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Studying the Electric Quadruple Moments and Electric Transitions Probability of Nd(A=146-154) Isotopes

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Abstract. In this research, the nuclear structure of the deformed even-even Nadinum(Nd)isotopes of mass region around A=(146-154Nd) have been studied theoretically using the framework of the first interacting boson model (IBM-1) to calculate the energy levels E(L), transitional energy (E_{Υ}),electric transition probability B(E2) and electric quadruple moments(Q_L), as well as the electric quadruple moments could be calculated to know the shape of the deformation. The model calculations gave a best fit with experimental results for energy levels.

INTRODUCTION

The interacting boson model (IBM) has been remarkably successful in describing the low –lying states in many medium and heavy even –even nuclei with (+ve) parity [1].

The interacting boson model-1 (IBM-1) which proposed by [2] and [3]is an important model that is used to study the nuclear properties of rotation ground state, nuclei are considered as systems composed of bosons. The total number of bosons (N) depends on the number of active nucleons(or hole) Pairs outside a closed shell and it can be calculated by adding the number of neutrons pairs and protons pairs of (s and d)bosons[4] i.e.

$$N = N_\pi + N_\upsilon \quad (1)$$

Where: N_π is the number of s-bosons

N_υ: the number of d-bosons.

The Nadinum is a nucleus with proton number (Z=60),the structure of Nd(A=146-154) isotopes in the N= 86 to 94 The Interacting Boson Model-1 is successful in studying the properties of nuclei when the total number of bosons N>>0, but it fails whenever N =2,8,20,28,50,82,and126 where N=0 because, there is no interacting between proton and neutron bosons (i.e there is no degree of freedom)

THE HAMILTONIAN OPERATOR IN (IBM-1)

The suitable formula of the Hamilton function of the (IBM-1) is the formula assumed by Arima and Iachello [5,6] :

$$\hat{H} = \varepsilon_s(\hat{s}^{+} \hat{s}) + \varepsilon_d\Big(\sum_m \hat{d}^{+} \hat{\tilde{d}}_m\Big) + \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{1/2} C_L \left[\left(d^{+} d\right)^{(L)} \left(\tilde{d}\,\tilde{d}\right)^{(L)} \right]^{(0)}$$
$$+U_2 \left[\left(d^\dagger d^\dagger\right)^{(2)} \left(\tilde{d}\,\tilde{s}\right)^{(2)}\right]^{(0)} + \left(\tfrac{1}{2}\right)U_0 \left[\left(\hat{s}^\dagger \hat{s}^\dagger\right)^{(0)}\left(\tilde{s}\,\tilde{S}\right)^{(0)}\right]^{(0)} \quad (2)$$

Where ($s^\dagger, d^\dagger$) and ($\tilde{s}, \tilde{d}$) are creation and annihilation operators for s- and d-bosons, respectively. The effect of the Hamiltonian function in equation (2) includes two parameters describing a single particle and The connection

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between energy levels E(L) parameters describing the two interacting particles represented by C_L(L=0,2,4) and four parameters clarifying the two interacting particles, represented by U_L (L = 0, 2), V_L(L = 0, 2), and all of these parameters based on the number of N bosons equal to $(n_s + n_d)$.The equation (2) can be written within various formulas, but the most general one is the formula [5,6]:

$$\hat{H} = \varepsilon \hat{n}_d + a_o(\hat{P}^{\dagger}.\hat{P}) + a_1(\hat{L}.^{\dagger}\hat{L}) + a_2(\hat{Q}^{\dagger}.\hat{Q}) + a_3(\hat{T}_3^{\dagger}.\hat{T}_3) + a_4(\hat{T}_4^{\dagger}.\hat{T}_4) \tag{3}$$

Where $\hat{n}_d$, $\hat{p}$, $\hat{L}$, $\hat{Q}$, $\hat{T}_3$ and $\hat{T}_4$ are the sum number of d-boson, pairing,angular momentum, quadrupole, octoupole and hexadecapole operators, respectively

