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Studying the nuclear structure of the ($^{162}_{66}Dy_{96}$) deformed nucleus by using (IBM-1) and (VAVM) Models

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Abstract. In this paper, the nuclear structure of the deformed even-even $^{162}_{66}Dy_{96}$ nucleus was studied using the first interacting boson model (IBM-1) to calculate the energy levels E(L) and the transitional energy (E_γ). The energy bands, as well as the quadrupole moment (Q_L) can be calculated to know the shape of the nucleus and the moment of inertia model, energy levels E(L), transitional energy (E_γ) and the bands intersection. The current study showed that at dynamic symmetry SU (3), there is no effect of inertia moment on the shape of the rotational nuclei due to the absence of the phenomenon of back bending that belongs to this dynamic symmetry, through calculating the energy levels ratios found that $^{162}_{66}Dy_{96}$ nucleus belongs to the rotational dynamic symmetry SU (3). A comparison of the available practical results with the results calculated by (VAVM) and (IBM-1) showed that the (VAVM) model are better than the results calculated by (IBM-1).

Keywords: Nuclear structure, Intersection, bosons, moment of inertia.

Introduction

Nuclear physics has had vast quantities of theoretical and experimental energy levels data and information related to the nuclei due to many researchers tried to penetrate into these nuclei or because of the attempt to disassemble these nuclei into their various components, so it has become the duty of nuclear physicists to develop a nuclear model, which is the first step to understand the observed and measured data, relate them and draw conclusions. In spite of the great success achieved by many of the proposed nuclear models in linking data and explaining the nuclear properties, they did not reach the stage of adopting a "one" model, that is, a comprehensive unified theory that can explain everything related to the nuclei in terms of structure and interactions. The most important basic nuclear models proposed to describe the interaction between nucleons and are currently applied are the shell model, the liquid drop model and the collective model. Each of these models is based on a set of assumptions and may be useful. Within certain limits, it can interpret a "specific" range of experimental energy levels data, but it may fail when applied to data outside that range. For example, the "shell model is appropriate" if it is assumed that the interaction between nucleons is a weak interaction, while the liquid drop model or the collective model is used to describe the strong interactions between nucleons, researchers (Arima and Iachello) in 1974 [1] presented a new "nuclear" model called the first Interacting Bosons model This model relied in many aspects on group theory. It is a system of bosons (L = 2) s, (L = 0) d interacting with each other. This model (IBM-1) does not distinguish in its first formula between protons and neutrons bosons.

IBM-1 model is one of the important subjects which use for the purpose of studying some nuclear properties of each odd -mass or even-mass nuclei .in this model we deal with the nucleus with even numbers both of neutrons and protons, i.e. the nuclei (even-even) as a system of neutrons or proton bosons the calculations resulting from this model have shown that there is agreement with practical experiences and the first interacting boson model has had a widely success in describing the fine structure of the nuclei, especially at low energy levels[2] . And the interacting boson model builds on the boson holes inside a nuclear closed shell or the interacting valance boson particles outside a closed shell.

N is dependent on the number of active nucleon or(hole) pairs outside the closed shell and the total number of bosons can be calculated by adding the number of proton pairs with the number of neutron pairs that can be their written in the following equation [3]:

$$N = n_s + n_d \quad \ldots\ldots\ldots\ldots(1)$$

Where : n_s is the number of s-bosons

n_d: the number of d-bosons.

The Hamiltonian operator in (IBM-1)

The most appropriate syntax for the operator of the Hamilton function is the formula postulated by Arima and Iachello [4,5] and Iachello [6]

$$\hat{H} = \varepsilon_s(\hat{s}^{+}\hat{s}) + \varepsilon_d(\sum_m \hat{d}^{+}\hat{\tilde{d}}_m) + \sum_{L=0,2,4}\frac{1}{2}(2L+1)^{1/2}C_L\left[\left(d^{+}d\right)^{(L)}\left(\tilde{d}\tilde{d}\right)^{(L)}\right]^{(0)}$$

$$+V_2\left[\left(\hat{d}^{\dagger}\hat{d}^{\dagger}\right)^{(2)}\left(\tilde{d}\tilde{s}\right)^{(2)} + \left(s^{\dagger}d^{\dagger}\right)^{(2)}\left(\tilde{d}\tilde{d}\right)^{(2)} + \left(\frac{1}{2}\right)^{1/2}V_0\left[(\hat{d}^{\dagger}\hat{d}^{\dagger})^{(0)}\left(\tilde{s}\tilde{s}\right)^{(0)} + \left(\hat{s}^{\dagger}\hat{s}^{\dagger}\right)^{(0)}\left(\tilde{d}\tilde{d}\right)^{(0)}\right]^{(0)}\right]$$

