*-*ß*c)-continuous function.

Assume that *f* is *Еc* (*δ*-*ß*c)-continuous function, therefore via (Corollary 2.2.1.4-part (ii) and (viii)), we have *f* is *Е* (*δ*-*ß*)-continuous function.

**Corollary 2.2.1.11:** Let *f*: *(X, T) → (Y, T\*)* be continuous function whenever *X* is *T1* space. Then, *f* is *Еc* (*δ*-*ß*c)-Continuous function.

***Proof*:** The proof is obvious it follows directly from(Remark 2.1.1.18).

**Theorem 2.2.1.12:** A function *f*: *(X, T) → (Y, T\*)* is *Еc* (*δ*-*ß*c)-continuous if and only if *f* is *Е* (*δ*-*ß*)-continuous function and for each and each open set *V* of *Y* such that *f()* *V*, there exists a closed set *F* *X* containing such that *f* (*F*) *V*.

***Proof*:** Let *f*: *(X, T) → (Y, T\*)* be *Еc* and *δ*-*ß*c-continuous function, then *f* is *Е* (*δ*-*ß*)-continuous, and also let and *V* be any open set of *Y* containing *f ().* Then via *Еc* (*δ*-*ß*c)-continuity, there exists *Еc* (*δ*-*ß*c)-open set *U* *X* containing such that *f* (*U*) *V*. Since *U* is *Еc* (*δ*-*ß*c)-open set. Then for each , there exists a closed set *F* of *X* such that Therefore, *f* (*F*) *f* (*U*) *V* and hence, we get *f* (*F*) *V*. And *Еc* (*δ*-*ß*c)-Continuous always implies *Е* (*δ*-*ß*)-continuous.

Let *V* be any open set of *Y*. We have to show that *f* −1(*V*) is *Еc*(*δ*-*ß*c)-open set in *X*. by hypothesis since *f* is *Е*(*δ*-*ß*)-continuous, then *f* −1(*V*) is *Е*(*δ*-*ß*)-open set in *X* and for any we have, *f()* *V.* By hypothesis, there exists a closed set *F* of *X* containing such that, *f* (*F*) Therefore, via (Definition 2.1.1.2), *f* −1(*V*) is  *Еc* (*δ*-*ß*c)-open set in *X* and thus, via (Theorem 2.2.1.3), *f* is *Е c*(*δ*-*ß*c)-continuous function .

Some characterizations concerning of the *Еc* (*resp*. *δ*-*ß*c)-continuous function are explained in the following Theorem.

**Theorem 2.2.1.13:** The following characterizations are equivalent for a function *f*: *(X, T) → (Y, T\*):*

**(i)-** *f* is *Еc* (*δ*-*ß*c)-continuous function .

**(ii)-** *f* −1(*V*) is *Еc*(*δ*-*ß*c)-open set in *X,* for every open set *V* of *Y*.

**(iii)-** *f* −1(*F*) is *Еc*(*δ*-*ß*c)-closed set in *X,* for every closed set *F* of *Y*.

**(iv)-** *f* (*Еc-Cl(A))* (*resp*. *δ*-*ß*c*-Cl(A)))* *Cl(f(A))*(*resp*. *Cl(f(A))),* for every subset *A* of *X*.

**(v)-** *Еc-Cl(f* −1(*B))* (*resp*. *δ*-*ß*c*-Cl(f* −1(*B))) f* −1(*Cl(B))* (*resp*. *f* −1(*Cl(B))),* for every subset *B* of *Y*.

**(vi)-** *f* −1(*Int(B))* *Еc-Int (f* −1(*B))* (*resp*. *δ*-*ß*c*-Int (f* −1(*B)))* for every subset *B* of *Y*.

**(vii)-** *Int(f(A))*  *f* (*Еc- Int(A))* (*resp*. *δ*-*ß*c*- Int(A)))* for every subset *A* of *X*.

***Proof*:** It is straightforward and obvious thus omitted.

**Theorem 2.2.1.14:** If *f*: *(X, T) → (Y, T\*)* is a strongly θ-continuous function, then *f* is *Еc* (*δ*-*ß*c)-continuous function.

***Proof*:** Suppose that *f* is a strongly θ-continuous function and *V* an open subset of *Y*, then via (Definition 2.2.1.2-part (*v*)), *f* −1(*V*) is θ-open set. Since every θ-open is *Еc* (*δ*-*ß*c)-open set hence, via (Theorem 2.1.1.21), *f* −1(*V*) is *Еc* (*δ*-*ß*c)-open set in *X*. Therefore, via (Theorem 2.2.1.3), *f* is *Еc* (*δ*-*ß*c)-continuous function.

**Theorem 2.2.1.15:** If *f*: *(X, T) → (Y, T\*)* is contra continuous and *Е* (*δ*-*ß*)-continuous function, then *f* is *Еc* (*δ*-*ß*c)-Continuous function.

***Proof*:** Assume that *f* is contra continuous and *Е* (*δ*-*ß*)-continuous function and consider *V* as open subset of *Y*, then via (Definition 2.2.1.2-part (*i, ii, iv*)), *f* −1(*V*) is closed and *Е* (*δ*-*ß*)-open set in *X* which implies, *f* −1(*V*) is *Еc*(*δ*-*ß*c)-open set thus, *f* is *Еc* (*δ*-*ß*c)-continuous function.

**Theorem 2.2.1.16:** Let *f*: *(X, T) → (Y, T\*)* be a perfectly (clopen)-continuous function, then *f* is *Еc* (*δ*-*ß*c)-continuous function.

***Proof*:** Assume that *f* is perfectly continuous function and let *V* be an open subset of *Y*, then via (Definition 2.2.1.2-part (*iii*)), *f* −1(*V*) is clopen set in *X* which implies, *f* −1(*V*) isopen set and so, *Е*(*δ*-*ß*)-open set and *f* −1(*V*) is closed set. Thus, *f* −1(*V*) is *Еc* (*δ*-*ß*c)-open set in *X* and so, *f* is *Еc*(*δ*-*ß*c)-continuous function.

**Theorem 2.2.1.17:** Let *(X, T)* be an Alexandroff space. Then, a function *f*: *(X, T) → (Y, T\*)* is a contra continuous and *Е* (*δ*-*ß*)-continuous function if and only if is *Еc* (*δ*-*ß*c)-continuous function.

***Proof*:**  Assume that *(X, T)* is an Alexandroff space and let *f*: *(X, T) → (Y, T\*)* is *Еc*(*δ*-*ß*c)-continuous function . Then *f* −1(*V*) is *Еc* (*δ*-*ß*c)-open set in *X,* for every open set *V* of *Y*, which implies *f* −1(*V*) is *Е*(*δ*-*ß*)-open set in *X,* and *f* −1(*V*) is a union of closed sets which is closed in an Alexandroff space *(X, T)*. So, *f* is *Е* (*δ*-*ß*)-continuous and contra continuous function.

The converse part follows immediately from(Theorem 2.2.1.15).

**Theorem 2.2.1.18:** Let *f*: *(X, T) → (Y, T\*)* be *RC*-continuous function, then *f* is *δ*-*ß*c-continuous function.

***Proof*:** suppose that *f* is *RC*-continuous function and let *V* be an open subset of *Y*, then via (Definition 2.2.1.2-part (*vi*)), *f* −1(*V*) is regular closed in *X* and via **(**Theorem 2.1.1.23), we have *f* −1(*V*) is (*δ*-*ß*c)-open set. So *f* is *δ*-*ß*c-continuous function.

**Theorem 2.2.1.19:** Let *f*: *(X, T) → (Y, T\*)* be a continuous function, such that *(X, T)* is a regular space. Then, *f* is *Еc* (*δ*-*ß*c)-continuous function.

***Proof*:** Let *f* be continuous function and *(X, T)* is regular space. Then *f* −1(*V*) is open set in a regular space *X* for any open subset *V* of *Y* and therefore, via (Theorem 2.1.1.25), *f* −1(*V*) is *Еc* (*δ*-*ß*c)-open set in *X*. Hence *f* is *Еc* (*δ*-*ß*c)-continuous function.

**2.2. 2. Fundamental Properties of *Еc* and*δ*-*ß*c -Continuous Functions**

In this part, we recall several fundamental properties concerning of the *Еc* (*resp*. *δ*-*ß*c)-continuous function.

**Theorem 2.2.2.1:** Let *f*: *(X, T) → (Y, T\*)* be *Еc* (*δ*-*ß*c)-continuous function such that *Y* *Z*. If *(Y, T\*)* is an open subspace of the topological space *(Z, T\*\*)*, then *f*: *(X, T) → (Z, T\*\*)* is *Еc* (*δ*-*ß*c)-continuous function.

***Proof*:** Let *V* be an open subset of *Z.* Then is open subset of *Y*. Since *f*: *(X, T) → (Y, T\*)* is *Еc* (*δ*-*ß*c)-continuous, via(Theorem 2.2.1.3), is *Еc* (*δ*-*ß*c)-open subset of *X*. Since hence is *Еc* (*δ*-*ß*c)-open subset of *X*. So via (Theorem 2.2.1.3), *f*: *(X, T) → (Z, T\*\*)*is *Еc* (*δ*-*ß*c)-continuous function.

