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Studying the Nuclear Structure of the ($^{172}_{70}Yb_{102}$) Deformed Nucleus using (IBM-1) and (GVMI) Models

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Abstract: In this research, some properties of the nuclear structure of even-even $^{172}_{70}Yb_{102}$ deformed nucleus were studied with dynamic symmetry SU (3).The energy levels E (L), excited energy levels (E_{γ}), and electric quadrupole moment(Q_L) were calculated using a set of parameters in the calculation to obtain a match with the practical values available using the first interacting boson model. IBM-1 and generalized variable moment of inertia (GVMI) model, The calculated results are in good agreement with the available practical values, especially, the values calculated according to the GVMI program. The phenomenon of band intersection ,and back bending were also studied, the results showed no effect of the moment of inertia on the shape of the nucleus under consideration ,because the back bending phenomenon was not occur.

1-Introduction

Several nuclear models study the nuclear structure, they are called "nuclear models", one of them is liquid drop model which proposed by (Von Weizsacker, 1934) [1] which explained the binding energy and the phenomenon of Nuclear fission, but it did not succeed in explaining the stability of the nucleus. As for the shell model proposed by (W. Elsasier, 1935) [2], practical experiments have shown that the stability of the nucleus is high when the number of nucleons is equal to one of the magic numbers [3]. In addition to that, it determined the angular momentum of energy levels, but at the same time failed to explain the nuclear spin of the ground level of the even-even nucleus, which always equals zero [4].

In 1974 Arima & Iachello [5] proposed a nuclear model in which it was able to describe the characteristics of the energy levels in the even-even nuclei and positive parity(π^+) of medium and heavy mass numbers by the pairs of nucleons outside the closed shell that were treated as bosons [6] so that it did not take into account the degrees of freedom for these bosons, they were called the first interacting boson model IBM-1. This model was developed by introducing degrees of freedom to bosons. As a result of this modification, new nuclear properties of the nuclei were revealed and it was called the second interacting boson model IBM-2 [7].

The first interacting boson model does not distinguish between proton bosons (s_π, d_π) and neutron bosons (s_ν, d_ν). The total number of bosons (N) is calculated from the sum of (N_π proton bosons and N_ν neutron bosons) as pairs of particles, Starting from the closest closed shell to the middle of the next shell, meaning that the number of bosons (N) is equal to the number of pairs of particles outside the closed shell, which is a fully preserved and constant quantity for each nucleus, but if it is more than half of the shell then (N_π) and (N_ν) is taken as the number of Hole Pairs [8,9].

In this model, we find that the S and d bosons can interact with each other, and as a result, the general formula for the Hamiltonian effect for this system is written after the creation effects (S^+ , d^+) and the annihilation effects (S, d) (Annihilation Operators) are defined. [10, 11]:-

$$\begin{aligned} H = {} & \varepsilon_s (s^+ . \tilde{s}) + \varepsilon_d (d^+ . \tilde{d}) + \sum_{L=0.2,4} \frac{1}{2} (2L+1)^{\frac{1}{2}} C_L [(d^+ \times d^+)^{(L)} \times (\tilde{d} \times \tilde{d})^{(L)}]^{(0)} \\ & + \frac{1}{\sqrt{2}} \tilde{V}_2 [(d^+ \times d^+)^{(2)} \times (\tilde{d} \times \tilde{s})^{(2)} + (d^+ \times s^+)^{(2)} \times (\tilde{d} \times \tilde{d})^{(2)}]^{(0)} \\ & + \frac{1}{2} \tilde{V}_0 [(d^+ \times d^+)^{(0)} \times (\tilde{s} \times \tilde{s})^{(0)} + (s^+ \times s^+)^{(0)} \times (\tilde{d} \times \tilde{d})^{(0)}]^{(0)} \\ & + U_2 [(d^+ \times s^+)^{(2)} \times (\tilde{d} \times \tilde{s})^{(2)}]^{(0)} + \frac{1}{2} U_0 [(s^+ \times s^+)^{(0)} \times (\tilde{s} \times \tilde{s})^{(0)}]^{(0)} \end{aligned} \quad \text{---- (1)}$$