$$+U_2\left[\left(d^{\dagger}d^{\dagger}\right)^{(2)}\left(\tilde{d}\tilde{s}\right)^{(2)}\right]^{(0)} + \left(\tfrac{1}{2}\right)U_0\left[\left(\hat{s}^{\dagger}\hat{s}^{\dagger}\right)^{(0)}\left(\tilde{s}\tilde{S}\right)^{(0)}\right]^{(0)} \quad \ldots\ldots\ldots\ldots(2)$$

Where ($s^{\dagger}, d^{\dagger}$) and ($\tilde{s}, \tilde{d}$) are creation and annihilation operators for s- and d-bosons, respectively. The effect of the Hamiltonian function in equation (1) includes two parameters describing a single particle and The relationship between energy levels E(L) parameters describing the two interacting particles represented by C_L(L=0,2,4) and four parameters describing the two interacting particles, represented by U_L (L = 0, 2), V_L (L = 0, 2), and all of these parameters depend on the number of N bosons equal to (n_s + n_d). The equation (1) can be written in several formulas, but the most popular one is the formula [4.5]:

$$\hat{H} = \varepsilon\hat{n}_d + a_o(\hat{P}^{\dagger}.\hat{P}) + a_1(\hat{L}.^{\dagger}\hat{L}) + a_2(\hat{Q}^{\dagger}.\hat{Q}) + a_3(\hat{T}_3^{\dagger}.\hat{T}_3) + a_4(\hat{T}_4^{\dagger}.\hat{T}_4) \quad \ldots\ldots\ldots(3)$$

where $\hat{n}_d$, $\hat{p}$, $\hat{L}$, $\hat{Q}$, $\hat{T}_3$ and $\hat{T}_4$ are the total number of d-boson, pairing, angular momentum, quadrupole, octoupole and hexadecapole operators, respectively

Since ($\varepsilon = \varepsilon_d - \varepsilon_s$) represents the difference between the energy of the bosons (d, s), and for ease it was considered that the energy of the s boson is equal to zero (ε_s = 0) and that

$$\left.\begin{aligned}
\hat{n}_d &= (\hat{d}^\dagger . \tilde{\hat{d}}) \\
\hat{P} &= 1/2(\tilde{\hat{d}} . \tilde{\hat{d}}) - 1/2(\tilde{\hat{s}} . \tilde{\hat{s}}) \\
\hat{L} &= \sqrt{10}[\hat{d}^\dagger \times \tilde{\hat{d}}]^{(\ell)} \\
\hat{Q} &= [(\hat{d}^\dagger \times \tilde{\hat{s}}) + (\tilde{\hat{s}}^\dagger \times \tilde{\hat{d}})] - \frac{\sqrt{7}}{2}[\hat{d}^\dagger \times \tilde{\hat{d}}]^{(2)} \\
\hat{T}_3 &= [\hat{d}^\dagger \times \tilde{\hat{d}}]^{(3)} \\
\hat{T}_4 &= [\hat{d}^\dagger \times \tilde{\hat{d}}]^{(4)}
\end{aligned}\right\} \quad ..(4)$$

The parameters a_0 .... a_4, they express the strength of the interaction of the pairs, angular momentum, electric quadrupole, octaloupole, hexadecpole are among the bosons, respectively.

The electric quadrupole moment (Q_L)

Electric quadrupole moment **(Q_L)**is defined as the amount of deviation from a symmetric spherical distribution with respect to the nuclear charge within the nucleus. The quadrupole electric moment takes the dimensions of the area and it is measured in barns unit or square meters unit (1barn = 10^{-28} m^2). The shape of the nucleus can be determined by relying on the value of **(Q_L)**where the shape of the nucleus is spherical at (Q = 0) and the shape of the nucleus is deformed prolate at (Q> 0) or is deformed oblate at (Q <0). To derive the electric quadrupole moment values , can be use the values of B (E2, Li→L_f), as shown in the following equation: [7,8,9]:

$$Q_L = [16\pi/5]^{1/2}[L(2L-1)/(2L+1)(L+1)(2L+3)]^{1/2}[B(E2, L_i \rightarrow \quad \text{……….(5)}$$

Where B (E2) is the electric transitions probability, L represents the angular momentum, L $_i$ is the initial angular momentum, L $_f$ is the final angular momentum**.**

Variable Anharmonic Vibrator (VAVM) Model

The two scientists Bonatsos D. and Klein A. in (1984) [10] suggested another new model called (VAVM) model is depend on the (GVMI) model by making the basic functions of the angular momentum (L) and the variable L(L-2). The (VMI) model failed to provide adequate results for some experimental energy levels measurements, where (VAVM) model gave good results in the regions SU(3), SU(5), O(6).