Since ($\epsilon = \epsilon_d - \epsilon_s$) refers to the difference between the energy of the bosons (d, s), and for comfort. was considered that the energy of the s boson is equal to zero (ϵ_s= 0) and that:

$$\left.\begin{aligned}
\hat{n}_d &= (\hat{d}^{\dagger}.\hat{\tilde{d}}) \\
\hat{P} &= 1/2(\hat{\tilde{d}}.\hat{\tilde{d}}) - 1/2(\hat{\tilde{s}}.\hat{\tilde{s}}) \\
\hat{L} &= \sqrt{10}[\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(\ell)} \\
\hat{Q} &= [(\hat{d}^{\dagger} \times \hat{\tilde{s}}) + (\hat{\tilde{s}}^{\dagger} \times \hat{\tilde{d}})] - \frac{\sqrt{7}}{2}[\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(2)} \\
\hat{T}_3 &= [\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(3)} \\
\hat{T}_4 &= [\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(4)}
\end{aligned}\right\} \tag{4}$$

The parameters α_2....... α_4, they express the power of the interaction of the pairs, angular momentum, electric quadruple, is among the bosons, respectively.

THE ELECTRIC QUADRUPOLE MOMENT (QL)

The nuclear electric quadrupole moment **(QL)** could be defined as the amount of deviation from a symmetric spherical distribution with respect to the nuclear charge within the isotopes . The quadrupole electric moment takes the dimensions of the area and it is measured in barns unit or square meters unit, while the form of the isotopes is spherical at (Q = 0) and the form of the isotopes is deformed prolate at (Q> 0) or is deformed oblate at (Q <0). To derive the electric quadrupole moment values , can be use the values of B (E2, Li→L_f), as in below equation: [7-9].

$$Q_L = [16\pi/5]^{1/2}[L(2L-1)/(2L+1)(L+1)(2L+3)]^{1/2}[B(E2, L_i \rightarrow \text{L+2}) \tag{5}$$

while B (E2) is the electric transitions probability, L represents the angular momentum, L $_i$ is the initial angular momentum, L $_f$ is the final angular momentum**.**

RESULTS AND DISCUSSION

In this paper, the properties of the isotopes $146 - 154_{Nd}$ died such as the energy levels E(L) , electric transitions probability B(E2), and exited energy states $E_{?}$, The calculated and experimental energy bands (g, β, γ) and energy ratio were used to find the classified and behaviour of the nuclei under study in order to choose the parameters of the Hamilton function the equation (3). These calculations found that the isotopes $146 - 154_{Nd}$ belongs to the O (6)-SU (5), O (6) , O(6), SU (3) –O (6),SU(3) . symmetries.

- **Energy levels and parameters of the Hamiltonian:**

The energy values in the IBM-1 model calculated from (IBM1. For) program through the Input File (BOS. INP), which contains seven parameters shown in Table (1), the values of these parameters are determined by fitting the values of practical energy levels with theoretical energy level values.

TABLE 1. The parameter values used in the (IBM .For) program measured in (MeV) unit.

Parameters / Nuclei	N_π	N_υ	N	EPS (MeV)	$\hat{P}^\dagger.\hat{P}$ (MeV)	$\hat{L}.\hat{L}$ (MeV)	$\hat{Q}.\hat{Q}$ (MeV)	$\hat{T}_3.\hat{T}_3$ (MeV)	$\hat{T}_4.\hat{T}_4$ (MeV)	CHI (MeV)
${}^{146}_{60}Nd_{86}$	2	5	7	0.000	0.080	0.010	0.000	0.256	0.000	0.000
${}^{148}_{60}Nd_{88}$	3	5	8	0.000	0.034	0.010	0.000	0.081	0.000	0.000
${}^{150}_{60}Nd_{90}$	4	5	9	0.000	0.034	0.000	0.000	0.081	0.000	0.000
${}^{152}_{60}Nd_{92}$	5	5	10	0.000	0.000	0.033	0.033	0.000	0.000	0.000
${}^{154}_{60}Nd_{94}$	5	6	11	0.011	0.000	0.012	0.011	0.000	0.000	0.000

- **The electric transitions probability B(E2):**

TABLE 2. The parameter values derived from B (E2) using (IBMT. For).

