The effect of the VMI, VAVM and GVMI moment of inertia and rotational motion on the nuclear structure of $^{56}_{26}Fe_{30}$ nucleus

***Dr.Iman Tarik Al-Alawy (proff.) *Dr.Khalid Salman Ibraheim (proff.) **Dr.Ali Kalaf Aobaid (Assist.proff.)**

***Physics Department, College of Science, Al-Mustansiriyah University**
****Physics Department, College of Science, Anbar University**

ABSTRACT

In the present work, the moment of inertia models (VMI, VAVM,GVMI) are used to study the energy levels, gamma transitions, moment of inertia as a function of angular momentum for each state ,($2\vartheta/\hbar^2$), rotational energy square $(\hbar\omega)^2$, standard deviation (Δ),softness(σ) ,and Chi-square($\chi 2$) for $^{56}_{26}Fe_{30}$ nucleus .Also this work, have been studied the energy band crossing and back bending phenomena.

دراسة تأثيرعزم القصور الذاتي والحركة الدورانية على التركيب النووي لنواة الحديد $^{56}_{26}Fe_{30}$

د.ايمان طارق العلوي (استاذ)* د.خالد سلمان ابراهيم(استاذ)*
د.علي خلف عبيد(استاذ مساعد)**

***قسم الفيزياء/كلية العلوم/الجامعة المستنصرية**
****قسم الفيزياء/كلية العلوم/جامعة الانبار**

المستخلص

تم في هذا البحث استخدام ثلاثة نماذج من نماذج عزم القصور الذاتي هي (VMI,AVM,GVMI) لغرض دراسة مستويات الطاقة وانتقالاتها الكامية بين أي مستويين متتالين وكذلك تم حساب كل من عزم القصور الذاتي لكل مستوي($2\vartheta/\hbar^2$) ومربع الطاقة الدورانية $(\hbar\omega)^2$ والانحراف المعياري (Δ) ومربع كآي لنواة الحديد $^{56}_{26}Fe_{30}$.

كما تم في هذا البحث أيضا دراسة ظاهرة تقاطع حزم الطاقة وظاهرة الانحناء الخلفي.

INTRODUCTION

Moment of inertia is one of the most important and complicated subjects that are used to explain some properties and structures of nucleus [1].

The theoretical studies carried out on the nuclear spectrum of all dynamical symmetries found out that the theoretical values of energy are different from that of experimental values .This difference increases if we propose that moment of inertia is an invariant, and the nucleus is rigid body [2], while the practical measurements show that the energy of gamma transitions (Eγ) for SU(3) nuclei between each two successive levels, does not increase linearly with that of angular momentum values. But there is a great difference at high excitation energies because the centrifugal force will increase with angular velocity .This leads; in turn to deformation of the nucleus (i.e. the nucleus is extended [1].

Deleplanque M.A., et al. (2002) [3] have shown that the moments of inertia at high spins along the yarest line are not rigid, and the shell effects, which produce the deviations from the rigid-body values, contribute to the moments of inertia up to the highest spins observed.

In this paper we used three models, they are as follows:

1. Variable Moment of Inertia (VMI) Model

The variable moment of inertia model has been proposed by Mariscotti M.A., et al. (1969)[4]to stand up to the difference in the practical and theoretical values of energy levels for SU(3) nuclei .They suggested a variable moment of inertia (VMI) model supposing that moment of inertia is a function of angular momentum.

Harris S.M.(1965)[5]concluded that the nuclear moment of inertia (ϑ) is a function of angular velocity (ω), that can be written as a power series of $(\hbar\omega)^2$. This model succeeded in SU(3) region, and it is also used to describe the energy levels of SU(5), and O(6) regions but in a limited way [1,6].

Scharff-Gold Haber G. et al. (1970) [7] used (VMI) model to study the magic number nuclei. They arrived to the conclusion that the quadrupole of the ground state bands in even-even nuclei starting from the deformed nuclei and ending with doubly magic nuclei. They found that the internal stress of the nucleus is what makes the nucleus preserve its spherical shape; it overcomes the increase in its angular momentum.

Elmasri Y. et al. (1977) [8] have studied the back bending for $^{156}_{66}Dy$ usin rotational-vibrational model and (VMI) model to explain this phenomenon.

Bonatsos D. (1985)[9] designed a simple model for back bending using (VMI) model for (g-band), (β-band) and (γ-band). A good description was obtained for back bending for intermediate and heavy nuclei.

2. Generalized Variable Moment of Inertia (GVMI) Model

Bonatsos D., and Klein A.(1984)[10] concluded a new model to calculate the energy levels of nuclei. This model is more comprehensive than (VMI).

The general formulation of energy levels in this model contains basic functions of (VMI) for angular momentum L and L(L+1). Because this equation consists of many terms, Bonatsos D., and Klein A., reduced these terms to two that depend on the same basic functions .They produced a new model called (GVMI). The theoretical results obtained from this model are in full accord with the experimental values for the energy levels of SU(3),SU(5) nuclei more than that of other models.

Alenicheva T.V. et al. (2004)[11] studied the rotational band for strong deformed nuclei using generalizing variable moment of inertia (GVMI) model, which is taking into account the decoupling effect for bands with K=12. He obtained a good description of rotational energies for the majority of rotational bands.

3. Variable Anharmonic Vibrator (VAVM) Model

Bonatsos D ., and Klein A.(1984)[10] proposed another model called (VAVM) that is based on (GVMI) model by making the basic functions of angular momentum (L) and L(L-2) variable.

This model gave good results in SU(3),SU(5) and O(6) regions, where(VMI) model failed to give suitable results for some experimental measurements.

Bonatsos D. (1988)[12] studied the two parameters of the variable moment of inertia (VMI) model and the three parameters of the variable anharmonic vibrator model (VAVM) as a function on the valence proton-neutron product Np.Nυ in several regions of the periodic table.

THEORETICAL PART

The simple formula of rotational energy levels in even-even nuclei is determined by [13]:

$$E = \frac{1}{2}\vartheta\omega^2 = \frac{L^2}{2\vartheta} \quad \text{.....(1)}$$

Where ω is a rotational or vibrational frequency, and it quantized to [3]

$$E(L) = \frac{\hbar^2}{2\vartheta} L(L+1) \quad \text{.....(2)}$$

Mariscotti M. et al. (1969)[4] proposed another formula from equation (2) as follows:

$$E(L)=\frac{1}{2}\left[\frac{L(L+1)}{\vartheta(L)}+C(\vartheta(L)-\vartheta_o)^2\right].....(3)$$

Where

ϑ (L) is a moment of inertia as a function of L, and C, ϑ_o are parameters which are suitable with fitted to the experimental data.