The Energy Band Intersection

The energy band intersection means that the moment of inertia with respect to a certain energy level in any band like (γor β) intersects, or takes the place of any energy level in any other band as a band (g). Bands intersection characteristic consider as an important characteristic for the purpose of studying the bending back phenomenon [11] which occurs between any two bands that have the same spin and but they differ in energy and moment of inertia.

Results and discussion

In this research, the properties of the nucleus $^{162}_{66}Dy_{96}$ are studied such as the electric transitions probability B(E2), energy levels E(L) and transitional energy E_γ and by energy bands (g, γ,β) these levels were classified and the behavior of the nucleus $^{162}_{66}Dy_{96}$ must be determined to be able to choose the parameters of the Hamilton function the equation (2) for the purpose of studying nuclear properties and found that the nucleus $^{162}_{66}Dy_{96}$ belongs to the SU(3) symmetry by comparing the ratios of practical energy values with the ratios of ideal energy values and the values of energy levels are calculated in the IBM-1 model using (IBM. For) program the Written with Fortran language through the Input File (BOS. INP), which contains seven parameters the shown in Table (1), the values of these parameters are determined by matching the values of practical energy levels with theoretical energy level values while the values of energy levels are calculated in the VAVM model using (VAVM. For) through the input file (PAR. INP), which contains The relationship between energy levels E(L) parameters are : (ϑ_0 / $\hbar^2$) measured by *(MeV)*$^{-1}$ *unit* ,C measured by *(MeV)* ^{3} *unit* and E_K measured by(MeV) unit , where E_k represents the energy of the head of the band in the ground state band (G-band) equals zero while in the β- band equals the angular momentum energy 0^+ for the excited level, but in the gamma band (ɤ-band) the value is variable and by matching the practical values with the theoretical values calculated in the PROGRAM (IBM-1) the values of the parameters shown in table (2) were chosen when the less value of the χ squared (chi-squared) is obtained from the following equation:

$$\chi^2 = \frac{(E_{Cal} - E_{\exp})}{E^2_{Cal}} \quad ....(6)$$

Table (1): the parameter values used in the (IBM .For) program measured in (MeV) unit

ε	*P^ † .P^*	*L^.L^*	*Q^.Q^*	*T^3 .T^ 3*	*T^4 .T^ 4*	*CHI*
0.0021	*0.0055*	*0.0099*	*-0.0091*	*0.0010*	*0.0011*	*-1.3200*

Table (2): parameter values used in the (VAVM) model with the chi-square (χ^2)

Band	ℓ_0(MeV)$^{-1}$	C(MeV)^3	E_K (MeV)	χ^2
G-band	91.0000	0.0111	0.0000	0.0046
$\gamma_1 -$ band	98.9999	0.0002	0.7199	0.0145
β_1-band	99.9900	49.2991	01.4000	0.0098
β_2-band	99.9999	0.0021	01.6660	0.0092

Table (3): quadrupole electric moment values derived from B (E2) parameter values using (IBMT. For) program

$\alpha_2\ (eb)$	$\beta_2 (eb)$
0.30000	0.41000

Figure (1) shows the relationship between E(L) practical energy level values compared with theoretical energy level values of the two models (IBM-1,VAVM) for ground state band as a function of angular momentum of a nucleus ($^{162}_{66}Dy_{96}$) and it was noted from the figure that there a quite match between practical energy level values with the values of theoretical energy levels calculated according to the two models(IBM-1,VAVM) for (L≤10) either the values of theoretical energy levels calculated by the (IBM-1)model do not match to the values of practical energy levels for(L>10) because the first interacting boson model do not distinguish between the neutron bosons N_π and the proton bosons N_v, and also because the first interacting boson model did not succeed in calculating the excited energy levels, while the values of energy levels calculated by (VAVM) model with the values of practical energy levels for (L>10) are still fully matched , and this match was the result of better selection of the three input parameter ℓ_0, C and E_K through the input file (PAR. INP) ,and also because the (VAVM) model distinguish between the neutron bosons N_π and the proton bosons N_v . And it was noticed from the figure is that the values of energy levels calculated by the (VAVM) model gave a better match with the values of practical energy levels than the values of theoretical energy levels calculated by the(IBM-1) model and this was the result of the optimal choice of the input parameter values through the input file (PAR. INP).