Nuclei	β_2 (eb)	α_2 (eb)
${}^{146}_{60}Nd_{86}$	0.01000	0.01000
${}^{148}_{60}Nd_{88}$	0.01300	0.01200
${}^{150}_{60}Nd_{90}$	0.03000	0.00667
${}^{152}_{60}Nd_{92}$	0.05170	0.05190
${}^{154}_{60}Nd_{94}$	0.22890	0.22890

The general form of B (E2) was written in equation (6) when L = 2 these transitions obey selection on the eqution (6) [10].

$$T(E2) = \alpha 2\, [d \times s + s \times d]^{(2)} + \beta 2\, [d \times d]^{(2)} \tag{6}$$

Through the transition effect of the electrice quadruple and determinig the values of the two workers (α_2 , β_2) for elementary and final cases .The calculation of the quadruple transitions is done on the equation B (E2) .

- **Electric quadrupole transition We can explain two type of deformation as follows [11].**
- **Prolate deformation:**

In this deformation the Isotope rotate around an axis perpendicular to the nuclear symmetry axis .This rotion is called collective rotation [12].

- **Oblate deformation:**

In this deformation the Isotope is rotating in axis parallel to the nuclear symmetric axis.This rotating happened for spherical nuclei and it is called non-collective rotation .

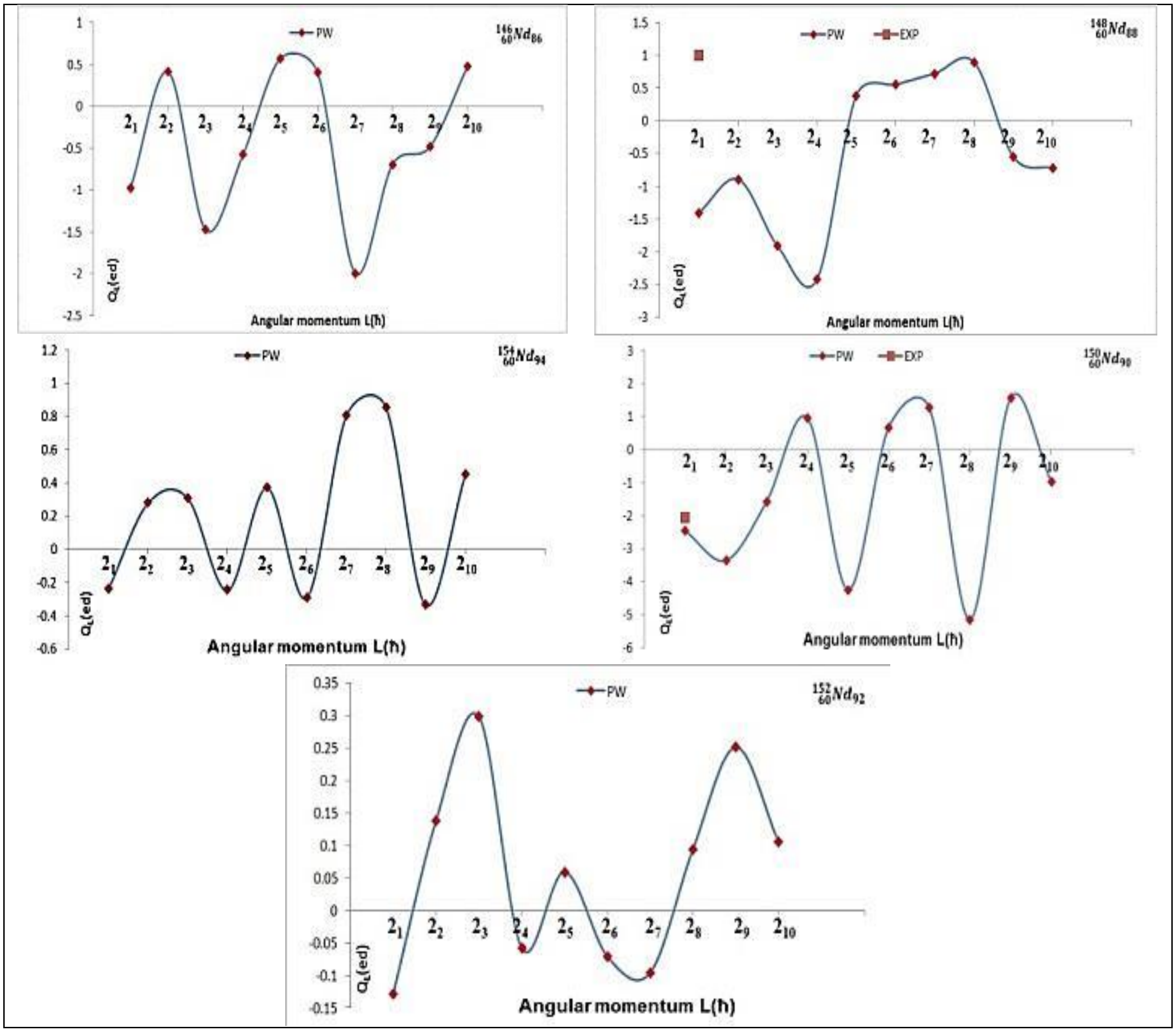

FIGURE 1. Shows the relationship between the practical Guardable Momentum (eb) compared with theoretical energy Angular Momentum L(h) by the (IBM-1) model for ground state band as a function of angular momentum, it was noted from this figure.

According to below graphic we explained the following:-

1. In Isotope ($^{146}_{60}Nd_{86}$) : the quadrpoie torque was found to be the first irritating levele Q (2_1^+) . its value is negative . that means the Isotop take (oblate oval shape). that is an isotopes on an axis that is parallel to the axis of symmetry . its value largest (-0.973) eb . the four.pole torque of the agitated plane has apostive value at Q (2_2^+) .that means the isotope take (prolate 0val shape). the value of the quadrpole is reduce as momentum increases as one move toward the closed shell. Until it because Zero .where the isotope is spherical. the isotope type take (prolat shape). At angulare momentum $(2^+_{3,}2^+_4, 2^+_5, 2^+_6)$. While take the (oblate value shape) at the angular momentum $(2^+_7, 2^+_8, 2^+_9, 2^+_{10})$