**Theorem 2.2.2.2:** The following properties are equivalent for a function *f*: *(X, T) → (Y, T\*):*

**(i)-** *f* is clopen-continuous function.

**(ii)-** *f* is *Еc*(*δ*-*ß*c)-continuous and contra-continuous function.

**(iii)-** *f* is *Е-*(*δ*-*ß*)-continuous and contra-continuous function.

***Proof*:** The proof followed from the (Theorem 2.1.1.37) immediately**.**

**In the following results,** we recall some conditions in which the restrictions of *Еc* (*δ*-*ß*c)-continuous functions on subspaces are *Еc* (*δ*-*ß*c)-continuous.

**Theorem 2.2.2.3:** Let *f*: *(X, T) → (Y, T\*)* be *Еc* (*δ*-*ß*c)-continuous function. If *A* is a clopen subset of *X*, then *f |A*: *A* *→ Y* is *Еc* (*δ*-*ß*c)-continuous in the subspace *A*.

***Proof*:** Let *V* be an open subset of *Y*. Then via(Theorem 2.2.1.3), *f* −1(*V*) is *Еc*(*δ*-*ß*c)-open subset of *X*. Since *A* is clopen subset of *X*, then via (Corollary 2.1.1.42), (*f* ***|****A*) -1*(V)* = *f* −1(*V*) *A* is *Еc* (*δ*-*ß*c)-open subset in the subspace of *A*. This shows that, *f |A*: *A* *→ Y* is *Еc* (*δ*-*ß*c)-continuous in the subspace *A*.

**Theorem 2.2.2.4:** A function *f*: *(X, T) → (Y, T\*)* is *Еc* (*δ*-*ß*c)-continuous. If , there exists a clopen set *A* of *X* containing such that *f |A*: *A* *→ Y* is *Еc* (*δ*-*ß*c)-continuous.

***Proof*:** Suppose that for each, there exists a clopen set *A* of *X* containing where *f* ***|****A*: *A* *→ Y* is *Еc* (*δ*-*ß*c)-continuous function. Let *V* be any open subset of *Y* containing *f* (), then there exists a *Еc* (*δ*-*ß*c)-open set *U* in *A* containing such that, *f* ***|****A(U)* *V*. Since *A* is clopen set, via (Theorem 2.1.1.40), *U* is *Еc*(*δ*-*ß*c)-open set in *X* and hence *f(U)* *V* . This shows that *f*: *(X, T) → (Y, T\*)* is *Еc* (*δ*-*ß*c)-continuous.

**Theorem 2.2.2.5:** Let *f*: *(X, T) → (Y, T\*)* be afunction where *X = A B* such that both *A* and *B* are clopen sets. If *f |A*: *A* *→ Y* and *f |B*: *B* *→ Y* are *Еc* (*δ*-*ß*c)-continuous functions, then *f* is *Еc* (*δ*-*ß*c)-continuous function.

***Proof*:** Suppose that *V* is an open subset of *Y*. then *f* −1(*V*) = (*f* ***|****A*) -1*(V)* (*f* ***|****B*) -1*(V)*. Since *f* ***|****A* and *f* ***|****B* are *Еc* (*δ*-*ß*c)-continuous, so via (Theorem 2.2.1.3), (*f* ***|****A*) -1*(V)* and (*f* ***|****B*) -1*(V)* are *Еc*(*δ*-*ß*c)-open sets in *A* and *B* respectively. Since *A* and *B* are clopen sets in *X*, so via (Theorem 2.1.1.40), (*f |A*) -1*(V)* and (*f |B*) -1*(V)* are *Еc* (*δ*-*ß*c)-open sets in *X*. Since the union of *Еc* (*δ*-*ß*c)-open sets is *Еc* (*δ*-*ß*c)-open, thus *f* −1(*V*) is *Еc*(*δ*-*ß*c)-open in *X*. Therefore, via (Theorem 2.2.1.3), *f* is *Еc* (*δ*-*ß*c)-continuous function.

**Theorem 2.2.2.6:** Let *f*, *g*: *(X, T) → (Y, T\*)* be two functions where *(Y, T\*)* is *Hausdorff*-space. If *f* is *Еc* (*δ*-*ß*c)-continuous and *g* is perfectly (clopen) continuous, then the set *E* = { : *f ()* = *g()*} is *Еc* (*δ*-*ß*c)-closed in *X*.

***Proof*:** Let Then *f ()* *g()*, since *(Y, T\*)* is Hausdorff, so there exist disjoint open sets *V1* and *V2* of *Y* such that . Since *f* is *Еc* (*δ*-*ß*c)-continuous, there exists *Еc* (*δ*-*ß*c)-open set *U1* of *X* containing *x* such that *f(U1)* *V1*. Since *g* is perfectly continuous, there exists clopen set *U2* of *X* containing such that *g(U2)* *V2*. Put *U = U1 U2* is *Еc*(*δ*-*ß*c)-open set of *X* containing via (Theorem 2.1.1.31). It follows that So that, *U* *X****|****E* and hence, *X|E* is *Еc* (*δ*-*ß*c)-open set. Therefore *E* is *Еc* (*δ*-*ß*c)-closed set in *X*.

**Corollary 2.2.2.7:** Let *f*, *g*: *(X, T) → (Y, T\*)* be two functions where *(Y, T\*)* is Urysohn space. If *f* is *Еc* (*δ*-*ß*c)-continuous and *g* is perfectly (clopen) continuous, then the set *E* = { : *f ()* = *g()*} is *Еc* (*δ*-*ß*c)-closed in *X*.

***Proof*:** The proof is obvious it follows immediately from (Proposition 1.18) and (Theorem 2.2.2.6).

**Theorem 2.2.2.8:** Let *f*: *(X1, T) → (Y, T\*)* and *g*: *(X2, T) → (Y, T\*)* be two *Еc* (*δ*-*ß*c)-continuous functions. If *(Y, T\*)* is Hausdorff-space, then the set *E* = is *Еc* (*δ*-*ß*c)-closed in .

***Proof*:** Let Then , since *(Y, T\*)* is Hausdorff, so there exist disjoint open sets *V1* and *V2* of *Y* such that . Since *f* and *g* are *Еc* (*δ*-*ß*c)-continuous functions, so there exists *Еc*(*δ*-*ß*c)-open sets *U1* and *U2* of *X1*and *X2* containing *ϰ1* and *2* such that *f (U1)* *V1* and *f (U2)* *V2* respectively.

Put , Thus via(Theorem 2.1.1.43), where is *Еc*(*δ*-*ß*c)-open set in and It follows that Therefore, we obtain and hence,is *Еc* (*δ*-*ß*c)-open set. Therefore *E* is *Еc* (*δ*-*ß*c)-closed set in

**Corollary 2.2.2.9:** Let *f*: *(X1, T) → (Y, T\*)* and *g*: *(X2, T) → (Y, T\*)* be two *Еc* (*δ*-*ß*c)-continuous functions. If *(Y, T\*)* is Urysohn space, then the set *E* = is *Еc* (*δ*-*ß*c)-closed in .

***Proof*:** The proof is obvious it follows immediately from **(**Proposition 1.18) and (Theorem 2.2.2.8).

**Theorem 2.2.2.10:** If *fi*: *(Xi , T) → (Yi , T\*)* is *Еc*(*δ*-*ß*c)-continuous functions for *i* = 1,2, and let *f*: *→*  be a function defend as follows:

*f (1, 2) = (f1 (1), f2 ( 2))*. Then *f* is *Еc* (*δ*-*ß*c)-continuous.

***Proof*:** Suppose that, , such that *Ri* is open sets in *Yi* for *i* = 1,2. Then, *f -1*(*R1* × *R2*) = *f1 -1*(*R1) × f2 -1*(*R2)*. Since *fi* is *Еc* (*δ*-*ß*c)-continuous functions for *i* = 1,2. Thus by (Theorem 2.2.1.3), and (Theorem 2.1.1.43), we get *f -1*(*R1* × *R2*) is *Еc* (*δ*-*ß*c)-open set in Therefore *f* is *Еc*(*δ*-*ß*c)-continuous function.

**Definition 2.2.2.11:** A function *f*: *(X, T) → (Y, T\*)* is said to be:

**(i)-** *Еc*-irresolute, if *f* −1(*K*) is *Еc*-open in *X* for every *Еc*-open subset *K* of *Y*.

**(ii)-** *δ*-*ß*c-irresolute, if *f* −1(*K*) is *δ*-*ß*c-open in *X* for every *δ*-*ß*c-open subset *K* of *Y*.

**(iii)-** *Еc*-Open, if*, f* (*K*) is *Еc*-open of *Y* for everyopen subset *K* of *X.*

**(iv)-** *δ*-*ß*c-Open, if*, f* (*K*) is *δ*-*ß*c-open of *Y* for everyopen subset *K* of *X.*

**Theorem 2.2.2.12:** The following properties hold for a functions *f*: *(X, T) → (Y, T\*)* and *g*: *(Y, T\*)* *→ (Z, T\*\*):*

**(i)-**If *f* is *Еc* (*δ*-*ß*c)-continuous and *g* is continuous, then *g o f* is *Еc* (*δ*-*ß*c)-continuous.

**(ii)-**If *f* is continuous and *g* is perfectly (clopen) continuous, then *g o f* is *Еc* (*δ*-*ß*c)-continuous.

**(iii)-**If *f* is *Еc* (*δ*-*ß*c)-irresolute and *g* is *Еc* (*δ*-*ß*c)-continuous, then *g o f* is *Еc*(*δ*-*ß*c)-continuous.