Where C_L (L = 0,2,4), V_L (L = 0,2), U_L (L = 0,2) describes the interactions of the bosons with each other, and these coefficients depend on the number of N bosons. As for the parentheses, they represent the angular momentum. Several other formulas are equivalent to the general formula for IBM-1 of Hamilton, where $\varepsilon = \varepsilon_d + \varepsilon_S$ represent the energy of the bosons. To simplify it, the energy of the bosons was considered to be zero, thus: $\varepsilon=\varepsilon_d$

So we can rewrite the equation (1) using the Multiple Expansion Formula [12]: -

$$\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \text{-------(2)}$$

Where $\mathcal{E}_s$ - $\mathcal{E}_d$= $\mathcal{E}$ represents the energy of the bosons, and for ease assumed that the energy of the boson S is equal to zero. As for the coefficients a_0, a_1, a_2, a_3, a_4, they express the strength of the interaction of duality, the angular momentum, the quadrupole, the octopole, and the sixteenth pole between the bosons respectively.

The general formula for the electric quadrupole transition effect can be written with the following relationship[13]:

$$T_\mu^{(E2)}=\alpha_2[d^\dagger s+s^\dagger d]_\mu^{(2)}+\beta_2[d^\dagger d]_\mu^{(2)} \ldots\ldots\ldots\ldots.(3)$$

Based on the values of the energy ratios and their comparison with the typical values, it was found that this core belongs to the rotational determination [14]. So we'll say a little bit about rotational dynamic symmetry.

The second determination presented by (Arima and Iachello) is in which the energy of the boson (ε) is much smaller than the reaction energy (V) i.e. (V >>ε), as (V) represents the energy of the quadrupole momentum reaction (Q.Q) between the bosons. Hamilton's function to determine by the equation[15]:

$$\hat{H}^{(II)} = a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} \ldots\ldots\ldots..(4)$$

From equation (4) we find that the influences ($\hat{\varepsilon}, \hat{P}, \hat{T}_3, \hat{T}_4$) are ineffective, as we notice that The angular momentum dipole interaction (L.L) in addition to the electrode quadrupole interaction (Q.Q) between bosons are dominant in this determination. As for Hamilton's eigenvalues equation SU (3), it is given by the following relationship [14]:

$$E\left|N,(\lambda,\mu),K,L,M\right\rangle = \frac{a_2}{2}(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)+(a_1-\frac{3a_2}{8}).L(L+1)-------(5)$$

The eigenvalues are known as the previously defined quantum numbers (M, L, N) over the two quantum numbers(μ, λ). As they represent the states of SU (3) and the quantum number **K** denotes the states that have equal values for and the figure (2 -2) shows a typical spectrum for the determination of SU(3)[12,15].

The reduced electromagnetic transition probability can be calculated from the relation [16]:

$$B(E2, L_i \rightarrow L_f) = \frac{1}{2Li+1}\left|\left\langle L_f \left\| \hat{T}^{(E2)} \right\| L_i \right\rangle\right| \text{-------------------}(6)$$

Where: $\left|\left\langle L_f \left\| \hat{T}^{(E2)} \right\| L_i \right\rangle\right|$ the reduced matrix elements for the electric quadrupole transition probability.

2- Electric quadrupole Momentum:

The moment of the electric quadrupole is another important property of the cores, and is defined as the amount of anomaly or deviation from the symmetric spherical distribution of the electrical charge inside the core. Knowing it gives an idea of the shape of the nucleus and calculates its value through the equation:

$$Q_L = <L, M_L = L \mid \sqrt{\frac{16\pi}{5}}\, T_0^{(E2)} \mid L, M_L = L> = \sqrt{\frac{16\pi}{5}} \begin{pmatrix} L & 2 & L \\ -L & 0 & L \end{pmatrix} < Lf \left\| T^{\wedge(E2)} \right\| L_i > \cdots (7)$$

Where it was found in practice that the value of the moment of the electric quadrupole is equal to zero for the spherical nuclei (closed shell), and as the number of bosons increases, the moment value of the electric quadrupole increases, meaning that the amount of distortion increases until the core becomes permanently deformed, and the nuclear deformation is of two types:

1-Prolate deformation :

In this type, the Nucleus rotates around an axis perpendicular to the axis of nuclear symmetry. This type has a positive Value of the of the electric quadrupole moment (+Q_L).

2- Oblete deformation :

In this type, the Nucleus rotates in an axis parallel to the symmetric nuclear axis. This type occurs in spherical cores, and has a negative value of the electric quadrupole moment ($-Q_L$).