2. In Isotope ($^{148}_{60}Nd_{88}$) : take the (oblate oval shape)at the angulare momentum $(2^+_1, 2^+_2, 2^+_3, 2^+_{4,}2^+_5)$. While takes the (prolate oval shape) at the angulare momentum $(2^+_6, 2^+_7, 2^+_8, 2^+_9, 2^+_{10})$. And the Largest deformation of the (oblate ovale) type is at the angulare momentum Q (2^+_4) . and its value (- 1.402) eb.

3. In Isotope (${}^{150}_{60}Nd_{90}$) : take the (prolate oval shape) at the angulare momentum ($2_1^+, 2_2^+, 2_3^+$) . while take the (0blate oval shape) at the angulare momentum ($2_4^+, 2_5^+, 2_6^+, 2_7^+, 2_9^+, 2_{10}^+$) . and the largest deformation of the angulare momentum Q(2_8^+) and its value (-2. 463) eb.
4. In Isotope (${}^{152}_{60}Nd_{92}$) : the highest deformation of the angulare momentum Q (2_1^+) .take the (oblate oval shape) at the angulare momentum ($2_6^+, 2_8^+, 2_{10}^+$). While take the (prolate ovale shape) at the angulare momentum ($2_2^+, 2_3^+, 2_4^+, 2_5^+, 2_7^+, 2_9^+$) and its vale (- 0.128) eb .
5. In Isotope (${}^{154}_{60}Nd_{94}$) : we note the isotope type in highest deformation of angulare momentum Q(2_1^+) .but that the (oblate ovale shape) at angulare momentum ($2_2^+, 2_3^+, 2_5^+, 2_7^+, 2_8^+, 2_{10}^+$) . While take (prolate oval shape) at the angulare momentum ($2_4^+, 2_6^+, 2_9^+$) . and its value (- 0. 241) eb .

The Graphs in Figure (2). This graphic Shows that the probability of electric Transition increases an angular momentum reaches its maximum value , which indicates that the probability of electric transition is the support . The probability of magnetic transition is too low or not allowed . The probability of electrical transition decreases rapidly as angular momentum increases. That means they are less likely to move Or The transition could be a combination of the electric transition (E2) and Magnetic transition (M1). That means the probability of dipole magnetic transition is the anvil.