The physical meanings of these parameters are [11]:

ϑ_o is a moment of inertia for ground state ,C: is a meaning of "elasticity".

$$\frac{\partial E(L)}{\partial \vartheta(L)}=-\frac{1}{2}\frac{L(L+1)}{\vartheta^2(L)}+C(\vartheta(L)-\vartheta_o).....(4)$$

The values of moment of inertia ϑ(L) are determined for each level from the equilibrium condition [4, 11, 7]:

$$\frac{\partial E(L)}{\partial \vartheta(L)}=0.....(5)$$

$$\vartheta^3(L)-\vartheta^2(L)\vartheta_o-\frac{L(L+1)}{2C}=0.....(6)$$

The solution of above equation is [1]:

$$\vartheta(L)=(x+y)^{1/3}+(x-y)^{1/3}+\frac{1}{3}\vartheta_o.....(7)$$

Where

$$x=\frac{L(L+1)}{4C}+\frac{\vartheta_o^3}{27}, y=\left[\frac{(L(L+1))^2}{16C^2}+\frac{\vartheta_o^3 L(L+1)}{54C}\right]^{1/2}.....(8)$$

The nuclear softness coefficient (σ) can be written as [4]:

$$\sigma=\frac{1}{\vartheta(L)}\left(\frac{d\vartheta}{dL}\right)_{\vartheta=0}.....(9)$$

σ can be written according to variable moment of inertia model as [1,4]

$$\sigma=\frac{1}{2C\vartheta_o^3}.....(10)$$

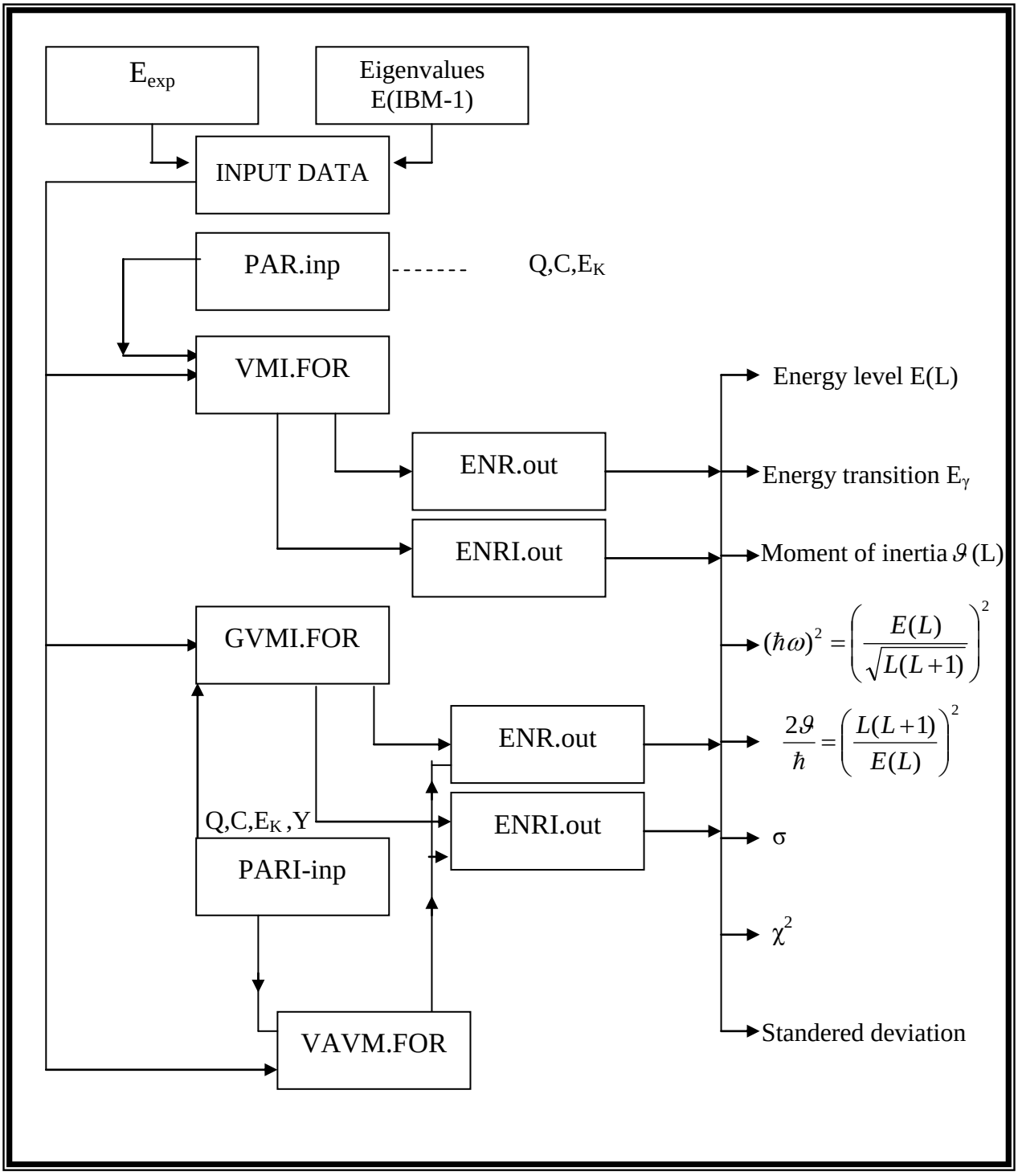


Figure (1): Block diagram of programmers' for VMI, GVMI, and VAVM models.

1. VMI program

The VMI.for program has been build to calculate the following:

1. Theoretical energy levels $E_{cal.}$ Using equation (3).

2. Moment of inertia as a function of (L) using equation (7).

3. Rotational energy square $(\hbar\omega)^2$ and$(2\vartheta/\hbar^2)$ as a function of angular momentum (L).

4. Chi-squared which can be calculated from the following equation [4]:

$$\chi^2 = \frac{(E_{cal} - E_{\exp})^2}{E^2{}_{cal}} \quad \text{.....(11)}$$

5. Nuclear softness parameters (σ) from equation (10).

6. Standard deviation (Δ), from the following equation [11, 14]:

$$deviation(\Delta) = \left[\frac{1}{N}\sum_{i=1}^{N}(E_{cal} - E_{\exp})^2\right]^{1/2} \quad \text{.....(12)}$$

In order to build this program, we need three files one of them called "*PAR.inp*". This file depends on three parameters:

ϑ_o, C are as defined earlier, E_k:is the band head energy[11].

The other files called, "*ENR.out*" and "*ENR1.out*".

Figure (1) shows the block diagram of this program.

2.(GVMI,VAVM) programs

These programs are used to calculate the same VMI.for program calculations. The calculations of these programs depend on four parameters that are:

Q, C, E_k : are as defined earlier.

Y: is the constant parameter fitted with experimental data.

The results of these programs were used in the present study in order to be compared with the results of VMI program. The comparison gives a very good agreement. Figure (1) shows the block diagram of these programs.

All programs have been written in Fortran 77 language using software Compaq Visual FORTRAN Version 6.6 (CVF6.6) for compiling, linking, and executing.

RESULTS AND DISCUSSION

1. Moment of Inertia and Rotational Motion in Even-Even Nuclei

One of the basic properties of the deformed nuclei is the presence of specific rotational bands in their excitation spectra. The excitation nuclear energy levels, which have high angular momentum decay to low energy levels by emitted gamma ray. Such nuclei tell us to know new facts about their nuclear structure [11].

The collective rotational motion of deformed nuclei depends on nucleons motion in a way coherently with the nuclear motion causing rotation of some nucleons around an axis different from nuclear symmetry axis .For this reason, we can explain two types of deformation as follows[2,4]:

1-Prolate deformation:

In this deformation the nucleus rotate around an axis perpendicular to the nuclear symmetry axis. This rotation is called *"collective rotation"*.