***Proof*: (i) -** Assume that *V* is open set in *Z*. Then *g*−1(*V*) is open in *Y* via continuity of *g*. Since *f* is *Еc*(*δ*-*ß*c)-continuous, *f* −1(*g*−1(*V*)) is *Еc*(*δ*-*ß*c)-open in *X* and thus, *(g o f)* −1*(V)* = *f* −1(*g*−1(*V*)) is *Еc* (*δ*-*ß*c)-open in *X.*  Therefore, *g o f* is *Еc*(*δ*-*ß*c)-continuous.

**(ii) -** suppose that *V* is open set in Z. Then *g*−1(*V*) is clopen in *Y* by perfect continuity of *g*. Since *f* is continuous, *f* −1(*g* −1(*V*)) is clopen in *X* and thus, *Еc*(*δ*-*ß*c)-open via (Theorem 2.1.1.36),therefore *(g o f)* −1*(V)* = *f* −1(*g*−1(*V*)) is *Еc*(*δ*-*ß*c)-open in *X.*  Hence, *g o f* is *Еc* (*δ*-*ß*c)-continuous.

**(iii) –** Let *V* be open set in Z. Then *g*−1(*V*) is *Еc* (*δ*-*ß*c)-open in *Y* via *Еc*(*δ*-*ß*c)-continuity of *g*. Since *f* is *Еc* (*δ*-*ß*c)-irresolute, *f* −1(*g* −1(*V*)) is *Еc* (*δ*-*ß*c)-open in *X* and thus *(g o f)* −1*(V)* = *f* −1(*g*−1(*V*)) is *Еc* (*δ*-*ß*c)-open in *X.* Hence, *g o f* is *Еc* (*δ*-*ß*c)-continuous.

**Theorem 2.2.2.13:** Let *f*: *(X, T) → (Y, T\*)* and *g*: *(Y, T\*)* *→ (Z, T\*\*)* be two functions. If *f* is *Еc* (*δ*-*ß*c)-open surjective function and *g o f* is *Еc* (*δ*-*ß*c)-continuous, then *g* is *Еc* (*δ*-*ß*c)-continuous.

***Proof*:** Let *V* be an open subset of *Z.* Since *g o f* is *Еc* (*δ*-*ß*c)-continuous, so *(g o f)* −1*(V)* = *f* −1(*g*−1(*V*)) is *Еc*(*δ*-*ß*c)-open in *X.* Since *f* is *Еc* (*δ*-*ß*c)-open and surjective, then *f (f* −1(*g*−1(*V*))) = *g*−1(*V*) is *Еc*(*δ*-*ß*c)-open set in *Y*. Thus, *g* is *Еc* (*δ*-*ß*c)-continuous.

**Corollary 2.2.2.14:** Let *f*: *(X, T) → (Y, T\*)* be *Еc*(*δ*-*ß*c)-irresolute and *Еc*(*δ*-*ß*c)-open surjective function and let *g*: *(Y, T\*)* *→ (Z, T\*\*)* be function. Then *g o f: (X, T) → (Z, T\*\*)* is *Еc* (*δ*-*ß*c)-continuous *if and only if* *g* is *Еc*(*δ*-*ß*c)-continuous.

***Proof*:** The proof follows from (Theorems 2.2.2.12-part-(iii))and(Theorems 2.2.2.13) immediately.

**Theorem 2.2.2.15:** Let *f*: *(X, T) → (Y, T\*)* be a function and let *G: X → X×Y* be the graph function of *f* defined via, *G()* = (*, f ())* where *G (f)* = {(*, f()):*  }. Then, *G* is *Еc* (*δ*-*ß*c)-continuous if and only if *f* is *Еc* (*δ*-*ß*c)-continuous.

***Proof*:** Suppose that *G* is *Еc* (*δ*-*ß*c)-continuous and *V* is an open subset of *Y*, then *X×V* is open set in *X×Y*. Since *G* is *Еc* (*δ*-*ß*c)-continuous, so *G -1*(*X×V*) is *Еc* (*δ*-*ß*c)-open set in *X*. Since *G -1*(*X×V*) = {: *G()* = *(, f()) X×V*} = { : *f() V*} = *f -1(V)* is *Еc* (*δ*-*ß*c)-open set in *X*. hence, via (Theorem 2.2.1.3), *f* is *Еc* (*δ*-*ß*c)-continuous.

Assume that *f* is *Еc* (*δ*-*ß*c)-continuous and let where *H* is an open subset of *X×Y* containing *G().* Since via (Theorem 1.17), *{}×Y* is homeomorphic to *Y* and H *({ }×Y)* is open in the subspace *{}×Y* containing *G(),* { : *(, y) H*} is open subset of *Y*. Since *f* is *Еc* (*δ*-*ß*c)-continuous, so *f -1*({*y*: *(, y) H*}) is *Еc* (*δ*-*ß*c)-open setin *X*. Since*f -1({y: (, y) H}) =* { *f -1(y) : (, y) H*} is *Еc* (*δ*-*ß*c)-open setin *X* and { *f -1(y) : (, y) H*} *G -1*(*H*). Therefore *G -1*(*H*) is *Еc* (*δ*-*ß*c)-open setin *X* and hence, by using (Theorem 2.2.1.3), *G* is *Еc* (*δ*-*ß*c)-continuous function.

**Theorem 2.2.2.16:** Let *f*: *(X, T) → (Y, T\*)* be *Еc* (*δ*-*ß*c)-continuous function where *(Y, T\*)* is Hausdorff-space, then *G (f)* = {(*, f()):*  } is *Еc*(*δ*-*ß*c)-closed set in *X×Y*.

***Proof*:** Let Then, , since *(Y, T\*)* is Hausdorff, so there exist disjoint open sets *V* and *H* of *Y* such that . Since *f* is *Еc*(*δ*-*ß*c)-continuous functions, so there exists *Еc*(*δ*-*ß*c)-open sets *U* in *X*containing where *f (U)* *H.* Thussuch that *U×V* is *Еc*(*δ*-*ß*c)-open set because *U* is *Еc*(*δ*-*ß*c)-open and *V* is open in Hausdorff and thus, by using (Theorem 2.1.1.10), *V* is *Еc*(*δ*-*ß*c)-open set. So is *Еc* (*δ*-*ß*c)-open set and hence, *G (f)* is *Еc* (*δ*-*ß*c)-closed set in *X×Y*.

**2. 3.** **Conclusions**

According of the present study in this chapter the following conclusions are obtained:

1. Generalized open and closed sets play a very prominent role in general topology and its applications. And many topologists worldwide are focusing their researches on these topics and this mounted to many important and useful results.
2. The importance of general topological spaces rapidly increases in both the pure and applied directions it plays a significant role in data mining. One can observe the influence of general topological spaces also in computer science and digital topology **[57]**.
3. In this chapter, we introduce and study new classes of generalized open sets called *Еc*-opеn and *δ*-*ß*c-open sеts which may have very important applications in quantum particle physics, high energy physics and superstring theory **[58].**
4. Since El-Naschie has shown that the notion of fuzzy topology has very important applications in quantum particle physics especially in related to both string theory and  theory. Therefore, the fuzzy topological version of the concepts and results introduced in this chapter are very important. **[59, 60].**
5. One can observe the influence made in the realms of applied research by general topological spaces, properties and structures. In digital topology, information systems, particle physic, computational topology for geometric design and molecular design **[61].** Thus we study new concepts of generalized continuous functions, called *Еc*-Continuous and *δ*-*ß*c-Continuous functions which may have very important applications in quantum particle physics and theoretical Physics, particularly in connections with string theory **[62].**
6. As well, the fuzzy topological versions of the concepts and results which concerning of *Еc*-Continuous and *δ-ßc*-Continuous functions which introduced and studied in this chapter are very important.

**CHAPTER THREE**

**On *Еc* and *δ-ßc-*Compact Spaces, *Еc and δ-ßc -* Separation Axioms Via *Еc* and *δ-ßc -*Open Sets**

Compactness is now one of the most important, useful, and fundamental notions not only of general topology but also of other advanced branches of mathematics. It is seen from the literature that a wide variety of topological properties neighbouring on compactness have been introduced and studied in detail by many researchers, For example, mention may be of H-closedness, near compactness, S-closedness, different variant forms of paracompactness….. etc. In literature, different classes of generalized compactness such as [**18, 19**] are studied. On the other hand, the class of generalized open and closed sets has a significant role in general topology, especially its suggestion of new generalized separation axioms which are useful in digital topology. In recent literature, we find many topologists had focused their research in the direction of investigating different types of separation axioms. Some of these have been found to be useful in computer science and digital topology [see for example [**22, 23**]. Dontcheve and Ganster [**22**] proved that the digital line is T3/4 space but not T1. Also, Navalagi [**24**] introduced semi generalized- Ti spaces, i= 0, 1, 2. In addition, in 2011, Ahu Açıkgöz [**25**] defined two new generalized separation axioms called *ß\**T1/2 and *ß\*\** T1/2 spaces as applications of *ß\**g−closed sets.

In order investigate the aim of this chapter methodically; it is divided into three sections as shown in the following figure:

***Contents of Chapter Three***

***Section Three***

***Conclusions of chapter Three***

**Section One**

***On Еc-(δ-ßc)-Compact Spaces via Еc-(δ-ßc)-open sets***

**Section Two**

***On Еc-(δ-ßc) - Separation Axioms via Еc-(δ-ßc)-open sets***

**Figure: (3.1). Overview of chapter Three**

**3.1. On *Е*c and *δ-ßc-*Compact Spaces Via *Еc* and *δ-ßc-*Open Sets**

**In recent years**, Benchalli and Patil [**63**] introduced and studied a new class of closed sets called *ωα*-closed sets and continuous functions in topological space. In [**3**] Hariwan Z. Ibrahim presented a new class of space named *Bc*-compact and gave some properties of *Bc*-compact space by using *Bc*- open sets. As well, P. G. Patil, in [**20**] introduced the concept of ***ω****α*-compactness in topological spaces and gave some characterization of ***ω****α*-compactness by using *ωα*-closed sets. On the other hand, recently, Sarika and Rayanagoudar [**21**] introduced a new concept called αg\*s-compactness in topological spaces and obtained some of their properties by using *α*g\*s-closed sets.