3- Generalized Variable Moment of Inertia (GVMI) Model

D. Bonatsos and A. Klein [17] proposed a new model for calculating energy levels of nuclei. This model is more comprehensive than the Variable Moment of Inertia (VMI) Model and the Harmonic Vibration Model. The general formula for energy levels in this model contains the basic functions of (VMI) that change with the change of angular momentum L and L(L+1). The energy levels as a function of angular momentum (L) is [18]:

$$E(L) = \frac{1}{2}\left[\frac{L(L+1)}{\vartheta(L)} + C(\vartheta(L) - \vartheta_0)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where :

$\vartheta(L)$ is a moment of inertia as a function of L, and C, ϑo are parameters which are suitable with fitted to the experimental data.

This equation consists of many terms, D. Bonatsos and A. Klein reduced these terms to two terms[17]. They are based on the same basic functions. The theoretical results obtained from this model are fully consistent with the experimental values of energy levels for SU (3), SU (5) for the nuclei more than those found in the other models.

A.M. Kalaf and others (2020) [19] used a variable moment of inertia model (VMI) to study identical bands (IB's) for mass number nuclei $158 \leq A \leq 170$. As results were obtained that matched the practical values.

4- Energy Band intersection:

The intersection of band means that the moment of inertia of a specific energy level for any band such as (β) or(ɤ) intersects or replaces any energy level of any other band such as a g-band . Between any two band have the same spin but differ in energy and moment of inertia[20].

5- Results and discussion:

In this paper, the properties of the even-even ($^{172}_{70}Yb_{102}$) nucleus were studied (such as energy levels, transition energy Eɤ, and the potential for electrical transfers), the energy levels of this core were arranged in the form of bands according to their appearance and the arrangement of the bands was as follows (g, β1, β2, ɤ1) and this arrangement indicates that the nucleus belongs to the symmetry SU (3) or SU (5), where the arrangement of these bands is very important for its use in calculating the behavior of the selected nuclei and the values of the moment of inertia for all the bands, and through the practical energy ratios it was found that the nucleus ($^{172}_{70}Yb_{102}$) belongs to the symmetry SU (3). The energy levels for this nucleus were also calculated using the first interacting boson model IBM-1 and the GVMI program. The energy levels for IBM-1 were calculated through the best interaction coefficients suitable for Equation

(2) shown in Table (1) of Matches theoretical values (pw) with practical values (exp.). While the GVMI values were calculated for any band of parameters (E_k). These measurements were chosen from the smallest value of chi-square (χ^2)[18] :

$$x^2 = \frac{\left(E_{cal} - E_{exp.}\right)^2}{E_{cal}^2} \dots\dots\dots\dots\dots (9)$$

where the parameters corresponding to the value of chi-square were given in Table (2) for all bands. This table shows that the parameter E_k is equal to zero for the ground band and takes the band head energy for the bands (β_1, β_2). As for the gamma band (γ_1-band), this value is variable.

Figure (1) shows the ground band where there is a match between the values of the practical energy levels and the IBM-1 values for the levels L<8. As for the levels L> 8, there is a convergence between the energy levels, this difference in values is attributed to the IBM-1 model does not distinguish between bosons Protons N_π and bosons N_V neutrons. As for the GVMI moment of inertia model, there is a great match between the theoretical value and the practical values for all levels of the ground band to choose the best parameters for this model.

Figure (2) shows the beta (β1) band where theoretical values for IBM-1 programs and L<6 levels are identical to the practical values. For high levels of L>6, the theoretical values are greater than the practical values. As for the GVMI model values, they are completely in agreement with the practical values for all levels.

Figure (3) shows that there is no good match between the theoretical values of (IBM-1) and the experimental values, as the results agree only at momentum L = 6, while there is a convergence of the GVMI values with the experimental values better than the IBM-1 values, which are greater or smaller than the rest of the values.

Figure (4) shows the calculated results for the gamma band (γ1) where the IBM-1 results are slightly higher than the other results at low levels and far apart at L> 6 levels because this model does not distinguish between proton bosons and neutron bosons. As for the results of GVMI, they agree with the experimental results at specific levels and diverge slightly at other levels.

The phenomenon of band intersection illustrated in Fig. (5) is an important topic because it is used to explain the phenomenon of back bending where bands intersect (β1, β2, γ1) at the angular momentum L = 14. The dissociation of a pair of nuclei crosses the bands and forms a back curvature phenomenon that was not seen in this nucleus. Band intersection also occurs due to a low moment of inertia at large angular momentum. The effect of the Curious force will reduce the energy of the nucleons, and the instability of these two excited states leads to the intersection of one beam with another.