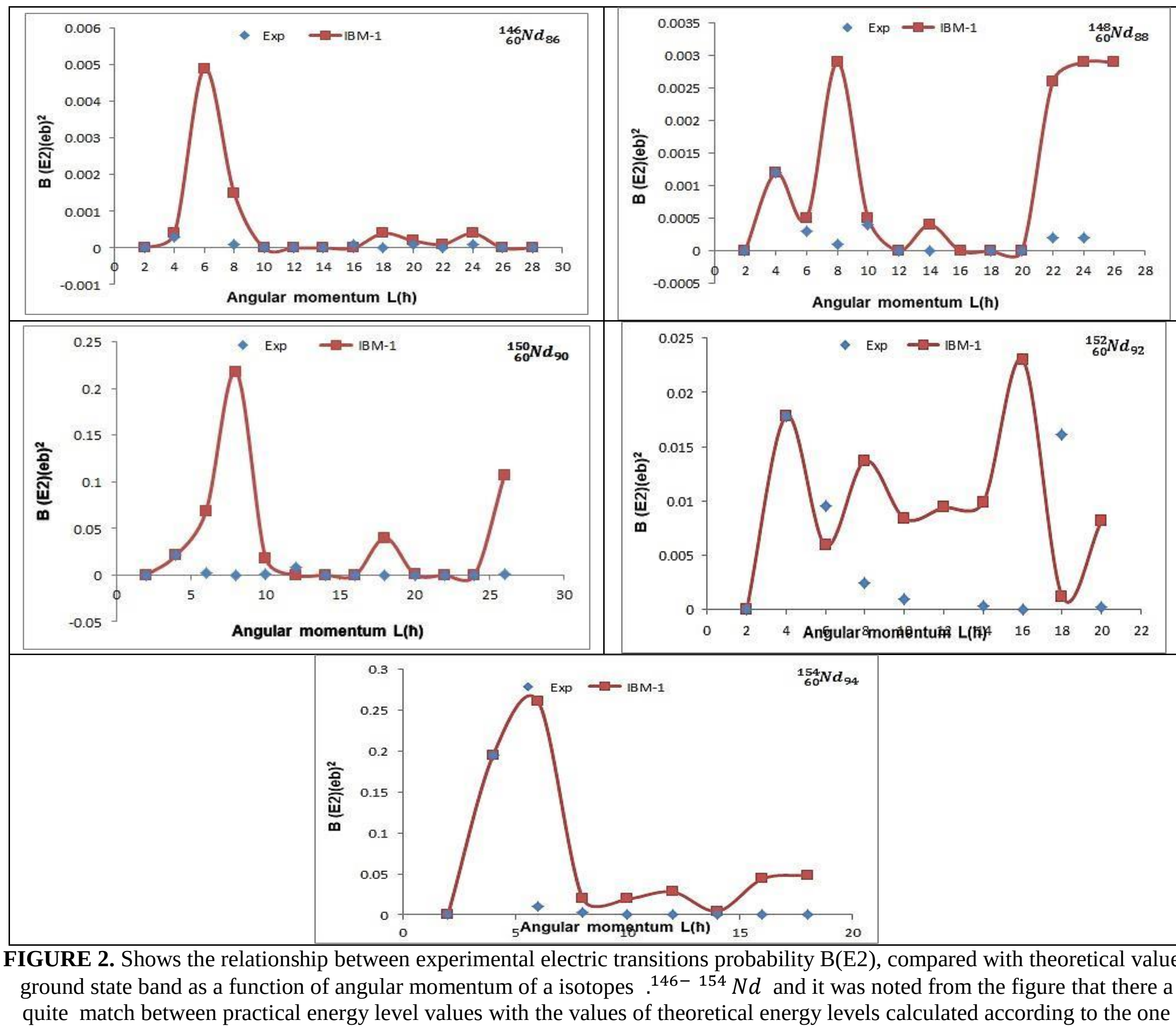

FIGURE 2. Shows the relationship between experimental electric transitions probability B(E2), compared with theoretical values ground state band as a function of angular momentum of a isotopes .${}^{146-154}Nd$ and it was noted from the figure that there a quite match between practical energy level values with the values of theoretical energy levels calculated according to the one model (IBM-1) for (L≤10) either the values of theoretical energy levels calculated by the (IBM-1)model .