The purpose of the section is to consider new classes of generalized compact spaces called *Еc-*(*resp*. *δ*-*ß*c)-compact spaces via *Еc-*(*resp*. *δ*-*ß*c)-open sets respectively. Several characterizations and fundamental properties concerning of these forms of spaces are obtained.

**Definition 3.1.1:** A topological space *X* is called:

**(a)-** *Еc* (*resp*. *δ*-*ß*c)- Compact if for every *Еc* (*resp*. *δ*-*ß*c)-open cover A finite sub-set

**(b)-**Nearly compact [**64**], if for each open cover of *X*, there exists a finite sub collection

**(c)-**Almost regular [**65**] If for each *T*- regularly closed subset disjoint - open sets

**(d)-** *Е- (resp. δ-ß)-*Compact [**66**] if every *E-(resp. δ-ß)-*open cover of *X* has a finite sub-cover.

**Remark 3.1.2:** The family of all regular open sets constitutes a base for a topology . This topology is known as the semi-regularization of *T*. We note that . [**65**]

**Definition 3.1.3:** A filter base in a space *X* is *Еc* (*resp*. *δ*-*ß*c)-Convergence to a point *Еc* (*resp*. *δ*-*ß*c)-open set

**Definition 3.1.4:** A filter base in a space *X* is *Еc* (*resp*. *δ*-*ß*c)- Accumulation of a point if

**Theorem 3.1.5:** Let be filter base of a space *X.* If is. *δ*-*ß*c- Convergence to a point then

***Proof:*** Assume that *δ*-*ß*c-Converges to Let be any regular closed set such that, *δ*-*ßCΣ(X)*. Since is *δ*-*ß*c-Convergence to thus an This explains that

**Remark 3.1.6:** The converse of (Theorem 3.1.5) is not necessarily true, as shown in the following example:

**Example 3.1.7:** Consider is the space of usual topology and let

. So is but doesn’t*δ*-*ß*c- because the set of is *δ*-*ß*c-open set containing 0, but , such that,

**Corollary 3.1.8:** Let be filter base of a space *X.* If *δ*-*ß*c-Accumulation of a point then

***Proof:*** Similar to that of(Theorem 3.1.5).

**Theorem 3.1.9:** Let be filter base of a space *X* and be any closed set containing . If such that Then *Еc* and *δ*-*ß*c- Convergence to a point

***Proof:*** Suppose that be any *Еc* (*resp*. *δ*-*ß*c)-open set containing, so a closed set such that via hypothesis, Thus, *Еc* (*resp*. *δ*-*ß*c)- Converges to

**Theorem 3.1.10:** Let be filter base of a space *X* and be any closed set containing , such that Then *Еc* (*resp*. *δ*-*ß*c)- Accumulation of a point

***Proof:*** Similar to that of **(**Theorem 3.1.9).

**Theorem 3.1.11:** Let be a topological space. If every closed cover of *X* has a finite sub-cover, then *X* is *Еc* (*resp*. *δ*-*ß*c)- Compact space.

***Proof:*** Let be any *Еc* (*resp*. *δ*-*ß*c)-open cover of *X*, and there exists a closed set

so the collection is a cover of *X* by closed set, then by hypothesis, this collection has a finite sub-cover such that: .

Thus therefore *X* is *Еc* (*resp*. *δ*-*ß*c)-Compact space.

**Theorem 3.1.12:** If a topological space is *Е*-(*resp*. *δ*-*ß*)-Compact, then its *Еc* (*resp*. *δ*-*ß*c)-Compact space.

***Proof:*** Let be any *Еc* (*resp*. *δ*-*ß*c)-open cover of *X*. So is *Е*(*resp*. *δ*-*ß*)-open cover of *X,* since *X* is *Е*-(*resp*. *δ*-*ß*)-Compact, so there exist a finite sub-set Thus,

*X* is *Еc* (*resp*. *δ*-*ß*c)-Compact space.

**Theorem 3.1.13:** Every *Еc* (*resp*. *δ*-*ß*c)-Compact is *Е*-(*resp*. *δ*-*ß*)-Compact space.

***Proof:*** Assume that *X* is *Еc* (*resp*. *δ*-*ß*c)-Compact and . Let be any *Е*(*resp*. *δ*-*ß*)-open cover of *X*. Since *X* is , and *X* is *Еc* (*resp*. *δ*-*ß*c)-Compact, thus there exists a finite subset Thus *X* is *Е*(*resp*. *δ*-*ß*)-Compact space.

The following corollary is directly consequence of Theorems-(3.1.12 and 3.1.13).

**Corollary 3.1.14:** Let *X* be a . Then *X* is *Еc* (*resp*. *δ*-*ß*c)-Compact *X* is *Е*-(*resp*. *δ*-*ß*)-Compact space.

**Theorem 3.1.15:** Let be a *δ*-*ß*c-Compact space, then *X* is .

***Proof:*** Suppose that is any regular closed cover of *X*, so byusing(Theorem 2.1.1.23), constitutes a*δ*-*ß*c-open cover of *X*. Since *X* is*δ*-*ß*c-Compact, hence there exists a finite subset Thus *X* is .

**Theorem 3.1.16:** Let be a regular space. If *X* is *Еc* (*resp*. *δ*-*ß*c)-Compact, then *X* is compact.

***Proof:*** Assume that is any open cover of *X*. Since *X* is regular, so via (Theorem 2.1.1.25), constitutesa *Еc* (*resp*. *δ*-*ß*c)-open cover of *X*. and since *X* is *Еc* (*resp*. *δ*-*ß*c)-Compact, thus there exists a finite sub-set Thus, *X* is Compact space.

**Theorem 3.1.17:** Let be an almost regular space. If *X* is *δ*-*ß*c-Compact, then *X* is nearly compact.

***Proof:*** Suppose that the collection is any regular open cover of a space . Since *X* is almost regular space, therefore for every

Via(Theorem 2.1.1.23),is constitutes a *δ*-*ß*c-open cover of *X*. Since *X* is *δ*-*ß*c-Compact, so there exists a sub-

**3.2. On *Еc* and *δ-ßc -*Separation Axioms by *Еc* and *δ-ßc -*Open Sets**

Recently, Hariwan Z. Ibrahim in [**26**] offered and investigated some weak separation axioms by using the notions of *Bc*-open sets and the *Bc*-closure operator. As well, in the same year Hussein A. Khaleefah[**27**] studied new types of separation axioms termed by, generalized b- Ri, i= 0, 1 and generalized b-Ti, i= 0, 1, 2 by using generalized b-open sets; relations among these types are investigated, and several properties and characterizations are provided. A.I. EL-Maghrabi and M.A. AL-Juhani [**28**] introduced and investigated a new class of separation axioms called M-Ti-spaces, i = 0, 1, 2. Furthermore, the M-regularity and the M-normality are examined in the context of these new concepts.

This section is devoted to introduce and investigate new classes of generalized separation axioms namely *Еc-(resp. δ-ßc)-* separation axioms such as *Еc-T0-*(*resp*. *δ*-*ß*c*-T0-*), *Еc-T1-*(*resp*. *δ*-*ß*c*-T1-*), *Еc-T2-*(*resp*. *δ*-*ß*c*-T2-*). Several fundamental properties and preservation properties concerning of these kinds of weak separation axioms are provided. Furthermore, the relationships among these types of separation axioms and other well-known types of spaces are discussed. Also, the *Еc-*(*resp*. *δ*-*ß*c)-Regularity and the *Еc-*(*resp*. *δ*-*ß*c)-Normality are studied in the context of these new classes and some of fundamental properties of them are introduced.

This section consists of two main parts as follows:

**3.2.1. Fundamental Properties of *Еc* and *δ-ßc* -Ti-Spaces (i = 0,1,2)**

In this part, several basic properties concerning of new kinds of separation axioms called*Еc* (*resp*. *δ*-*ß*c)- separation axioms such as *Еc-T0-*(*resp*. *δ*-*ß*c*-T0-*), *Еc-T1-*(*resp*. *δ*-*ß*c*-T1-*) *Еc-T2-*(*resp*. *δ*-*ß*c*-T2-*) are given, as well the relationships among these kinds of generalized spaces and other well- known spaces are investigated.