Figure (6) shows the shape of the Nucleus and the amount of Deformation; where it was observed that the shape of the Nucleus takes the shape of an Elongated Oval at Momentum (21+, 23+, 25+, 26+, 28+, 210+). While it takes a flat ellipse shape at Momentum (22+, 24+, 27+, 29+) and has the highest deformation at Angular Momentum (21+) and its Value (1.964 eb).

Figure (7) shows the relationship between the moment of inertia and the square of the rotational energy, Where it is noticed from the figure that there is no back bending in the bundles of this core, which indicates the absence of an effect of the moment of inertia on the shape of the nucleus.

References:

[1] C.F. Von Weizsacker, Z. Phys. Vol. 6, P.431, (1935).

[2] W.J. Elsasser, Phys. Vol. 4, P.549, (1933).

[3] W. Meyerholf, "Elements of Nuclear Phys.", Mc Graw-Hill, (1967).

[4] H. A. Enge. " Introduction to nuclear Phys.", (1983).

[5] F. Iachello, and A. Arima, Phys., Lett. B53, P.309, (1974).

[6] A. Arima, T. Otsuka, F. Iachello, and I. Talmi, Phys. , Lett. B66, P.205, (1977).

[7] R.F. Casten, A. Gelberg and P. Von Brentano, Phys. Rev. C36, P.1225, (1987).

[8] S.H.Trier," A study of nuclear Structure of 160-180Hf even-even Isotopes by the IBM-1.", M. Sc. Thesis, Babylon University, (2006).

[9] O.Scholten, F. Iachello, and A. Arima, "Interacting Bosons Model of Collective Nuclear States III. The Transition Form SU(5) to SU(3) .", Ann.phys.115, (1978).

[10] W.Greiner,J.A.Maruhn, "Nuclear Models.",Spring-Verlag Berlin Heidelberg NewYork,(1996).

[11] F. Iachello," An Introduction to the Interacting Boson Model .", Plenum Press,(1980).

[12] R. Casten and D. Warner, Rev. Mod. Phys. 60, 389, (1988).

[13] A. Arima and F. Iachello , Ann. Phys.,123 ,468 ,(1979).

[14] K. Abrahams, K. Allaart, and A. Dieperink, Dieperink nuclear structure, New York and London,67,53, (1981).

[15] A. Arima, and F. Iachello, Ann. Phys . 111 , 201 , (1978).

[16] J . Jabber , "Decay Schemes From the (n,γ) Reaction on 151Eu and 181Ta" , Ph. D Thesis , London University , (1989).

[17] D. Bonatsos and A. Klein; phys. Rev. C,Vol.29,P.1879 (1984).

[18] M. A. J. Mariscotti , Gertude Scharff – Gold Haber , and Brain Buck ; Phys. Rev. vol. 178 , p.1864 (1969).

[19] A. M Khalaf.,Asmaa Abdelsalam ,"Identical bands in doubly even nuclei in framework of variable moment of inertia (VMI) and interacting boson models" vol 997,(2020).

[20] J .p Elliott, , and A. P. white Phys. Lett. B vol.97.p.169. (1980).

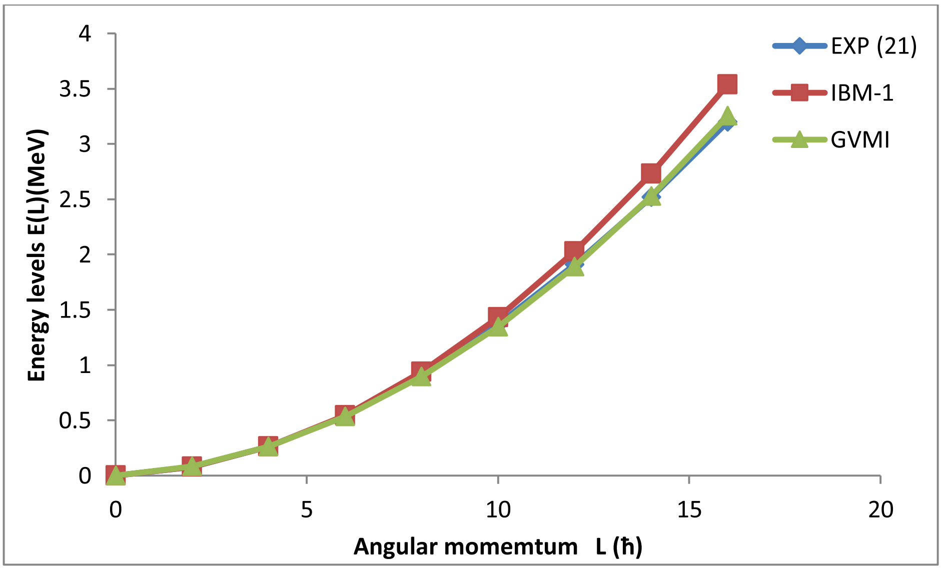
[21] Balraj Singh , Nuclear Data Sheets 75, 199, P.214 (1995).