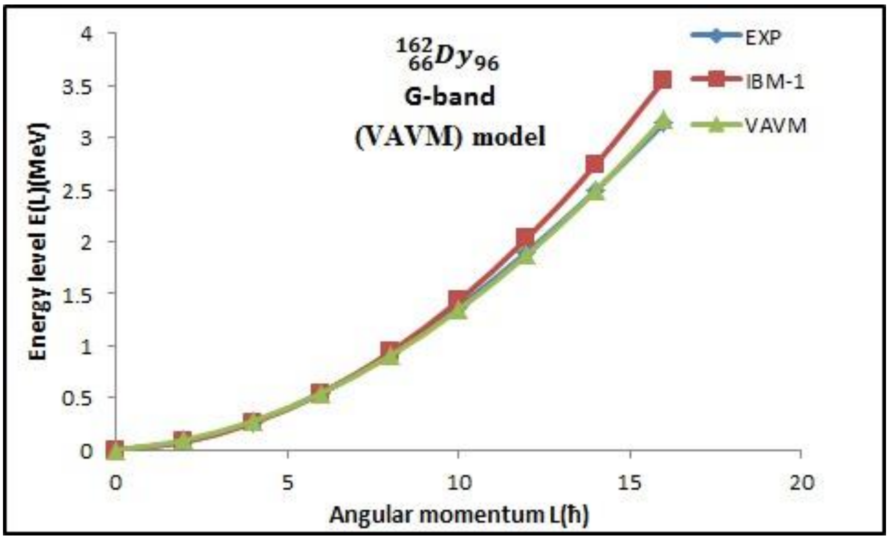

Figure 1. The relationship between energy levels E(L) calculated IBM-1,VAVM and experimental energy levels E(L)versus L for g-band

Figure(2) shows that IBM-1's theoretical energy level values are fully matched, with the gamma -band practical energy level values for (L≤8), the reason for the match is that the first interacting boson model is very successful in calculating low energy levels, either for(L>8) where there is no match between the values of the theoretical energy levels calculated by the(IBM-1) model with the values of the practical energy levels, and a mismatch because the first interacting boson model do not distinguish between the neutron bosons N_π and the proton bosons N_v, and also because the first interacting boson model did not succeed in calculating the excited energy levels, while all the values of the theoretical energy levels calculated by the (VAVM)model are fully matched with all the practical energy level values for(L>8), and this match was the result of better selection of the three input parameter ℓ_0, C and E_K through the input file (PAR. INP) ,and also because the (VAVM) model distinguish between the neutron bosons N_π and the proton bosons N_v .

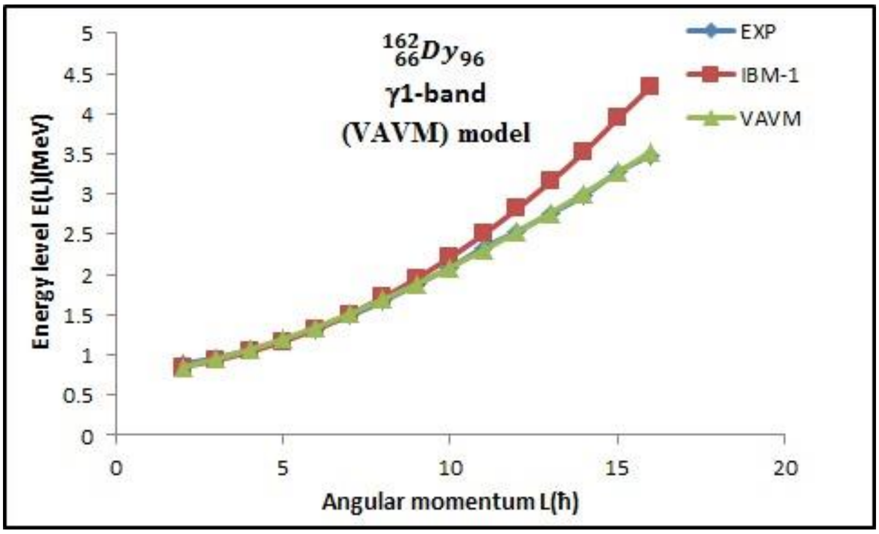

Figure 2. The relationship between energy levels E(L) calculated IBM-1,VAVM and experimental energy levels E(L)versus L for for γ-band