**Definition 3.2.1.1:** *A* function *f*: *(X, T) → (Y, T\*)*is said to be:

**(i)-** *Еc*-Irresolute, if *f* −1() is *Еc*-open in *X* for every *Еc*-open sub-set .

**(ii)-** *δ*-*ß*c-Irresolute, if *f* −1() is *δ*-*ß*c-open in *X* for every *δ*-*ß*c-open sub-set .

**(iii)-** *Еc*-open, if the image of each open set of *(X, T)* is *Еc*-open of *(Y, T\*).*

**(iv)-** *δ*-*ß*c-open function, if the image of each open set of *(X, T)* is *δ*-*ß*c-open of *(Y, T\*).*

**(v)-** *Еc-*closed function, if the image of each closed set of *(X, T)* is *Еc-*closed of *(Y, T\*).*

**(vi)-** *δ-ßc-*closed function, if the image of each closed set of *(X, T)* is *δ-ßc*-closed of *(Y, T\*).*

**(vii)-** *Еc*-continuous, if *f* −1() is *Еc*-open in *X* for every open sub-set

**(viii)-** *δ*-*ß*c-continuous function, if *f* −1() is *δ*-*ß*c-open in *X* for every open sub-set

**(ix)-**Pre-*Еc*-open function, if the image of each *Еc*-open set of *(X, T)* is *Еc*-open of *(Y, T\*).*

**(x)-**Pre-*δ*-*ß*c-open function, if the image of each *δ*-*ß*c-open set of *(X, T)* is *δ*-*ß*c-open of *(Y, T\*).*

**(xi)-** *Еc*-Homeomrphism, if, *f* is bijective, *Еc*-irresolute and pre-*Еc*-open.

**(xii)-** *δ*-*ß*c-Homeomrphism, if, *f* is bijective, *δ*-*ß*c-irresolute and pre-*δ*-*ß*c-open.

**Definition 3.2.1.2:** *A*topological space is called:

**(a)-** if for every distinct points *x* and *y* of *X*, there is *Еc*(*resp.* *δ*-*ß*c)-open set containing one of them but not the other.

**(b)-** if for every pair of distinct

.

**(c)-**

if for each pair of distinct points, there exist two disjoint*Еc*(*resp.* *δ*-*ß*c)-open sets

**Remark 3.2.1.3:**From the respective definitions, we have the following diagram.However, none of these implications is reversible as shown via examples of [**67, 4, 5**].

***bc-open set***

***b-open set***

***Pre-open set***

***Pc-open set***

***Pc-open set***

***ß*c-*open set***

***ß*-open set**

***δ*-*ß*c-open set**

***δ*-*ß*-open set**

***E-open set***

***Ec-open set***

***δ-Pre-open***

***Ec-open set***

**Figure: (3.2). The relationships among some well-known types of** **generalized open sets in topological spaces**

**Theorem 3.2.1.4:**The following properties are hold in a topological space:

**(a)-**Every is .

**(b)-**Every is .

**(c)-**Every is

**(d)-**Every is

**(e)-**Every is

***Proof:*** The proof is obvious it immediately follows from their respective definitions.

**Remark 3.2.1.5:**From the respective definitions, the relationships among *Еc (resp*. *δ-ßc)*- and some other well-known forms of spaces shown in the following figure:

***T2-Space***

***Ec- T2-Space***

***δ-ßc -T2-Space***

***T1-Space***

***T0-Space***

***Ec - T1-Space***

***δ-ßc -T1-Space***

***Ec - T0-Space***

***δ-ßc -T0-Space***

**Figure: (3.3). Relationships among *Еc (resp*. *δ-ßc)*- and some other well-known forms of spaces**

However none of these implications is reversible as shown in the following examples.

**Example 3.2.1.6:**Let *Х* = {*a, b, c, d*} with a topology *T* = {, {*a*}, {*b*}, {*a, b*}, {*b*, *d*}, {*a, b, d*}, *Х*}. Then, *ECΣ(Х, T) =* {, {*a, c*}, {*b*, *c*, *d*}, {*a, c, d*}, *Х*}.

And, *δ*-*ßCΣ(X, T) =* {, {*a, c*}, {*c, d*}, {*b*, *c*, *d*}, {*a, c, d*}, *Х*}. Then *X* is , but it is neither nor .

**Example 3.2.1.7:**Consider *X* any infinite set with the co-finite topology (such that the closed sets are *X* and the finite sub-sets). Since is *Еc* (*resp.* *δ*-*ß*c)-open, therefore *X* is, . But there is not non empty *Еc* (*resp.* *δ*-*ß*c)-open sets are disjoint, so *X* cannot be .

**Theorem 3.2.1.8:**The following statements are equivalent for a space :

**(a)-** *X* is an

**(b)-**For every two distinct points *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).*

***Proof:*** **Necessity.** Suppose that and for each there exists an*Еc* (*resp.* *δ*-*ß*c)-open set , where is *Еc(resp. δ-ßc)-*closed which does not contain but contains . Since *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* is the smallest *Еc*(*resp.* *δ*-*ß*c)-closed set containing so *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*  and hence *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).* Consequently *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).*

**Sufficiency:** Assume thatand*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).* Let *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* but *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).* We prove that *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).* Suppose that *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})),* so *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})),*which implies that, *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* and thus *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*  which is a contradiction with the fact of *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*, hence *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* which implies that, *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).* Consequently *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* is an *Еc*(*resp.* *δ*-*ß*c)-open set containing but not . Therefore, *X* is

**Theorem 3.2.1.9:**Let be a topological space. Then the following statements are equivalent:

**(a)-** *X* is an .

**(b)-**For each point the singleton set is *Еc* (*resp.* *δ*-*ß*c)-closed set,

**(c)-**For each point *Еc-D({})* (*resp*. *δ*-*ß*c*-D({})).*

***Proof:***  Suppose that *X* is for each there exists *Еc* (*resp.* *δ*-*ß*c)-open set Consequently, . Thus which is the union of an *Еc* (*resp.* *δ*-*ß*c)-open sets. Then is an *Еc* (*resp.* *δ*-*ß*c)-open sets. Thus  *Еc* (*resp.* *δ*-*ß*c)- closed sets.

Assume that is *Еc* (*resp.* *δ*-*ß*c)-closed for each . So via hypothesis for each are *Еc* (*resp.* *δ*-*ß*c)- closed sets. Hence are *Еc* (*resp.* *δ*-*ß*c)- open sets such that Therefore *X* is .

Assume that is*Еc* (*resp.* *δ*-*ß*c)-closed set for each . Thus,

***=*** *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) = Еc-D({})* (*resp*. *δ*-*ß*c*-D({})).* Therefore,

*Еc-D({})* (*resp*. *δ*-*ß*c*-D({}) )*

Let *Еc-D({})* (*resp*. *δ*-*ß*c*-D({}))* for each . Since

*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) = Еc-D({})* (*resp*. *δ*-*ß*c*-D({}))*. Thus,

*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) =*  *Еc*(*resp.* *δ*-*ß*c)-closed set.

Suppose that *X* is and assume that *Еc-D({})* (*resp*. *δ*-*ß*c*-D({}))* for some *Еc-D({})* (*resp*. *δ*-*ß*c*-D({}))* andSince *X* is *Еc*(*resp.* *δ*-*ß*c)-open set which implies, and thus *Еc-D({})* (*resp*. *δ*-*ß*c*-D({}))* which a contradiction with the hypothesis. Hence,  *Еc-D({})* (*resp*. *δ*-*ß*c*-D({})).*

Let *Еc-D({})* (*resp*. *δ*-*ß*c*-D({}))*, , so

*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) = Еc-D({})* (*resp*. *δ*-*ß*c*-D({})) =*  which implies, is *Еc*(*resp.* *δ*-*ß*c)-closed set and thus by using (part (a) and (b)) *X* is

**Theorem 3.2.1.10:**If is a topological space, then the following properties are equivalent:

**(a)-** *X* is an

**(b)-**

***Proof:*** since *X* is so

*Еc* (*resp.* *δ*-*ß*c)-open sets Thus, put then is *Еc* (*resp.* *δ*-*ß*c)-closed set, is *Еc* (*resp.* *δ*-*ß*c)-closed set and =*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).*

Assume that by hypothesis, there exists *Еc* (*resp.* *δ*-*ß*c)-open set *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).* Hence (*Еc-Cl({})*(*resp*. *δ*-*ß*c***-****Cl({})))* which is *Еc*(*resp.* *δ*-*ß*c)-open and (*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))).* As well,

(*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})))* So, *X* is

**Definition 3.2.1.11:**Let be a topological space and. Then, the intersection of all *Еc* (*resp.* *δ*-*ß*c)-open subsets of containing is called the *Еc-***(***resp.**δ*-*ß*c-) of and it’s denoted via *Еc-(resp. δ-ßc- of (i. e) :*

**Theorem 3.2.1.12:**Let be a topological space and then,  *Еc-(resp.**δ-ßc- Еc-(resp. δ-ßc-.*

***Proof:*** Assume that *Еc-(resp.**δ-ßc-.* So, there exists *Еc*(*resp.* *δ*-*ß*c)-open set . Thus we get  *Еc-(resp. δ-ßc-.* Similarly we can prove the converse case.

**Theorem 3.2.1.13:**Let a topological space

Then, *Еc-(resp.**δ-ßc-=: Еc-(resp. δ-ßc-*

***Proof:*** Suppose that *Еc-(resp.**δ-ßc-* and

*Еc-(resp. δ-ßc-.* Thus (*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})))* which is an *Еc*(*resp.* *δ*-*ß*c)-open set . This case is not possible, since  *Еc-(resp.**δ-ßc-.* therefore*Еc-(resp. δ-ßc-.* Now suppose that such that*Еc-(resp. δ-ßc- and Еc-(resp. δ-ßc-.* therefore*,* there exists an *Еc*(*resp.* *δ*-*ß*c)-open set Let *Еc****-****(resp.**δ-ßc-.* Hence, is an *Еc*(*resp.* *δ*-*ß*c)-Neighbourhood of which does not contain . So via this contradiction we get  *Еc-(resp.**δ-ßc-* and this is the request.