2-Oblate deformation:

In this deformation, the nucleus is rotating in axis parallel to the nuclear symmetric axis .This rotating happened for spherical nuclei, and it is called*" non-collective rotation"*[2].

In the present work, we propose that the nucleus is a rigid body in order to calculate the results.

Table (1) shows the corresponding parameters ($\vartheta_o/\hbar^2$, Y, C, E_k) of the three models (VMI, VAVM, and GVMI) for each band.

These parameters have been obtained by the best fitting using the experimental energy levels, which can be known from smaller Chi-square (χ^2) as written in equation (11). In this table we noticed that the zero value of Chi-square (χ^2) are in β_2, $\beta 3$ for (VAVM) model.

In general, the Chi-squared value of (VAVM) model is smaller than (VMI, GVMI) models. This table also contains the calculated values of standard deviation.

2. Moment of Inertia and Softness Factor for (g, β, γ) Bands

The energy bands arrangement (g, β, γ) are very important to calculate the total moment of inertia of any nuclear model as a function of angular momentum. These bands and parameters ($\vartheta_o/\hbar^2$, C ,Y, E_k)are shown in table (1) and used in (VMI,VAVM, and GVMI) programs to

calculate the moment of inertia (ϑ) and nuclear softness coefficient (σ). This table shows the nuclear softness coefficient depends inversely on the moment of inertia of the ground state (ϑ_0) and the parameter(C).

Table (2) shows the moment of inertia as a function of angular momentum for (L=0 to 26)for g and β band and(L=2 to15)for γ band with their comparison for three models.

3. The Energy Band Intersection

The band intersection phenomenon happened between any two bands having same spin, and different in energy and moment of inertia.

The increasing of rotational energy between any two excited states due to reduce moment of inertia at large angular momentum. Here, Corielis force effect will increase with the increase of rotational energy between these bands. This effect leads to reduce the energy of two nucleons, which occupy these states because, the depairing of the nucleon pairs. In this case, the direction of angular momentum of these nucleons is in same direction with the rotational axis with low rotational energy and maximum angular momentum. The unstable of these two excited states leads to crossing band with any other band at certain angular momentum called *"L_{cross}."*.

The results of the Calculations are listed in table (2). This table shows the values of energy levels E(L) in (MeV) for the three models, which are used to study the energy band crossing.

Figure (2) indicates the energy band crossing for $^{56}_{26}Fe_{30}$ nucleus using (VMI, VAVM, and GVMI) models.

It is noticed in this figure that the $(g,\beta_2),(g,\beta_3),(\beta_1, \beta_2,\gamma_2 ,\gamma_3)$ of the $^{56}_{26}Fe_{30}$ nucleus using (VMI)model will intersect at angular momentum L_c=(14,7,5) respectively, and the bands$(g, \beta_3),(g,\gamma_1), (\beta_1,\beta_2,\gamma_2,\gamma_3)$ of VAVM intersect at angular momentum L_c=(15,7,5) respectively ,while the bands $(g,\beta_3, \gamma_1).(\beta_1,\beta_2,\gamma_2,\gamma_3)$of GVMI crossing at L_c=14,6 respectively.

4. The Back Bending Phenomenon

In order to study the back bending phenomenon, we must calculated and drawn the values of moments of inertia ($2\vartheta/\hbar^2$) and rotational energy squared $(\hbar\omega)^2$ for the three models as in table (2).

Figure (3) shows the back bending phenomena by drawing the moment of inertia ($2\vartheta/\hbar^2$) as a function of rotational energy squared $(\hbar\omega)^2$ using these three models. This figure shows that the back bending using VMI model

occurs in γ_1-band at angular momentum L_c=9,while it is not occurs in the other bands of this model and VAVM, While the back bending using (GVMI) model occurs in (g, β_1,β_3) bands at angular momentum L_c=(8,8,18) respectively.

We noticed from these figures in which the back bending appearance, the moment of inertia ($2\Im/\hbar^2$) increases with rotational energy squared $(\hbar\omega)^2$, but at certain angular momentum (L_c) the rotational energy squared$(\hbar\omega)^2$ decreases, while the moment of inertia ($2\Im/\hbar^2$) increases ,causing the back bending phenomena .This reason belonging to *"*Corielis *antipairing effect"* .In this case the moment of inertia will effect on nuclear structure of these nuclei .

The figures in which no back bending appearance; this means that there is no effect of moment of inertia on nuclear structure of these nuclei.

Table(1) : The corresponding parameters of (VMI),GVMI,VAVM) model for even-even nuclei.

Nuclei	band	parameter / Model	$\vartheta_o/\hbar^2$ (MeV)$^{-1}$	C (MeV)^3	E_k (MeV)	Y (MeV)	σ	Standard deviation	χ^2
$^{56}_{26}Fe_{30}$ O(6)	g	VMI	1.8100	**0.0470**	0.0000	———	1.7941	0.0709	0.0068
		GVMI	0.0280	**0.044**	0.0000	0.4680	51.78	0.0763	0.0075
		VAVM	**24.1800**	**0.01000**	0.0000	0.4280	-0.0141	0.0603	0.0054
	β_1	VMI	6.0000	**5.0000**	2.9417	———	0.0005	0.3956	0.1605
		GVMI	9.1200	**7.8800**	2.9417	1.0000	-0.0002	0.1666	0.0278
		VAVM	7.9000	**18.0000**	2.9417	0.0280	-0.0002	0.0464	0.0019
	β_2	VMI	**13.0000**	**8.8000**	3.5992	———	0.0003	0.1305	0.0132
		GVMI	**54.0000**	**0.0020**	3.5992	5.2000	-0.0298	0.0198	0.0003
		VAVM	**10.1600**	**12.0000**	3.5992	0.0016	-0.0001	0.0004	0.0000
	β_3	VMI	**27.9000**	**0.08000**	4.3002	———	0.0003	0.0529	0.0019
		GVMI	**93.4000**	0.0110	4.3002	3.9900	-0.0008	0.0010	0.000001
		VAVM	**23.2000**	**11.0000**	0.0096	4.3002	-0.00001	0.0007	0.00000
	γ_1	VMI	**12.4000**	2.0160	2.3800	———	0.00001	0.1960	0.0632
		GVMI	**17.9000**	3.9800	2.6575	0.8400	-0.00003	0.1879	0.05950
		VAVM	0.2800	0.0008	2.6460	**0.00588**	-0.00004	0.6688	06776

Table(1):To be continued (2/2).