Table (1): The best fitted interaction parameters of equation (2) for the energies in(MeV)units

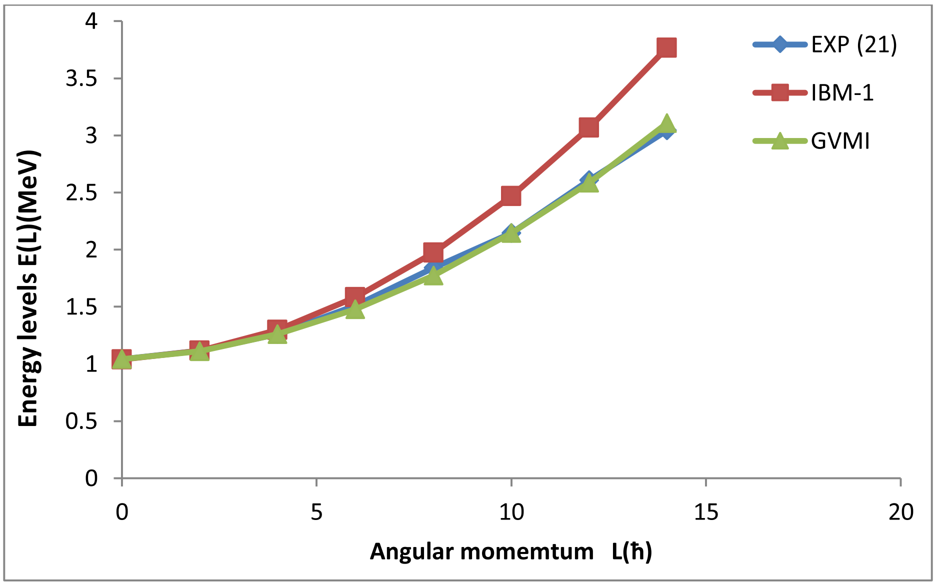
E	*P̂ † .P̂*	*L̂.L̂*	*Q̂.Q̂*	*T̂3 .T̂ 3*	*T̂4 .T̂ 4*	*CHI*
0.0001	*0.0000*	*0.0088*	*-0.0112*	*0.0000*	*0.0000*	*-1.3200*

Table (2):The best fitted parameters of equation(5)with ($\chi 2$) value.

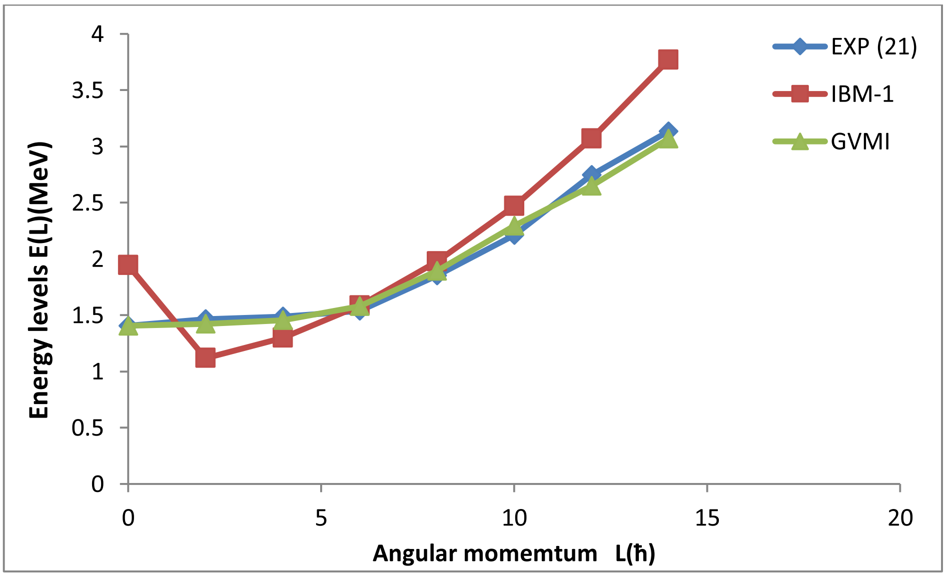
Band	ℓ_0(MeV)$^{-1}$	C(MeV)^3	E_k(MeV)	χ^2
G	23.3000	56.0000	0.0000	0.0027
β1	28.0100	56.5000	1.0420	0.0045
β2	35.2000	57.0000	1.4060	0.0276
γ1	8.9999	52.1000	1.1390	0.0870