**Theorem 3.2.1.14:**The following properties hold for the subsetsof a topological space:

**(a)-** *Еc-(resp.**δ-ßc-.*

**(b)-** *Еc-(resp.**δ-ßc-* *Еc-(resp.**δ-ßc-.*

**(c)-**If is *Еc* **(***resp.**δ*-*ß*c)-open of , then = *Еc-(resp.**δ-ßc-*

**(d)-** *Еc- (resp.**δ-ßc- = Еc-(resp. δ-ßc-*

***Proof:*** The proof of parts (**a), (b)** and **(c),** are directly consequences of (Definition 3.2.1.11).Now we prove part **(d),** first via parts **(a)** and **(b)** we have:

*Еc-sp.**δ-ßc- Еc-. δ-ßc-* If *Еc-sp.**δ-ßc- So*  *ECΣ(Х)* (*resp*.*δ*-*ßCΣ(X))* such that

, *Еc-(resp.**δ-ßc-* and so we get:

*Еc- (resp.**δ-ßc-* Therefore,

*Еc-(resp.**δ-ßc-*

**Theorem 3.2.1.15:**for any two distinct points in a topological space the following properties hold:

**(a)-** *Еc-(resp. δ-ßc-* *Еc-(resp. δ-ßc-.*

**(b)-** *Еc-(resp. δ-ßc- Еc-(resp. δ-ßc-*

***Proof:*** Assume that *Еc-(resp. δ-ßc-* *Еc-(resp. δ-ßc-.* So there exists a point  *Еc-(resp. δ-ßc-* and *Еc-(resp. δ-ßc-.* Since *Еc-(resp. δ-ßc-* Consequently that *Еc-(resp. δ-ßc-* *Еc-(resp. δ-ßc-* utilize *Еc-(resp. δ-ßc-* we get *Еc-(resp. δ-ßc-* Since *Еc-(resp. δ-ßc-,* so*, Еc-(resp. δ-ßc-* *Еc-(resp. δ-ßc-* and *Еc-(resp. δ-ßc-* Thus, it follows that *Еc-(resp. δ-ßc- Еc-(resp. δ-ßc-* So, *Еc-(resp. δ-ßc-* *Еc-(resp. δ-ßc-* implies that *Еc-(resp. δ-ßc- Еc-(resp. δ-ßc-*

Suppose that*Еc-(resp. δ-ßc- Еc-(resp. δ-ßc-* So there exists a point *Еc-(resp. δ-ßc-* and  *Еc-(resp. δ-ßc-* Then, there exists an *Еc* (*resp.* *δ*-*ß*c)-open set containing but not , namely, *Еc-(resp. δ-ßc-* and hence *Еc-(resp. δ-ßc-* *Еc-(resp. δ-ßc-.*

**Theorem 3.2.1.16:** Suppose that *f*: *(X, T) → (Y, T\*)*isan injective *Еc* and *δ*-*ß*c**)**-continuous function and is then is *Еc (resp*. *δ-ßc)*-

***Proof:*** We prove that the theorem for and the other are similar.

Assume that since *f* is injective, then But,, then there exist an open set

since *f* is *Еc***(***resp.**δ*-*ß*c)-continuous, so is *Еc***(***resp.**δ*-*ß*c)-open set of such that: , Thus is

**Theorem 3.2.1.17:** Let *f*: *(X, T) → (Y, T\*)*bean injective *Еc* **(***resp.**δ*-*ß*c)-irresolute function and is an *Еc (resp*. *δ-ßc)*- then is *Еc (resp*. *δ-ßc)*-

***Proof:*** We prove that the theorem for and the other are similar.

Suppose that since is injective, then But is an so there exist two disjoint *Еc***(***resp.**δ*-*ß*c**)**-open sets By utilizing *Еc* **(***resp.**δ*-*ß*c**)**-irresolute of we get, are *Еc***(***resp.**δ*-*ß*c**)**-open set of *X* such that:

**Theorem 3.2.1.18:** Suppose that *f*: *(X, T) → (Y, T\*)*isa bijective *Еc* **(***resp.**δ*-*ß*c**)**-open function and is then is *Еc (resp*. *δ-ßc)*-

***Proof:*** We prove that the theorem for and the other are similar.

Let since is bijective, so there exist such that, . Since , then there exist two disjoint open sets Since is *Еc* **(***resp.**δ*-*ß*c**)**-open function, then are *Еc***(***resp.**δ*-*ß*c**)**-open sets of with therefore is .

**Theorem 3.2.1.19:**  property.

***Proof:*** Suppose that a function *f*: *(X, T) → (Y, T\*)*is , and since *f* is injective, so . Since *X* is an *Еc* **(***resp.**δ*-*ß*c**)**-open set Since *f* is *pre*-*Еc***(***resp.**pre*-*δ*-*ß*c**)**-open , then is *Еc***(***resp.**δ*-*ß*c**)**-open set in *Y* such that

***Proof:*** Suppose that a function *f*: *(X, T) → (Y, T\*)*is , and since *f* is injective, then. Since *X* is two *Еc***(***resp.**δ*-*ß*c**)**-open sets Since *f* is *pre*-*Еc***(***resp.**pre*-*δ*-*ß*c**)**-open , then are *Еc***(***resp.**δ*-*ß*c**)**-open set in *Y*  Thus *Y* is

**Theorem 3.2.1.21:** A is a topological property.

***Proof:*** Suppose that a function *f*: *(X, T) → (Y, T\*)*is , and since *f* is injective, then. Since *X* is two disjoint *Еc***(***resp.**δ*-*ß*c**)**-open sets Since *f* is *pre*-*Еc***(***resp.**pre*-*δ*-*ß*c**)**-open , then are two disjoint *Еc***(***resp.**δ*-*ß*c**)**-open sets in *Y*  Thus *Y* is

**3.2.2. Some Fundamental Properties of the *Еc* and *δ-ßc*-Regular and the *Еc* and *δ-ßc*-Normal Spaces**

This part is devoted to introduce and investigate the *Еc (resp*. *δ-ßc)*-Regularity and the *Еc (resp*. *δ-ßc)*-Normality. Additional, some of fundamental properties concerning of regular and normal spaces are discussed.

**Definition 3.2.2.1:** A topological space *(X, T)* is said to be *Еc* **(***resp.**δ*-*ß*c**)**-Regular if for every closed set and each point such that , there exist two disjoint *Еc* **(***resp.**δ*-*ß*c**)**-open sets

**Theorem 3.2.2.2:** the following statements are equivalent for a space:

**(i)-** *X* is *Еc (resp*. *δ-ßc)*-Regular,

**(ii)-**For each closed set and , there exists an *Еc***(***resp.**δ*-*ß*c**)**-open set such that *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*

***Proof:*** Let *X* be an *Еc (resp*. *δ-ßc)*-Regular space, and . there exist two disjoint *Еc* (*resp.**δ*-*ß*c**)**-open sets *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*. Since *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*, then *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* . Thus, *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*

Let and be closed set such that *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* So *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*, which is an *Еc* **(***resp.**δ*-*ß*c**)**-open set and disjoint with . Thus *X* is *Еc (resp*. *δ-ßc)*-Regular.

**Theorem 3.2.2.3:** Let be an *Еc (resp*. *δ-ßc)*-Regular space, for any two points then either *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) = Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) OR*

*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*.

***Proof:*** Suppose that *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* then either  *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) OR Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).* Assume that *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})).* Since *X* is *Еc (resp*. *δ-ßc)*-Regular,then there exists an *Еc***(***resp.**δ*-*ß*c**)**-open set such that *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* andWhere is *Еc***(***resp.**δ*-*ß*c**)**-closed and *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*

Thus *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})) Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*

.

**Theorem 3.2.2.4:** Suppose that *f*: *(X, T) → (Y, T\*)*isa bijective continuous and *pre*-*Еc* **(***resp.**pre*-*δ*-*ß*c**)**-open function and *X* is *Еc (resp*. *δ-ßc)*-Regular space, then *Y* is *Еc (resp*. *δ-ßc)*-Regular.

***Proof:*** Assume that is a closed set andSince *f* is bijective continuous, consequently is closed of *X*. Put so Since *X* is *Еc (resp*. *δ-ßc)*-Regular space, so there exist two disjoint *Еc***(***resp.**δ*-*ß*c**)**-open sets Since *f* is bijective and *pre*-*Еc***(***resp.**pre*-*δ*-*ß*c**)**-open function, therefore Thus *Y* is *Еc (resp*. *δ-ßc)*-Regular space.

**Theorem 3.2.2.5:** Let *f*: *(X, T) → (Y, T\*)*bean injective *Еc* **(***resp.**δ*-*ß*c)-irresolute and closed function and is an *Еc (resp*. *δ-ßc)*-Regular space, then *X* is *Еc (resp*. *δ-ßc)*-Regular.

***Proof:*** Suppose that, is a closed set andSince *f* is injective closed function, so is closed of *Y and*, thus Since *Y* is *Еc (resp*. *δ-ßc)*-Regular space, so there exist two disjoint *Еc* **(***resp.**δ*-*ß*c**)**-open sets Since *f* is *Еc***(***resp.**δ*-*ß*c)-irresolute function, therefore Thus *X* is *Еc (resp*. *δ-ßc)*-Regular.