Nuclei	band	parameter / Model	$\vartheta_o/\hbar^2$ $(MeV)^{-1}$	C $(MeV)^3$	E_k (MeV)	Y (MeV)	σ	Standard deviation	χ^2
$^{56}_{26}Fe_{30}$ O(6)	γ_2	VMI	6.4000	0.6920	3.0000	———	0.0028	0.1626	0.0233
		GVMI	83.0200	0.9900	3.3697	7.9960	-0.00003	0.1579	0.0198
		VAVM	8.2000	6.6000	3.2600	0.0020	-0.0005	0.1278	0.0137
	γ_3	VMI	8.8600	0.6900	3.5400	———	0.0010	0.0956	0.0070
		GVMI	99.8800	1.0000	3.7480	7.6000	-0.00001	0.0944	0.0066
		VAVM	11.0000	8.8800	3.6900	0.0010	-0.0002	0.0741	0.0042
	γ_4	VMI	8.8800	4.0900	3.6800	———	0.00002	0.0550	0.0022
		GVMI	85.0400	1.0000	3.8303	7.4800	-0.00002	0.0517	0.0019
		VAVM	11.0000	4.8800	3.7200	0.0360	-0.0003	0.0392	0.0011

Table (2): Theoretical energy levels; moment of inertia, transition energy, rotation energy of (VMI, VAVM,GVMI) models.

Nuclei $^{56}_{26}Fe_{30}$					Band	N_π	N_υ	N	Dynamical symmetry								
					g	1	1	2	O(6)								
L^+	VMI (Pw)		VAVM (Pw)		GVMI (Pw)		$L_i^+ - L_f^+$	VMI (Pw)			VAVM (Pw)			GVMI (Pw)		
	Energy (MeV)	ϑ $(MeV)^{-1}$	Energy (MeV)	ϑ $(MeV)^{-1}$	Energy (MeV)	ϑ $(MeV)^{-1}$		Eγ (MeV)	$\hbar^2\omega^2$ $(MeV)^2$	$2\vartheta/\hbar^2$ $(MeV)^{-1}$	Eγ (MeV)	$\hbar^2\omega^2$ $(MeV)^2$	$2\vartheta/\hbar^2$ $(MeV)^{-1}$	Eγ (MeV)	$\hbar^2\omega^2$ $(MeV)^2$	$2\vartheta/\hbar^2$ $(MeV)^{-1}$
0^+	0.0000	1.8100	0.0000	24.1800	0.0000	0.0280	-	-	-	-	-	-	-	-	-	-
2^+	0.8346	4.6998	0.8560	24.1800	0.8362	3.5782	$2^+ - 0^+$	0.8346	0.1161	7.1893	0.8560	0.1221	7.0093	0.8362	0.1165	7.1751
4^+	2.0542	6.6378	2.0344	25.4182	2.0659	5.6134	$4^+ - 2^+$	1.2196	0.3636	11.4784	1.1784	0.3394	11.8805	1.2296	0.3696	11.3853
6^+	3.5199	8.2982	3.4957	27.3812	3.5239	7.3257	$6^+ - 4^+$	1.4657	0.5325	15.0100	1.4613	0.5293	15.0546	1.4580	0.5269	15.0889
8^+	5.1737	9.7940	5.1925	29.6427	5.1545	8.8560	$8^+ - 6^+$	1.6538	0.6806	18.1404	1.6967	0.7165	1706810	1.6306	0.6617	18.3986
10^+	6.9827	11.1819	7.0857	31.9950	6.9270	10.2634	$10^+ - 8^+$	1.8090	0.8158	21.0059	1.8933	0.8936	20.0711	1.7726	0.7833	21.4377
12^+	8.9254	12.4832	9.1466	34.3501	8.8219	11.5800	$12^+ - 10^+$	1.9427	0.9417	23.6781	2.0608	1.0597	22.3211	1.8948	0.8959	24.2765
14^+	10.9865	13.6768	11.3534	36.6721	10.8249	12.8255	$14^+ - 12^+$	2.0611	1.0605	26.1999	2.2068	1.2158	24.4697	2.0030	1.0017	26.9588
16^+	13.1542	14.8540	13.6897	38.9471	12.9256	14.0130	$16^+ - 14^+$	2.1677	1.1735	28.6019	2.3363	1.3632	26.5374	2.1007	1.1021	29.5142
18^+	15.4194	15.9843	16.1427	41.1708	15.1156	15.1522	$18^+ - 16^+$	2.2652	1.2817	30.9025	2.4530	1.5030	28.5370	2.1899	1.1980	31.9637
20^+	17.7747	17.0744	18.7019	43.3431	17.3881	16.2500	$20^+ - 18^+$	2.3553	1.3859	33.1174	2.5593	1.6364	30.4776	2.2725	1.2903	34.3228
22^+	20.2138	18.1295	21.3590	45.4657	19.7376	17.3119	$22^+ - 20^+$	2.4392	1.4866	35.2580	2.6571	1.7641	32.3663	2.3495	1.3793	36.6035
24^+	22.7317	19.1536	24.1069	47.5411	22.1594	18.3421	$24^+ - 22^+$	2.5178	1.5842	37.3336	2.7478	1.8868	34.2085	2.4217	1.4655	38.8154
26^+	25.3237	20.1502	26.9395	49.5725	24.6492	19.3441	$26^+ - 24^+$	2.5920	1.6790	39.3511	2.8326	2.0052	36.0089	2.4898	1.5493	40.9660

Table (2): To be continued (2/6).

Nuclei $^{56}_{26}Fe_{30}$			Band	N_π	N_υ	N	Dynamical symmetry								
			β_1	1	1	2	O(6)								
L^+	VMI (Pw)		VAVM (Pw)		GVMI (Pw)		$L_i^+ - L_f^+$	VMI (Pw)			VAVM (Pw)			GVMI (Pw)		
	Energy (MeV)	ϑ $(MeV)^{-1}$	Energy (MeV)	ϑ $(MeV)^{-1}$	Energy (MeV)	ϑ $(MeV)^{-1}$		Eγ (MeV)	$h^2\omega^2$ $(MeV)^2$	$2\vartheta/h^2$ $(MeV)^{-1}$	Eγ (MeV)	$h^2\omega^2$ $(MeV)^2$	$2\vartheta/h^2$ $(MeV)^{-1}$	Eγ (MeV)	$h^2\omega^2$ $(MeV)^2$	$2\vartheta/h^2$ $(MeV)^{-1}$
0^+	2.9417	6.0000	2.9417	7.9000	2.9417	9.1200	-	-	-	-	-	-	-	-	-	-
2^+	3.4410	6.0166	2.9977	7.9000	3.1610	9.1230	$2^+ - 0^+$	0.4993	0.0415	12.0166	0.0560	0.0005	107.14	0.2193	0.0080	27.3646
4^+	4.6008	6.0545	4.0659	7.9071	4.2562	9.1382	$4^+ - 2^+$	1.1598	0.3288	12.0712	1.0682	0.2789	13.1061	1.0952	0.2932	12.7829
6^+	6.4089	6.1124	6.1436	7.9212	6.2230	9.1653	$6^+ - 4^+$	1.8081	0.8103	12.1672	2.0777	1.0699	10.5887	1.9668	0.9588	11.1855
8^+	8.8478	6.1880	9.2254	7.9422	9.0538	9.2039	$8^+ - 6^+$	2.4388	1.4803	12.3010	3.0818	2.3636	9.7346	2.8308	1.9943	10.5977
10^+	11.8957	6.2790	13.3034	7.9699	12.7380	9.2534	$10^+ - 8^+$	3.0479	2.3159	12.4676	4.0780	4.1460	9.3182	3.6842	3.3837	10.3144
12^+	15.5284	6.329	18.3675	8.0041	17.2622	9.31331	$12^+ - 10^+$	3.6327	3.2929	12.6627	5.0641	6.3991	9.0835	4.5242	5.1074	10.1675
14^+	19.7205	6.4974	24.2054	8.0442	22.6110	9.3824	$14^+ - 12^+$	4.1921	4.3875	12.8812	6.0379	9.1015	8.9435	5.3488	7.1426	10.0957
16^+	24.4465	6.6205	31.4031	8.0901	28.7671	9.4603	$16^+ - 14^+$	4.7260	5.5779	13.1190	6.9976	12.2290	8.8601	6.1561	9.4646	10.0712
18^+	29.6813	6.7505	39.3449	8.14141	35.7120	9.5461	$18^+ - 16^+$	5.2348	6.8451	13.3721	7.9418	15.7552	8.8141	6.9449	12.0469	10.0794
20^+	35.4008	6.8858	48.2141	8.1976	43.4262	9.6390	$20^+ - 18^+$	5.7196	8.1729	13.6374	8.869	19.6525	8.7945	7.7141	14.8671	10.1113
22^+	41.5825	7.0252	57.9928	8.2584	51.8895	9.7382	$22^+ - 20^+$	6.1816	9.5480	13.9121	9.7788	23.8931	8.7945	8.4633	17.8972	10.1615
24^+	48.2049	7.1678	68.6627	8.3234	61.0817	9.8430	$24^+ - 22^+$	6.6224	10.9592	14.1942	10.6699	28.4487	8.8098	9.1922	21.1170	10.2260
26^+	55.2483	7.31	80.2048	8.3922	70.9826	9.9527	$26^+ - 24^+$	7.0434	12.3976	14.4816	11.5421	33.2921	8.8372	9.9008	24.4973	10.3021