Figure (3) shows the comparison between of the theoretical energy level values E(L) calculated by the two models (IBM-1,VAVM) with the values of practical energy levels as a function of the angular momentum of the β1-band and shows from the shape for (L≤4) there is a match between the values of the practical energy levels with theoretical energy level values calculated according to the two models (IBM-1,VAVM), while for(L>4) is no match between the values of practical energy levels with the values of theoretical energy levels calculated according to the model (IBM-1) while the values of the theoretical energy levels calculated by (VAVM) model is still matching with practical energy level values for (L>4).

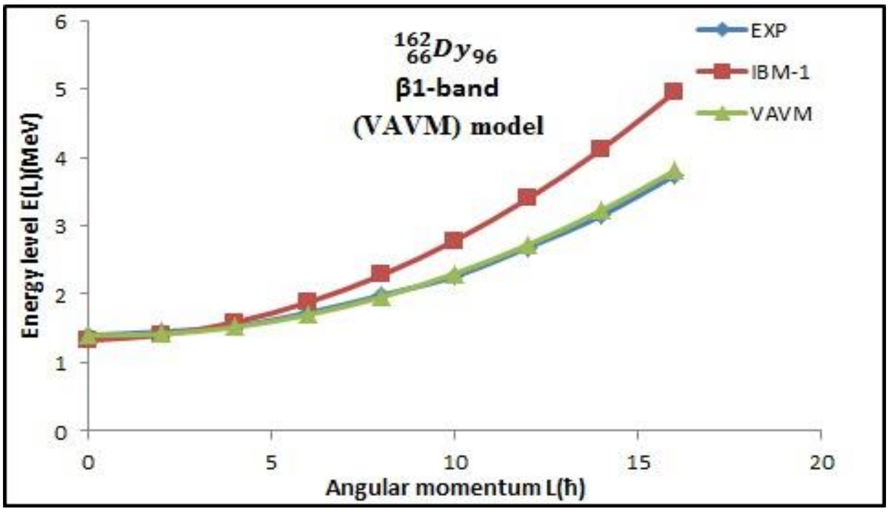

Figure 3. The relationship between energy levels E(L) calculated IBM-1,VAVM and experimental energy levels E(L)versus L for for β1-band

Figure (4) shows that there is a great match between all practical energy level values with all theoretical energy level values calculated by the(VAVM) model of the Beta 2 - band, the values of theoretical energy levels calculated by(IBM-1) model are slightly greater than the values of practical energy levels at low energy levels for (L≤8) while significantly moving away from the values of practical energy levels at excited energy levels for (L>8).

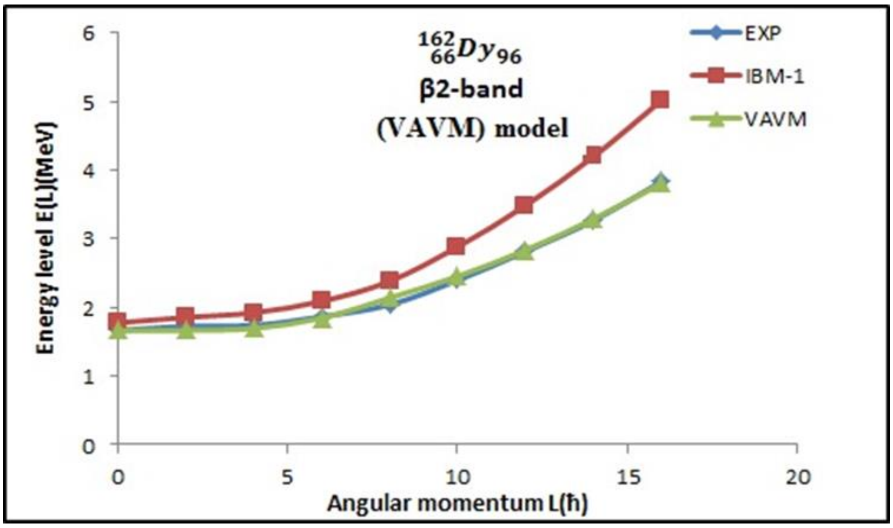

Figure 4. The relationship between energy levels E(L) calculated IBM-1,VAVM and experimental energy levels E(L)versus L for β2-band