- **Calculation of Energy levels using (IBM-1) model**

The relation shape between the values of the theoretical energy levels calculated under the (IBM-1) model was drawn in comparison with the practical values of the Earths band as a function of angular momentum for all the isotopes studied . it was noted from figure (3) that the similarity between the values computed under the (IBM-1) model .

1. for Isotopes (${}^{146}_{60}Nd_{86}$) and (${}^{148}_{60}Nd_{88}$). Was observed Give a match to the practical values.
2. in Isotopes (${}^{150}_{60}Nd_{90}$) with (${}^{152}_{60}Nd_{92}$) , however , they provide the best practical matches over those computed by (IBM-1) model.

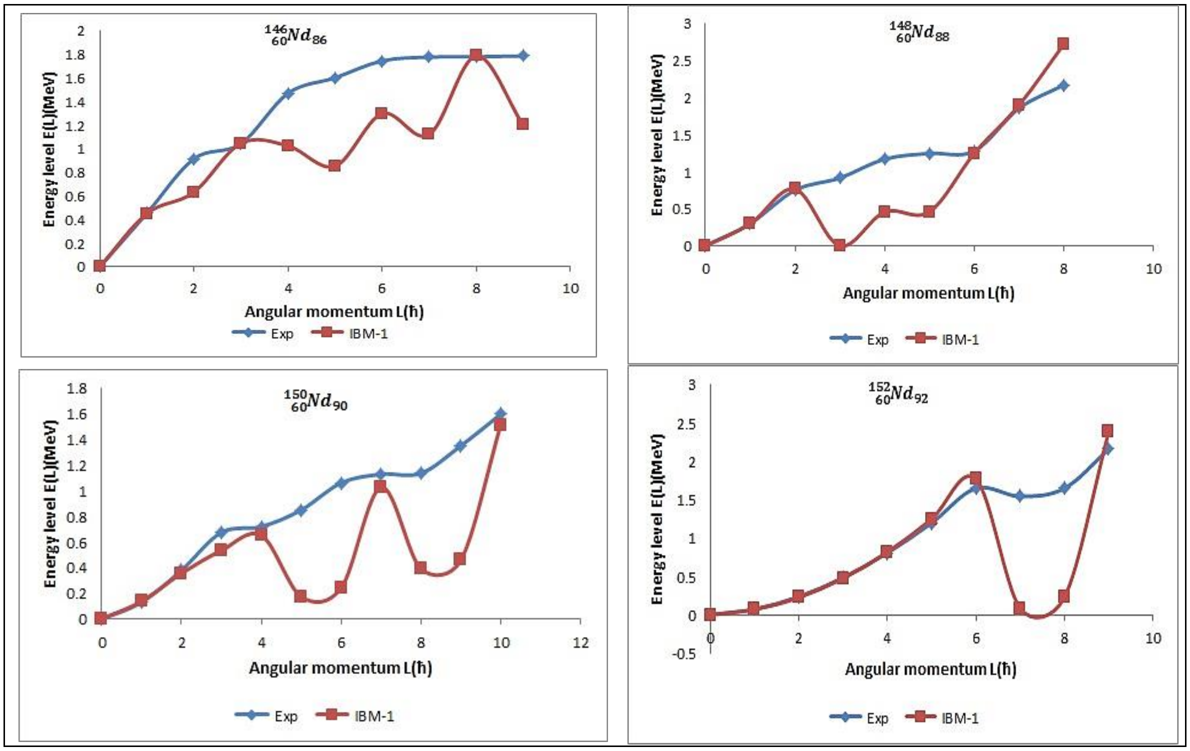

FIGURE 3. The relationship between the energy levels E(L) calculated (IBM-1) and experimental energy levels E(L) versus L for g- band.

CONCLUSION

I have Studied the transition of probability B (E2) and the quadruple momentum of the Isotopes Nd = 146 - 154 by using (IBM-1) model and by using the Practical , Theoretical values and energy transition with angular momentum.

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