Table (2): To be continued (3/6).

Nuclei $^{56}_{26}Fe_{30}$			Band	N_π	N_υ	N	Dynamical symmetry								
			β_2	1	1	2	O(6)								
L^+	VMI (Pw)		VAVM (Pw)		GVMI (Pw)		$L_i^+ - L_f^+$	VMI (Pw)			VAVM (Pw)			GVMI (Pw)		
	Energy (MeV)	ϑ (MeV)$^{-1}$	Energy (MeV)	ϑ (MeV)$^{-1}$	Energy (MeV)	ϑ (MeV)$^{-1}$		Eγ (MeV)	$\hbar^2\omega^2$ (MeV)^2	$2\vartheta/\hbar^2$ (MeV)$^{-1}$	Eγ (MeV)	$\hbar^2\omega^2$ (MeV)^2	$2\vartheta/\hbar^2$ (MeV)$^{-1}$	Eγ (MeV)	$\hbar^2\omega^2$ (MeV)^2	$2\vartheta/\hbar^2$ (MeV)$^{-1}$
0^+	3.5992	13.8000	3.5992	10.1600	3.5992	54.000	-	-	-	-	-	-	-	-	-	-
2^+	3.8164	13.8262	3.6024	10.1600	3.6361	54.3387	$2^+ - 0^+$	0.2172	0.0079	27.6262	0.0032	0.0000	18.2134	0.0369	0.0002	162.51
4^+	4.3216	13.8864	4.3927	10.1664	4.3951	60.2755	$4^+ - 2^+$	0.5052	0.0624	27.7127	0.7903	0.1527	17.7136	0.7589	0.1408	18.4456
6^+	5.1111	13.9791	5.9688	10.1793	5.7187	68.1015	$6^+ - 4^+$	0.7895	0.1545	27.8658	1.5760	0.6156	13.9593	1.3236	0.4342	16.6212
8^+	6.1794	14.1017	8.3275	10.1985	7.4726	76.1887	$8^+ - 6^+$	1.0683	0.2840	28.0813	2.3587	1.3846	12.7188	1.7539	0.7655	17.1047
10^+	7.5196	14.2513	11.4645	10.2238	9.5706	84.1089	$10^+ - 8^+$	1.3402	0.4478	28.3538	3.1370	2.4533	12.1133	2.0980	1.0973	18.0026
12^+	9.1237	14.4248	15.3742	10.2551	11.9561	91.7642	$12^+ - 10^+$	1.6041	0.6420	28.6771	3.9097	3.8141	11.7657	2.3855	1.4200	19.2829
14^+	10.9829	14.6189	20.0495	10.2922	14.5898	99.1469	$14^+ - 12^+$	1.8592	0.8629	29.0448	4.6753	5.4572	11.5499	2.6336	1.7317	20.5037
16^+	13.0881	14.8306	25.4825	10.3348	17.4427	107.231	$16^+ - 14^+$	2.1052	1.1068	29.4508	5.4330	7.3716	11.4118	2.8529	2.0326	21.7326
18^+	15.4301	15.0571	31.6640	10.3826	20.4925	115.121	$18^+ - 16^+$	2.3420	1.3701	29.8891	6.1815	9.5451	11.3240	3.0498	2.3235	22.9519
20^+	17.9997	15.2959	38.5842	10.4355	23.7218	123.411	$20^+ - 18^+$	2.5696	1.6496	30.3546	6.9202	11.9644	11.2713	3.2293	2.6054	24.1539
22^+	20.7880	15.5450	46.2324	10.4930	27.1163	131.521	$22^+ - 20^+$	2.7883	1.9427	30.8426	7.6482	14.616	11.2444	3.3944	2.8790	25.3355
24^+	23.7865	15.8023	54.5974	10.5549	30.6640	138.112	$24^+ - 22^+$	2.9985	2.2467	31.3490	8.3650	17.4853	11.2373	3.5478	3.1452	26.4956
26^+	26.9870	16.0663	63.6674	10.6210	34.3551	146.231	$26^+ - 24^+$	3.2005	2.5598	31.8704	9.0701	20.5584	11.2458	3.6910	3.4046	27.6344

Table (2) : To be continued (4/6).