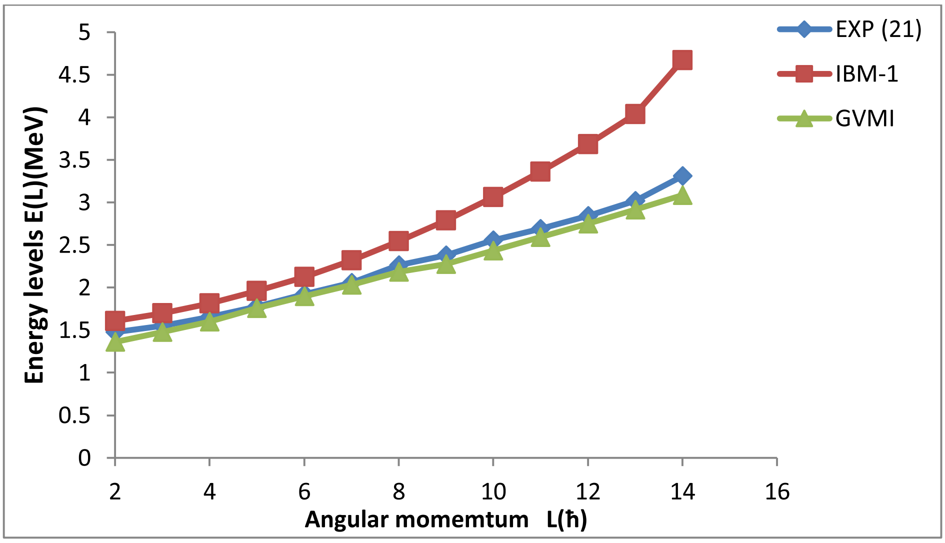
Figure(1): Three calculated IBM-1, GVMI, and experimental[21] E(L)versus L for g-band



Figure(2): Three calculated IBM-1, GVMI, and experimental[21] E(L)versus L for B1-band



Figure(3): Three calculated IBM-1, GVMI, and experimental[21] E(L)versus L for B2-band



Figure(4): Three calculated IBM-1, GVMI, and experimental[21] E(L)versus L for gamma 1-band

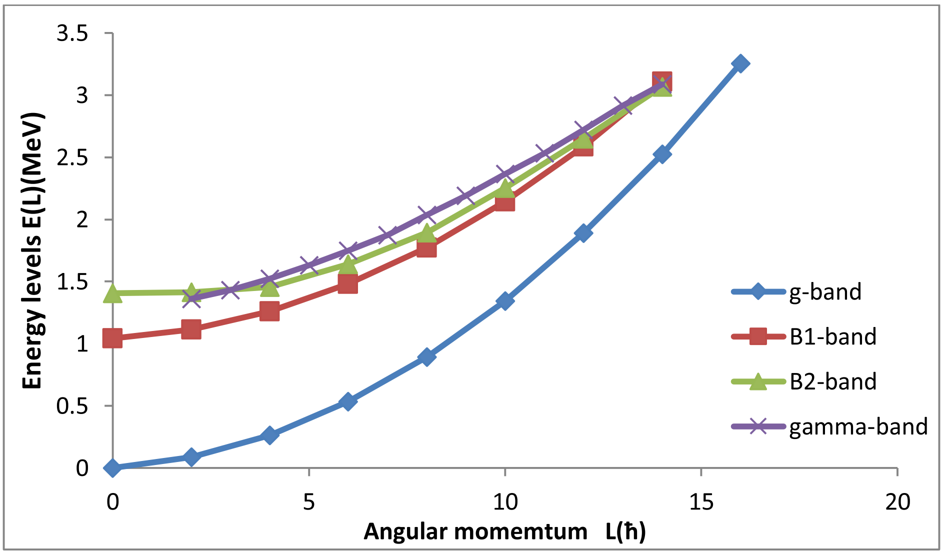


Figure (5): The energy band crossing [E(L) as a function of angular momentum(L)]using the GVMI model. The parameters used for calculations are in Table (2)