**Theorem 3.2.2.6:** A*Еc (resp*. *δ-ßc)*-Regular space is a topological property.

***Proof:*** Suppose that a function *f*: *(X, T) → (Y, T\*)*is . Then *f* is a bijective *pre*-*Еc* **(***resp.**pre*-*δ*-*ß*c**)**-open continuous function. Let be a closed set andso is closed set of *X* and Since *X* is *Еc (resp*. *δ-ßc)*-Regular space, so there exist two disjoint *Еc* **(***resp.**δ*-*ß*c**)**-open sets Since *f* is *pre*-*Еc* **(***resp.**pre*-*δ*-*ß*c**)**-open,

So *Y* is *Еc (resp*. *δ-ßc)*-Regular.

**Definition 3.2.2.7:** A Topological space *(X, T)* is said to be *Еc***(***resp.**δ*-*ß*c**)**-Normal if for every pair of disjoint closed sets there exist two disjoint *Еc* **(***resp.**δ*-*ß*c**)**-open sets

**Theorem 3.2.2.8:** the following statements are equivalent for a space:

**(i)-** *X* is *Еc (resp*. *δ-ßc)*-Normal,

**(ii)-**For every pair of open sets, there exists an *Еc* **(***resp.**δ*-*ß*c**)**-closed sets

**(iii)-**For every closed set and every open set containing , there exists an *Еc* **(***resp.**δ*-*ß*c**)**-open set such that *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*

***Proof:*** Suppose that are two open sets in *Еc (resp*. *δ-ßc)*-Normal space *X* . So are disjoint closed sets. Since *X* is *Еc (resp*. *δ-ßc)*-Normal space, so there exist two disjoint *Еc* **(***resp.**δ*-*ß*c**)**-open sets Assume that, . Therefore are *Еc* **(***resp.**δ*-*ß*c**)**-closed sets

Suppose that is a closed set and be an open set containing So and are open sets So via part **(ii)** there exist two *Еc* **(***resp.**δ*-*ß*c**)**-closed sets Then

Let . Thus are disjoint *Еc* **(***resp.**δ*-*ß*c**)**-open sets such that  So *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*

be two disjoint closed sets . Put , so where is an open set. By part (iii) there exist an *Еc* **(***resp.**δ*-*ß*c**)**-open set  *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))* Consequently that

*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*  Then there exist two *Еc***(***resp.**δ*-*ß*c**)**-open sets

So, *X* is *Еc (resp*. *δ-ßc)*-Normal.

**Theorem 3.2.2.9:** Let *f*: *(X, T) → (Y, T\*)*bea surjective *pre*-*Еc* **(***resp.**pre*-*δ*-*ß*c**)**-open continuous and *Еc* **(***resp.**δ*-*ß*c)-irresolute function from an *Еc (resp*. *δ-ßc)*-Normal space *X* onto *Y*, then *Y* is *Еc (resp*. *δ-ßc)*-Normal.

***Proof:*** Suppose that is a closed set and be an open set containing So via continuity of *f*, we get is closed and is open of *X* such that . Via *Еc (resp*. *δ-ßc)*-Normality of *X* and by using (Theorem 3.2.2.8),there exists an *Еc* **(***resp.**δ*-*ß*c**)**-open set *Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({}))*  Then, *f* (*Еc-Cl({})* (*resp*. *δ*-*ß*c*-Cl({})))* Since *f* is surjective *pre*-*Еc* **(***resp.**pre*-*δ*-*ß*c**)**-open and *Еc***(***resp.**δ*-*ß*c)-irresolute function, so we get, *Еc-Cl(* (*resp*. *δ*-*ß*c*-Cl({}))*  So, *Y* is *Еc (resp*. *δ-ßc)*-Normal space.

**Theorem 3.2.2.10:** Let *f*: *(X, T) → (Y, T\*)*bea bijective continuous and *pre*-*Еc* **(***resp.**pre*-*δ*-*ß*c**)**-open function from an *Еc (resp*. *δ-ßc)*-Normal space *X* onto *Y*, then *Y* is *Еc (resp*. *δ-ßc)*-Normal.

***Proof:*** are two disjoint closed sets of *Y*. Since *f* is continuous, so are disjoint closed sets of *X*. Since *X* is *Еc (resp*. *δ-ßc)*-Normal space, then there exist two disjoint *Еc***(***resp.**δ*-*ß*c**)**-open sets Via bijective and *pre*-*Еc* **(***resp.**pre*-*δ*-*ß*c**)**-open of a function *f* , we obtain So, *Y* is *Еc (resp*. *δ-ßc)*-Normal space.

**Theorem 3.2.2.11:** Let *f*: *(X, T) → (Y, T\*)*bean injective closed and *Еc* **(***resp.**δ*-*ß*c)-irresolute function and *Y* be *Еc (resp*. *δ-ßc)*-Normal, then *X is Еc (resp*. *δ-ßc)*-Normal.

***Proof:*** are two disjoint closed sets of *X*. Since *f* is closed function, so are disjoint closed sets of *Y*. Since *Y* is *Еc (resp*. *δ-ßc)*-Normal space, there exist two disjoint *Еc* **(***resp.**δ*-*ß*c**)**-open sets By using injective and *Еc***(***resp.**δ*-*ß*c)-irresolute of a function *f* , we obtain

So, *X* is *Еc (resp*. *δ-ßc)*-Normal.

**3. 3.** **Conclusions**

According to the present study in this chapter the following conclusions are obtained:

**1-** Compactness is the generalization to topological spaces of the property of closed and bounded subsets of the real line.

**2-** The notions of compactness are useful and fundamental notions not only of general topology but also of other advanced branches of mathematics and many researchers have studied and investigated the basic properties of compactness.

**3-** In this chapter, we propose and investigated new kinds of classes of generalized compact spaces called *Еc-*(*resp*. *δ*-*ß*c)-compact spaces via *Еc-*(*resp*. *δ*-*ß*c)-open sets respectively.

**4-** Several characterizations and fundamental properties concerning of these forms of spaces are obtained.

**5-** Moreover, the class of generalized open and closed sets has an important role in general topology, especially its suggestion of new separation axioms which are useful in digital topology **[68, 69].**

**6-** The investigation on generalization of open and closed sets has lead to significant contribution to the theory of generalized separation axioms.

**7-**Therefore, some new types of generalized separation axioms in topological spaces namely *Еc-*(*resp*. *δ*-*ß*c)-separation axioms such as *Еc-T0-*(*resp*. *δ*-*ß*c*-T0*), *Еc-T1-*(*resp*. *δ*-*ß*c*-T1*) and *Еc-T2-*(*resp*. *δ*-*ß*c*-T2*)-spaces via *Еc (resp. δ-ßc)-*open sets are introduced and studied.

**8-** Several fundamental properties concerning of these classes of generalized separation axioms are provided.

**9-**Furthermore, the relationships among these types of generalized separation axioms and other well-known types of spaces are discussed.

**10-** The concepts of regularity and normality are important topological properties and hence they are of significance both from intrinsic interest as well as from applications view point to obtain factorizations of regularity and normality in terms of generalized topological properties.

**11-** Therefore, the *Еc-*(*resp*. *δ*-*ß*c)-Regularity and the *Еc-*(*resp*. *δ*-*ß*c)-Normality are studied in the context of generalized separation axioms.

**12-** Further, some of the fundamental properties of *Еc-*(*resp*. *δ*-*ß*c)-Regularity and the *Еc-*(*resp*. *δ*-*ß*c)-Normality are investigated.

**13-** Since El-Naschie has shown that the notion of fuzzy topology has very important applications in quantum particle physics especially in related to both string theory and theory **[70].** Therefore, the fuzzy topological version of the concepts and results introduced in section one and section two of chapter four is very significant.

**Suggestion for Future Works**

In this thesis, a new concepts and open problems will introduce to be a future works for researchers specialized in this field.

**1-** In chapter **Two (section one),** we presented and investigated some new classes of generalized open sets called *Еc*-open and *δ*-*ß*c-open sets. Several characterizations and fundamental properties concerning of these new forms of generalized open sets are obtained. Furthermore, the relationships among *Еc*-open and *δ*-*ß*c-open sets and other well-known types of generalized open sets are also examined.

**As a future works,** a new kinds of generalized open sets could be introduced namely [*Ec-δ*-open sets and *δ-ßc-*δ-open sets in topological spaces], then obtaining several new characterizations for such these of generalized open sets. In addition, clarifying the relationship of these types of generalized open sets with other well-known forms of generalized open sets which are shown by previous researches

**2-** In chapter **Two (section two),** new notions of generalized continuous functions in a topological space, called *Еc*-continuous and *δ*-*ß*c-continuous functions are introduced and investigated utilize generalized of open sets called *Еc*-open and *δ*-*ß*c-open sets respectively. Several characterizations and fundamental properties concerning of these forms of generalized continuous functions are obtained. Furthermore, the relationships among *Еc*-continuous and *δ*-*ß*c-continuous functions and other well-known forms of generalized continuous functions are also discussed.

**As a future works,** new notions of generalized continuous functions in a topological space could be introduced namely, *E*c-*δ*-continuous and *δ*-*ß*c-*δ*-continuous functions by using new of generalized open sets which have been introduced in chapter three section one. Then, we can obtain several characterizations and fundamental properties concerning of these forms of generalized continuous functions. Moreover, we can discuss the relationships among *E*c-*δ*-continuous and *δ*-*ß*c-*δ*-continuous functions and other well-known forms of generalized continuous functions which are shown by previous researches.

.