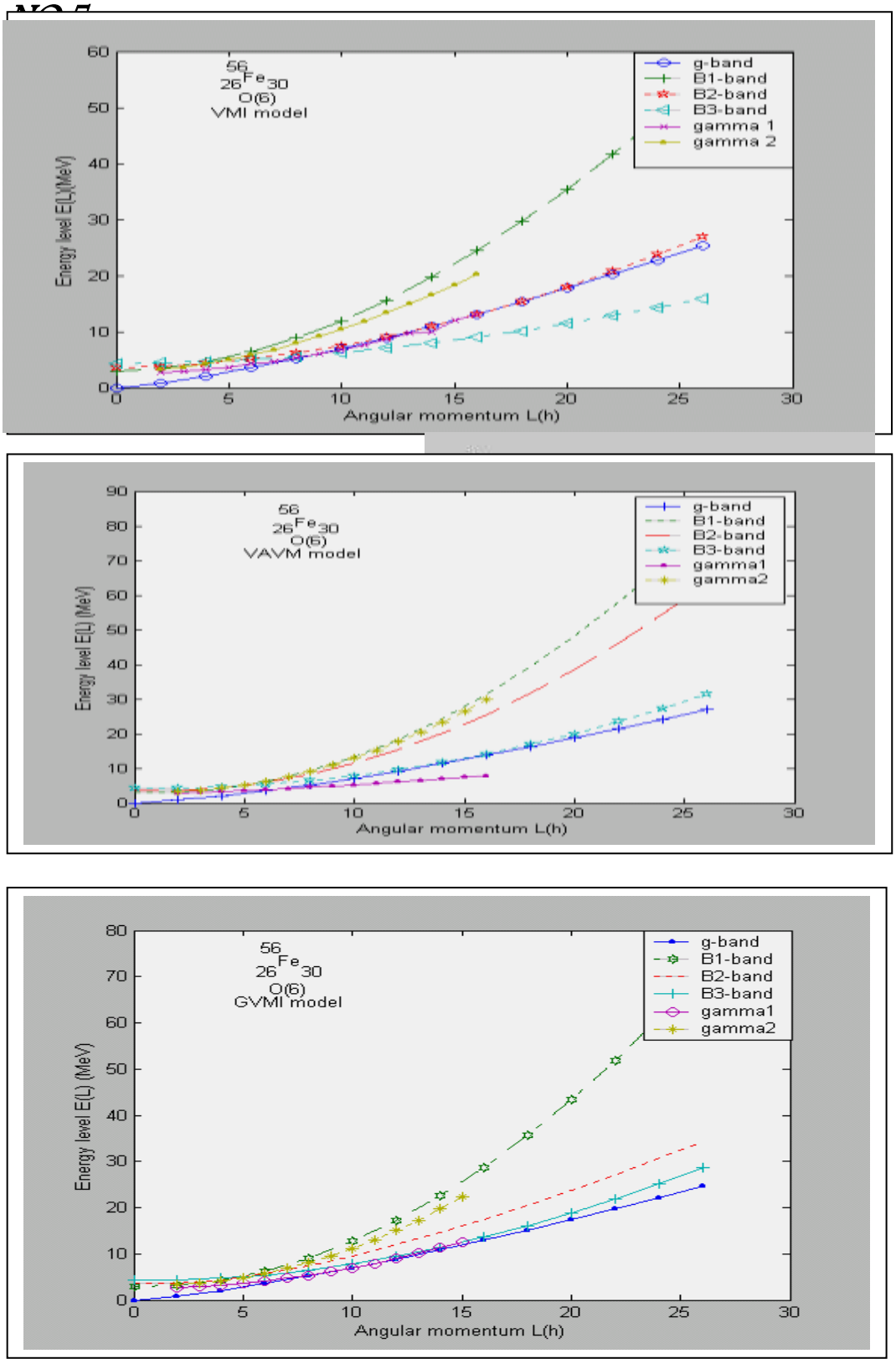
Nuclei $^{56}_{26}Fe_{30}$					Band	N_π	N_υ	N	Dynamical symmetry								
					β_3	1	1	2	O(6)								
L^+	VMI (Pw)		VAVM (Pw)		GVMI (Pw)		$L_1^+ - L_f^+$	VMI (Pw)			VAVM (Pw)			GVMI (Pw)			
	Energy (MeV)	ϑ (MeV)$^{-1}$	Energy (MeV)	ϑ (MeV)$^{-1}$	Energy (MeV)	ϑ (MeV)$^{-1}$		Eγ (MeV)	$\hbar^2\omega^2$ (MeV)^2	$2\vartheta/\hbar^2$ (MeV)$^{-1}$	Eγ (MeV)	$\hbar^2\omega^2$ (MeV)^2	$2\vartheta/\hbar^2$ (MeV)$^{-1}$	Eγ (MeV)	$\hbar^2\omega^2$ (MeV)^2	$2\vartheta/\hbar^2$ (MeV)$^{-1}$
0^+	4.3002	27.9000	4.3002	23.2000	4.3002	93.400	-	-	-	-	-	-	-	-	-	-	-
2^+	4.4076	27.9480	4.3194	23.2000	4.3216	93.4208	$2^+ - 0^+$	0.1074	0.0019	55.8481	0.0192	0.0001	312.5	0.0214	0.0001	28.02	
4^+	4.6576	28.0588	4.6834	23.2013	4.6840	93.7714	$4^+ - 2^+$	0.2500	0.0153	56.0069	0.3640	0.0324	38.4597	0.3624	0.0321	38.6306	
6^+	5.0484	28.2294	5.3922	23.2040	5.3836	94.4373	$6^+ - 4^+$	0.3908	0.0378	56.2887	0.7088	0.1245	31.0395	0.6996	0.1213	31.4447	
8^+	5.5777	28.4557	6.4456	23.2081	6.4136	95.3932	$8^+ - 6^+$	0.5292	0.0697	56.6861	1.0534	0.2762	28.4789	1.0299	0.2640	29.1277	
10^+	6.2421	28.7328	7.8434	23.2135	7.7644	96.6066	$10^+ - 8^+$	0.6644	0.1100	57.7898	1.3979	0.4871	27.1842	1.3508	0.4549	28.1318	
12^+	7.0381	29.0549	9.5855	23.2202	9.4247	98.0418	$12^+ - 10^+$	0.7960	0.1581	57.7894	1.7421	0.7573	26.4051	1.6603	0.6879	27.7051	
14^+	7.9616	29.4167	11.6716	23.2283	11.3823	99.6632	$14^+ - 12^+$	0.9235	0.2129	58.4737	2.0860	1.0863	25.8868	1.9575	0.9567	27.5856	
16^+	9.0083	29.8127	14.1011	23.2377	13.6242	100.541	$16^+ - 14^+$	1.0467	0.2736	59.2318	2.4296	1.8741	25.519	2.2419	1.2552	27.6550	
18^+	10.1740	30.2378	16.8738	23.2484	16.1376	١٠١,١٢٢	$18^+ - 16^+$	1.1656	0.3394	60.0531	2.7727	1.9204	25.2461	2.5134	1.5780	27.8505	
20^+	11.4542	30.6874	19.9892	23.2605	18.9100	١٠٢,٦٣١	$20^+ - 18^+$	1.2802	0.4094	60.9281	3.1154	2.4248	25.0371	2.7724	1.9203	28.1344	
22^+	12.8447	31.1576	23.4467	23.2738	21.9294	١٠٣,٨١٥	$22^+ - 20^+$	1.3905	0.4831	61.8481	3.4575	2.9870	24.8733	3.0194	2.779	28.4825	
24^+	14.3414	31.6448	27.2458	23.2885	25.1844	١٠٤,٣٤١	$24^+ - 22^+$	1.4967	0.5598	62.8055	3.7991	3.6066	24.7429	3.2551	2.6476	28.8781	
26^+	15.9403	32.1459	31.3858	23.3044	28.6646	١٠٥,٩٠١	$26^+ - 24^+$	1.5989	0.6388	63.7938	4.1400	4.2832	24.6377	3.4801	3.0266	29.3096	

Table (2) : To be continued (5/6).