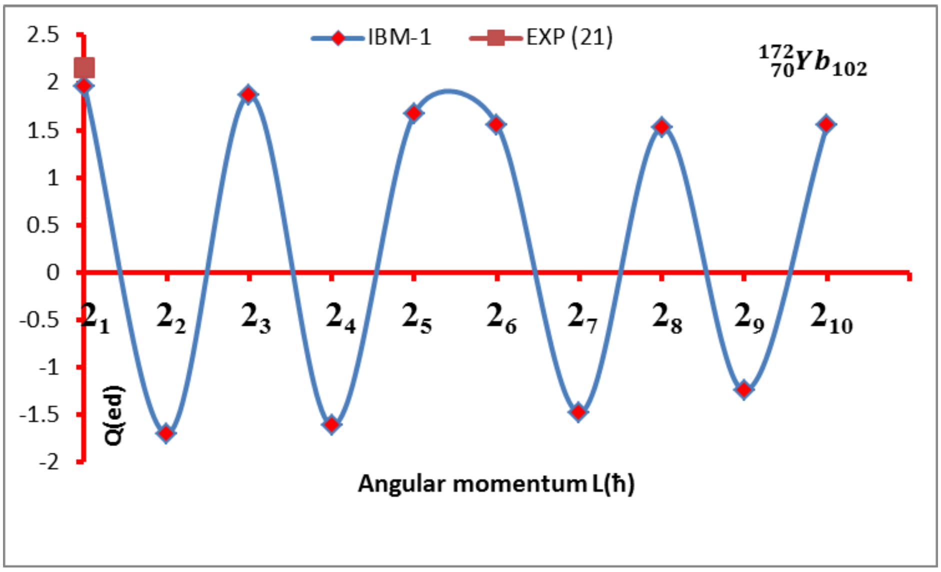


Figure (6): The electric quadrupole moment as a function of angular momentum L=2_1 to 2_{10}.

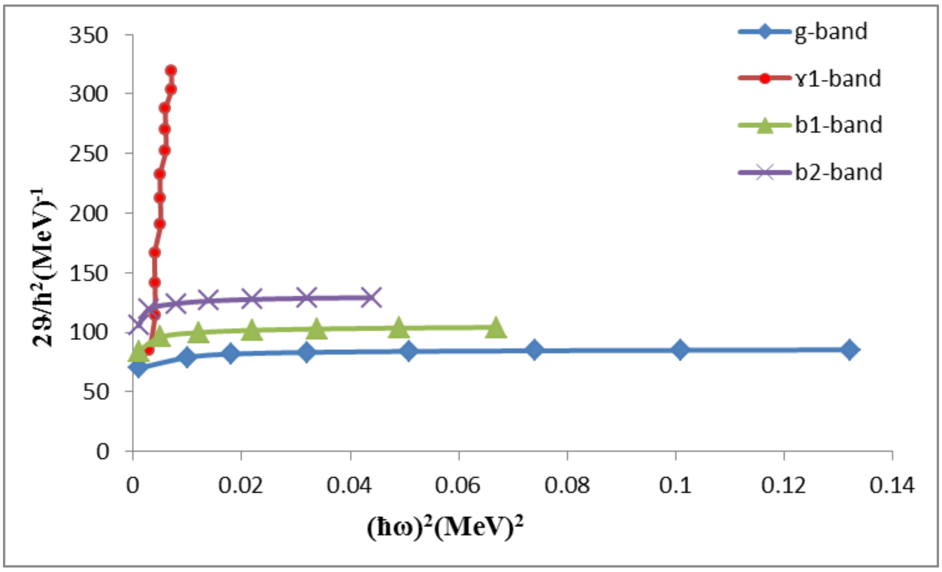


Figure (7): The back bending ($2\vartheta/\hbar^2$) versus $(\hbar\omega)^2$ plot for chosen nuclei using (GVMI) model.