**SUMMARY AND CONCLUSIONS**

**On *Еc* (*δ*-*ß*c)-Continuous functions in Topological Sp via *Еc* (*δ*-*ß*c)-open sets**

**On *Еc -(δ-ßc)-*Compact Spaces and *Еc -(δ-ßc) -* Separation Axioms**

**In recent years,** Hariwan presented a new class of space named *Bc*-compact and gave some properties of *Bc*-compact. As well, Patil, introduced ***ω****α*-compactness and gave some characterization of ***ω****α*-compactness. Also, Sarika introduced αg\*s-compactness and obtained some of their properties. Recently, Hussein studied new types of separation axioms termed by, generalized b- Ri, i= 0, 1 and generalized b-Ti, i= 0, 1, 2. EL-Maghrabi and AL-Juhani investigated a new class of separation axioms called M-Ti-spaces, i = 0, 1, 2.

***Certain Spaces and Functions by Using Еc and δ-ßc-Open Sets***

**Recently,** Hariwan presented a new class of ƅ-open sets callеd Ɓc-open and investigated some properties of Ɓc-continuity, also, Alias and Zanyar, introduced a Sc-open, and investigated some properties of Sc-continuity, Moreover , Zanyar, presented a new class of continuity namely *Pc*-continuous functions, and investigated some properties of *Pc*-continuity by using *Pc-*open. As well, Ayman, studied a ***ß*c**-continuity as a new class of continuous functions and gave several characterizations of these functions.

The concepts that depending on to getting the new generalized open sets, new generalized continuous functions, new generalized compact spaces and some new generalized separation axioms are: *Еc-*(*resp*. *δ*-*ß*c)- open sets, *Еc-(resp. δ-ßc)-*continuous functions, *Еc-*(*resp*. *δ*-*ß*c)-Compact space , *Еc-*(*resp*. *δ*-*ß*c)-separation axioms, *Еc (resp*. *δ-ßc)*-Regularity and the *Еc (resp*. *δ-ßc)*-Normality.

**The main concepts that used in this thesis**

**Chapter**

**3**

**Chapter**

**2**

**The Purpose**

**Previous**

**Study**

**Previous**

**Study**

**The Purpose**

**The purpose** of the present chapter is to consider new classes of generalized compact spaces called *Еc-*(*resp*. *δ*-*ß*c)-compact spaces via *Еc-*(*resp*. *δ*-*ß*c)-open sets respectively. Also to introduce and investigate new classes of generalized separation axioms namely ***Еc-(resp. δ-ßc)-*** separation axioms such as ***Еc-T0-*(*resp*. *δ*-*ß*c*-T0-*), *Еc-T1-*(*resp*. *δ*-*ß*c*-T1-*) *Еc-T2-*(*resp*. *δ*-*ß*c*-T2-*). As well** introduce and investigate the *Еc (resp*. *δ-ßc)*-Regular spaces and the *Еc (resp*. *δ-ßc)*-Normal spaces.

.

**New classes** of generalized open sets called *Е*c-open and *δ-ßc*-open sets and obtain of several characterizations concerning of these forms of generalized open sets. Furthermore, discuss the relations among *Еc-*(*resp*. *δ*-*ß*c)-open sets and other forms of generalized open sets. Also, introduce and investigate new notions of generalized continuous functions in a topological space, called *Еc*-continuous and *δ*-*ß*c-continuous functions by using new generalized of open sets.

**Conclusion**

**(Chapter-2)**

The notions of compactness are useful and fundamental notions not only of general topology but also of other advanced branches of mathematics and many researchers have studied and investigated the basic properties of compactness, thus we proposed and investigated new kinds of classes of generalized compact spaces called *Еc-(resp. δ-ßc)-*compact spaces. Moreover, the class of generalized open and closed sets has an important role in general topology, especially its suggestion of new separation axioms, the investigation on generalization of open and closed sets has lead to significant contribution to the theory of generalized separation axioms. Therefore, we introduce and study some new types of generalized separation axioms in topological spaces namely *Еc-(resp. δ-ßc)-*separation axioms

The field of mathematical science which goes under the name of topology is concerned with all questions directly or indirectly concerning of generalized open sets, continuity, compactness and separation Axioms…etc. These concepts are significant and fundamental subjects in the study of general topology as well as all branches of mathematics and quantum physics has been researched and investigated by several mathematicians and quantum physicists from the different points of views, of course its weak forms and strong forms are important, too. El-Naschie has indicated in many researches that topology plays important role in quantum physics, high energy physics and superstring theory, one can observe the influence made in the realms of applied research by general Topological spaces, properties and structures, in digital Ttopology, information systems, computational topology for geometric design and, since El-Naschie has shown that the notion of fuzzy topology has very important applications in quantum particle physics especially in related to both string theory and theory, Thus we introduce several new classes of generalized open sets, generalized continuity, generalized compactness and generalized separation Axioms which may have possible applications in quantum particle physics, theoretical physics, particularly in connections with string theory and theory, additionally, the fuzzy topological version of the concepts and results which introduced in this thesis is very important.

**Practical applications in the other fields**

Generalized open and closed sets play a very prominent role in general topology and its applications. And many topologists worldwide are focusing their researches on these topics and this mounted to many important and useful results. Indeed a significant theme in General Topology, Real analysis and many other branches of mathematics, thus we introduced and studied new classes of generalized open sets called *Еc*-opеn and *δ*-*ß*c-open sеts. Furthermore, it is well-known that the branch of mathematics called topology is related to all questions directly or indirectly concern with continuity. Therefore, generalization of continuity is one of the most important subjects in topology. Thus we study new concepts of generalized continuous functions, called ***Еc***-Continuous and ***δ*-*ß*c**-Continuous Functions

**Conclusion**

**(Chapter-3)**

**Figure: (3.4). Summary and Conclusions**

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**رسالــة مقدمــة**

**إلـى مجلس كلية التربية للعلوم الصرفة/ جامعة الانبار**

**كجزء من متطلبات نيل درجة ماجستير في الرياضيات**

**من قبل**

**ســاره حقــي عبـد الواحــد**

**بكالوريوس رياضيات- كليـة التربية للعلوم الصرفـة جامعـة الأنبار 2008**

**بإشراف**

**الأسـتاذ المساعد الدكتـور**

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***م* 2020 *1442ـھ***

**الخلاصـــة**

ان مفاهيم الدوال المستمرة, التراص وبديهيات الفصل من المواضيع المهمة والأساسية ليس فقط لنظرية التبولوجبا لمجموعة النقاط الكلاسيكية ولكن أيضًا في فروع الرياضيات المتقدمة الأخرى, وبالتأكيد فــأن اشكال الأستمرارية الضعيفة والقوية مهمة ايضاً.

, المجموعـــات pre المجموعات المفتوحـة -,α المجموعات شبـــه المفتوحـــة, المجموعات المفتوحة-

, المجموعـــات δ-pre ,المجموعات المفتوحـة –b, المجموعـــات المفتوحـة –pre شبـــه المفتوحـــة -

والمجموعـــات المفتوحـــة *Еc*- المجموعات المفتوحـــة,e- ,المجموعــات المفتوحـــة e\* المفتوحـــة -

تلعب دوراً مهماً فـي تعميم الأستمرارية, التراص وبديهيات الفصل. قـــدَم العديـــد من الباحثيــن,*δ-ßc*-

وناقشوا انواعـاً مختلفة من التعديلات على الــدوال المستمرة, التراص وبديهيات الفصل بأستخـــدام هــذه المجموعات المفتوحـــة المعممة وظهر عدد كبير من الأبحاث التي تتناول هذه المفاهيـــم.

أضافة الى ذلك ، تلعب المجموعـات المفتوحـــة المعممة دورًا مهمًا للغايــة في التبولوجيــا العامــة وهـي موضوعــات بحثية للعديد من علماء التبولوجيا في جميع أنحاء العالم.إنها بالفعل مهمــة فــي التبولوجيـــا العامــة والتحليـل الحقيقي والعديــد من فــــروع الرياضــيات الأخــرى التــي تتعلق بالأشكـــال المعدلــة المختلفـة من الاستمراريــــة ,التراص وبديهيات الفصل من خــــلال استخدام المجموعـــات المفتوحــــة والمغلقــة المعممــة.

في رسالتنا سنقدم ونناقش بعض الأصنــاف الجديدة من المجموعــات المفتوحــة المعممـــة والتي يطلق . *Еc-*open and *δ-ßc-*open sets يطلــق عليها

ومن خلال استخدام هذه الأنواع من المجموعات المفتوحة المعممة سنقدم مفاهيــــم جديــدة من الــدوال

المستمـــرة المعممـــة فــي الفضـــاءات التبولوجيــــة والتـي يطلـــق عليها

, أضافة الى ذلك نقدم أصناف جديـدة من *Еc-* continuous and *δ-ßc-* continuous functions

وذلك بأستخـدام  *Еc-* compact and *δ-ßc-* compact spaces الفضاءات المتراصة تسمى على التوالي بالاضافة الى ذلك *Еc-*open and *δ-ßc-*open sets مجموعات معممة جديدة يطلق عليه نقدم بعض الأنــواع الجديدة المعممة من بديهيات الفصل في الفضاءات التبولوجيــــة والتي اطلقنا عليها وذلك بأستخـــدام المجموعـــات *Еc-* separation axioms and *δ-ßc-* separation axioms أسم

كذلك بأستحدام *Еc-*open and *δ-ßc-*open sets

هـــذه المجموعـــات الجديــــدة المعممة تم تقديم مفاهيــم جديــدة في الفضاءات التبولوجيـــة اطلقنا .*Еc-* normality and *δ-ßc-* normality وكذلك  *Еc-* regularity and *δ-ßc-* regularity عليها