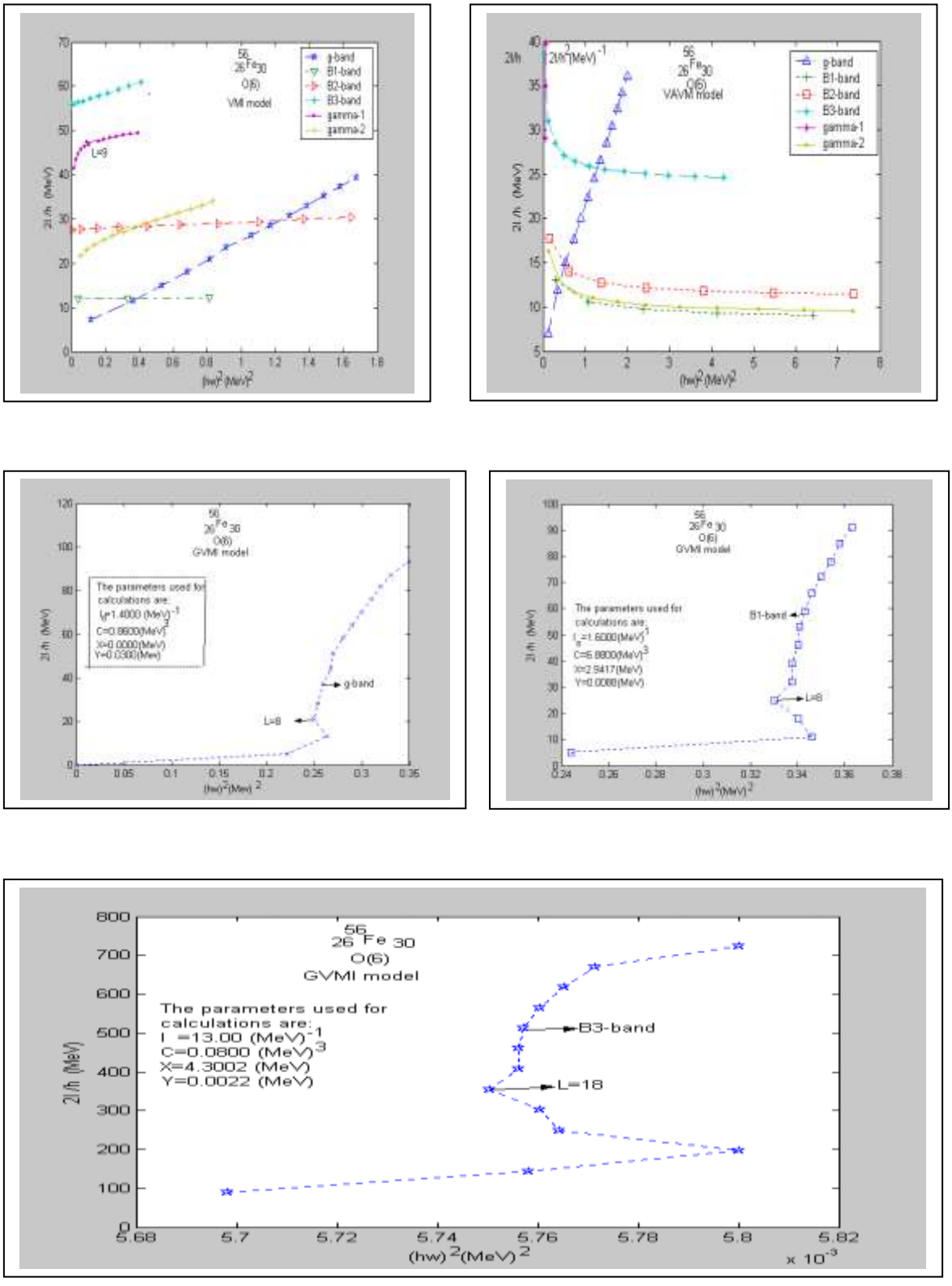
Nuclei $^{56}_{26}Fe_{30}$					Band	N_π	N_υ	N	Dynamical symmetry								
					γ_1	1	1	2	O(6)								
L^+	VMI (Pw)		VAVM (Pw)		GVMI (Pw)		$L_1^+ - L_f^+$	VMI (Pw)			VAVM (Pw)			GVMI (Pw)		
	Energy (MeV)	ϑ $(MeV)^{-1}$	Energy (MeV)	ϑ $(MeV)^{-1}$	Energy (MeV)	ϑ $(MeV)^{-1}$		Eγ (MeV)	$h^2\omega^2$ $(MeV)^2$	$2\vartheta/h^2$ $(MeV)^{-1}$	Eγ (MeV)	$h^2\omega^2$ $(MeV)^2$	$2\vartheta/h^2$ $(MeV)^{-1}$	Eγ (MeV)	$h^2\omega^2$ $(MeV)^2$	$2\vartheta/h^2$ $(MeV)^{-1}$
2^+	2.6808	12.3903	2.6577	0.8800	2.7692	17.9016	-	-	-	-	-	-	-	-	-	-	-
3^+	2.9226	12.3999	2.9425	15.8296	2.9658	17.9043	$3^+ - 2^+$	0.2418	0.0139	41.3602	0.2848	0.0788	21.0671	0.1967	0.0092	50.8506
4^+	3.2447	12.4128	3.2115	21.8377	3.2562	17.9084	$4^+ - 3^+$	0.3221	0.0253	43.4676	0.2690	0.0712	29.7404	0.2904	0.0206	48.2094
5^+	3.6468	12.4288	3.5038	26.8598	3.6404	17.9138	$5^+ - 4^+$	0.4021	0.0399	44.7613	0.2923	0.0846	34.2130	0.3841	0.0364	46.8604
6^+	4.1287	12.4479	3.8182	31.3657	4.1181	17.9205	$6^+ - 5^+$	0.4819	0.0575	45.6544	0.3144	0.0981	38.1724	0.4778	0.0566	46.0487
7^+	4.6899	12.4700	4.1525	35.5298	4.6894	17.9284	$7^+ - 6^+$	0.5613	0.0783	46.3237	0.3343	0.1112	41.8793	0.5713	0.0811	45.5113
8^+	5.3302	12.4951	4.5048	39.4420	5.3541	17.9377	$8^+ - 7^+$	0.6402	0.1020	46.8574	0.3523	0.1236	45.4112	0.6647	0.1099	45.1336
9^+	6.0489	12.5231	4.8736	43.1564	6.1120	17.9483	$9^+ - 8^+$	0.7187	0.0887	47.3046	0.3688	0.1356	48.8048	0.7580	0.1431	44.8575
10^+	6.8457	12.5539	5.2576	46.7092	6.9631	17.9601	$10^+ - 9^+$	0.7967	0.1582	47.6943	0.3840	0.1471	52.0824	0.8510	0.1806	44.6505
11^+	7.7198	12.5875	5.6558	50.1261	7.9071	17.9732	$11^+ - 10^+$	0.8742	0.1906	48.054	0.3981	0.1582	55.2598	0.9440	0.2223	44.4930
12^+	8.6709	12.6237	6.0671	53.4263	8.9437	17.9876	$12^+ - 11^+$	0.9510	0.2257	48.3693	0.4113	0.1688	58.3488	1.0367	0.2682	44.3723
13^+	9.6981	12.6625	6.4908	56.6245	10.0729	18.0032	$13^+ - 12^+$	1.0272	0.2634	48.6750	0.4237	0.1793	61.3587	0.1292	0.3182	44.2800
14^+	10.8008	12.7038	6.9263	59.7326	11.2944	18.0200	$14^+ - 13^+$	1.1027	0.3036	48.9681	0.4355	0.1894	64.2972	1.2214	0.3725	44.2103
15^+	11.9784	12.7474	7.3729	62.7600	12.6078	18.0381	$15^+ - 14^+$	1.1776	0.3463	49.2528	0.4466	0.1992	67.1708	1.3134	0.4308	44.1590

Table (2) : To be continued (6/6).