Figure (5) illustrates the band intersection phenomenon, which is an important subject because it is use to explain the back bending phenomenon observed from figure (5) that the bands (g1,β1) intersects at the angular momentum $L_{C=}14{,}16$. And the disintegration of a pair of nucleus generates the energy band intersection and be the back-banding phenomenon that did not appear in the nucleus ${}^{162}_{66}Dy_{96}$. Due to the decrease in inertial moment at the large angular momentum, the bands intersection also occurs, and the effect of the curious force effect generates a decrease in the energy of the two nucleons, and therefore, the instability in these two states leads to the intersection of one band with another.

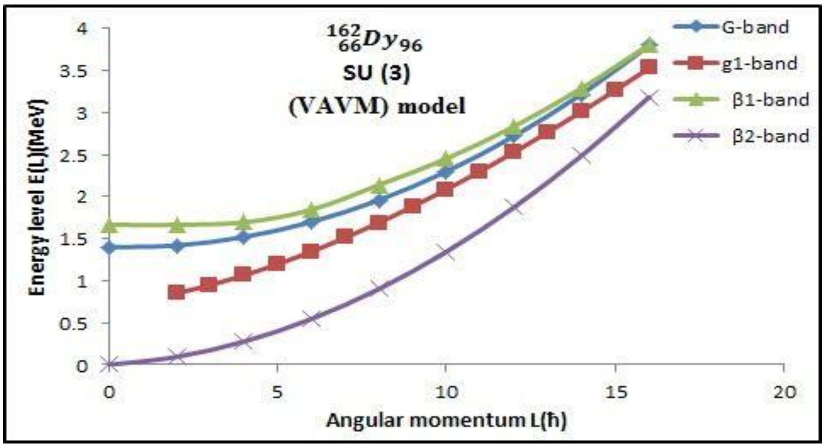

Figure 5. Energy band Intersection E(L)as a function of the angular momentum (L) using (VAVM) model.

The quadrupole moment values (Q_L) were calculated in IBM-1 model using of the(IBMT) program. For via entry file BE2. Dat which requires the determination of the two parameters (β_2,α_2) measured by the unit (eb) shown in table (3).

We note from figure(6)that the shape of the nucleus in the nucleus ${}^{162}_{66}Dy_{96}$ takes the prolate oval shape at the angular momentum ($2_1^+,2_2^+,2_3^+,2_5^+,2_6^+,2_7^+,2_{10}^+$) while takes the oblate oval shape at the angular momentum ($2_4^+,2_8^+,2_9^+$), and the largest deformation of the prolate oval type is at the angular momentum Q(2_8^+) and its value (-3.729 eb) and the highest deformation of the oblate oval type is at the angular momentum Q(2_3^+) and its value (4.15 eb).

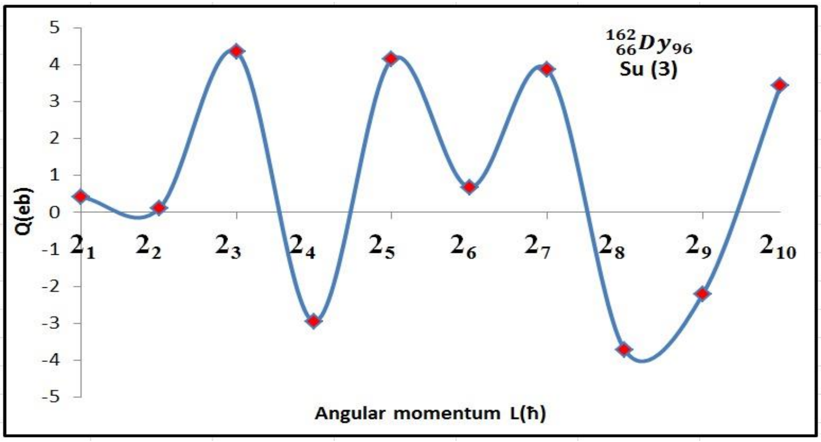

Figure 6. Electric quadrupole moment (Q_L)as a function of the angular momentum (L) from 2_1 *to* 2_{10} .

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