Nuclei $^{56}_{26}Fe_{30}$					Band	N_π	N_υ	N	Dynamical symmetry								
					γ_2	1	1	2	O(6)								
L^+	VMI (Pw)		VAVM (Pw)		GVMI (Pw)		$L_1^+ - L_f^+$	VMI (Pw)			VAVM (Pw)			GVMI (Pw)		
	Energy (MeV)	ϑ $(MeV)^{-1}$	Energy (MeV)	ϑ $(MeV)^{-1}$	Energy (MeV)	ϑ $(MeV)^{-1}$		Eγ (MeV)	$\hbar^2\omega^2$ $(MeV)^2$	$2\vartheta/\hbar^2$ $(MeV)^{-1}$	Eγ (MeV)	$\hbar^2\omega^2$ $(MeV)^2$	$2\vartheta/\hbar^2$ $(MeV)^{-1}$	Eγ (MeV)	$\hbar^2\omega^2$ $(MeV)^2$	$2\vartheta/\hbar^2$ $(MeV)^{-1}$
2^+	3.1644	6.2778	3.264	8.2000	3.3938	83.0202	-	-	-	-	-	-	-	-	-	-
3^+	3.6266	6.3843	3.6317	8.2067	3.6948	83.0240	$3^+ - 2^+$	0.4622	0.0508	21.6371	0.3677	0.1313	16.3175	0.3010	0.215	33.2248
4^+	4.2324	6.5163	4.2425	8.2179	4.1883	83.0300	$4^+ - 3^+$	0.6057	0.0897	23.1122	0.6108	0.3672	13.0967	0.4936	0.595	28.3645
5^+	4.9744	6.6684	5.0955	8.2335	4.8745	83.0383	$5^+ - 4^+$	0.7420	0.1359	24.2571	0.8530	0.7202	11.7235	0.6861	0.1162	26.2342
6^+	5.8452	6.8356	6.1893	8.2534	5.7531	83.0490	$6^+ - 5^+$	0.8708	0.1879	25.2641	1.0938	1.1880	10.9711	0.8786	0.1913	25.0392
7^+	6.8373	7.0139	7.5221	8.2774	6.8241	83.0620	$7^+ - 6^+$	0.9921	0.2446	26.2069	1.3328	1.7674	10.5038	1.0710	0.2850	24.2754
8^+	7.9436	7.2001	9.0920	8.3054	8.0875	83.0773	$8^+ - 7^+$	1.1063	0.3046	27.1168	1.5699	2.4548	10.1918	1.2633	0.3972	23.7460
9^+	9.1576	7.3919	10.8966	8.3374	9.5431	83.0950	$9^+ - 8^+$	1.2139	0.3671	28.0078	1.8046	3.2464	9.9748	1.4556	0.5278	23.3581
10^+	10.4731	7.5873	12.9333	8.3729	11.1908	83.1150	$10^+ - 9^+$	1.3155	0.4314	28.8862	2.0366	4.1376	9.8199	1.6477	0.6768	23.0625
11^+	11.8846	7.7851	15.1992	8.4120	13.0305	83.1373	$11^+ - 10^+$	1.4115	0.4970	29.7550	2.2659	5.1238	9.7090	1.8397	0.8441	22.8303
12^+	13.3871	7.9842	17.6914	8.4544	15.0619	83.1618	$12^+ - 11^+$	1.5025	0.5633	30.6153	2.4921	6.2000	9.6302	2.0314	1.0297	22.6410
13^+	14.9761	8.1838	20.4065	8.4999	17.2850	83.1888	$13^+ - 12^+$	1.5889	0.6302	31.4675	2.7152	7.3612	9.5758	2.2231	1.2336	22.4643
14^+	16.4673	8.3800	23.3414	8.5483	19.6996	83.2180	$14^+ - 13^+$	1.6712	0.6973	32.3120	2.9348	8.6022	9.5405	2.4146	1.4555	22.3679
15^+	18.3970	8.5826	26.4924	8.5995	22.3054	83.2494	$15^+ - 14^+$	1.7497	0.7644	33.1486	3.1510	9.9181	9.5206	2.6058	1.6955	22.2579



Figure(2):The energy band crossing E(L) as a function of angular momentum L using (VMI,VAVM,GVMI) model. The parameters used for calculations are in table(1).



Figure(3):The back bending($2\vartheta/\hbar^2$) versus $(\hbar\omega)^2$ plot for $^{56}_{26}Fe_{30}$ nucleus using (VMI,VAVM,GVMI) model .The parameters used for calculations are in table(1)and in same figure .

REFERENCES

1-Bonatsos D.,"Interaction boson model of nuclear structure," Ed.David. Stanford ,pub. In the united state by oxford university press., New York7th P.1,2.(1988).

2-Krane K.S.," Introductory nuclear physics," Ed, Halliday,D. , Pub. John, Wiley, P.141-144 (1987).

3-Deleplanque M.A.,Frauenderf S., Pashkevich V.V., Chu S.Y., and Unzhakova A.,"Gross shell structure of moments of inertia,"Lawrence Berkeley National Laboratory,(LBNI), University of California, Vol. 52316 (2002).

4-Mariscotti M.A.J., Gertude Scharff-Gold Haber, and Brain Buck; phys. Rev.Vol.178,P.1864 (1969).

5-Harris S.M.; phys. Rev. B,Vol.138,P.509 (1965).

6-Alenicheva T.V., Kabina P., Mitropolsky I.A., and Tyukavina T.M. ; IAEA Nuclear data section, Wagramer strasse5,A-1400 Viena , INDC(CCP),P.439 (2004).

7-Scharff-Gold Haber G., and Gold Haber A.S.; phys. Rev. Let. , Vol.24,P.1349 (1970).

8-Elmasri Y., and Vervier J., and Faessler A.;Nucl. Phys.,A,Vol.279, P.223 (1977).

9-Bonatsos D.; phys. Rev.,C,Vol.31,P.2265 (1985).

10-Bonatsos D. and Klein A.; phys. Rev. C,Vol.29,P.1879 (1984).

11-Alenicheva T.V., Kabina P., Mitropolsky I.A., and Tyukavina T.M. ; IAEA Nuclear data section, Wagramer strasse5,A-1400 Viena , INDC(CCP),P.439 (2004).

12-Bonatsos D.;J.Phys.,G, Nucl. Phys.Vol.14,P.569-588 (1988).

13-Elloek B.,"Analysis and description of nuclear rotational spectra," Neutron capture-Gamma ray spectroscopy, P.443 (1969).

14-Murray R., Spiegel and Larry J. Stephens," schaum,s out lines statistics," Ed. Mc Graw Hill (1999).