Theoretical study of energy Levels as a comprehensive description of the nuclear structure for some even-even nuclei

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ABSTRACT

In the present work, the interacting boson model (IBM-١) was used in the calculation of the energy levels as a function of angular momentum E(L) for some even-even deformed nuclei. The calculated results are compared with the available experimental data and they found to be in a good agreement, especially at low-lying states, while at high angular momentum, some theoretical values are somehow larger than experimental.

المستخلص

في البحث الحالي تم استخدام نموذج البوزونات المتفاعلة الاول (IBM-١) لحساب مستويات الطاقة كدالة للزخم الزاوي لبعض الانوية المشوهة الزوجية– الزوجية وقد تم مقارنة هذه النتائج مع النتائج العملية المتوفرة وكانت النتائج متطابقة بشكل جيد خصوصا عند مستويات الطاقة الواطئة .أما عند المستويات العالية فكانت القيم النظرية اكبر من القيم العملية.

INTRODUCTION

Nuclear transmutations are strongly affected by the angular momentum of the initial and final systems because, they have to satisfy the angular momentum conservation law, and this allows determinations of angular momentum (L) in some cases [١]. In the absence of any detailed knowledge of the nuclear force and nuclear structure, many theoretical descriptions of nuclei have been centered on “nuclear models” [٢].One of them is the interacting boson model (IBM).

The interacting boson model (IBM-١) was proposed, early by Feshbach I., and Iachello F. (١٩٧٣)[٣], and it was developed after that by Arima A., and Iachello F. (١٩٧٤)[٤]. Dukelsky J.,and Pittels S. study the exact solution for interacting boson model with repulsive pairing and shows a new and unexpected mechanism for sd dominance [٥].

The (IBM-١) is suitable for describing the collective structure of even – even medium and heavy deformed nuclei [٦]. They assumed that the shell model reveals that the low – lying collective state of such nuclei arises from interacting nucleon pairs coupled with angular momentum L = ٠ or ٢ (called s and d) bosons with energies ε_s , ε_d respectively, the energy difference of s and d bosons is equal to $\varepsilon_d - \varepsilon_s$ where ε_s is almost Zero [٧,٨].

The (IBM) is rooted in the spherical shell model and geometrical collective model of atomic nucleus [٩] and based on the following assumptions:

-The pairs of active nuclear particles or holes near closed shells are treated as "*bosons*" i.e. pairs of fermions [١٠].

-This model depends on the total number of bosons (N).

Where $N=N_\pi+N_\upsilon$,

N_π =number of proton boson, N_υ =number of neutron boson [١١]

-The multitude of shell which appears in the shell model is reduced to the simple s-shell (L=٠) and d-shell (L=٢) only, which are composed vectorially by s and d bosons analogously to the shell model technique[٩].

There are four versions of the (IBM) ,called IBM-١,٢,٣ and ٤.In this paper the researchers are concerned with version one of this model i.e"*IBM-١*". This version does not distinguish between neutron and proton bosons.

THEORETICAL PART

١. The Hamiltonian operator in (IBM-١)

The most commonly used form of IBM-١ Hamiltonian is [٨]:

$$\hat{H} = \in \hat{n}_d + a_o(\hat{P}^{\dagger}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \qquad (1)$$

Where :

$$\hat{n}_d = (\hat{d}^{\dagger}.\hat{\tilde{d}}) \quad \text{d-bosons number operator}$$

$$\hat{P} = 1/2(\hat{\tilde{d}}.\hat{\tilde{d}}) - 1/2(\hat{\tilde{s}}.\hat{\tilde{s}}) \quad \text{pairing operator}$$

$$\hat{L} = \sqrt{10}[\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(\lambda)} \quad \text{angular momentum operator}$$

ℓ=٠,١,…٤

$$\hat{Q} = [(\hat{d}^{\dagger} \times \hat{\tilde{s}}) + (\hat{\tilde{s}}^{\dagger} \times \hat{\tilde{d}})] - \frac{\sqrt{7}}{2}[\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(2)} \quad \text{quadrupole moment operator}$$

$$\hat{T}_3 = [\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(3)} \quad \text{octupole operator}$$

$$\hat{T}_4 = [\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{(4)} \quad \text{hexadecapole operator}$$

.(2)

and a_o,…..$a_٤$ are the strengths of $(\hat{P},\hat{L},\hat{Q},\hat{T}_3,\hat{T}_4)$ interacting between bosons respectively.

The s(L=٠) and d(L=٢) bosons can be described in terms of "*unitary group*" in ٦-components called U(٦), and the different reductions of U(٦) leads to "*three dynamical symmetries*" end in O(٢).

The analysis of U(٦) as below [٧]:

$$U(6) \supset \begin{Bmatrix} SU(5) \supset O(5) \\ SU(3) \\ O(6) \supset O(5) \end{Bmatrix} \supset O(3) \supset O(2) \begin{matrix} \ldots I \\ \ldots II \\ \ldots III \end{matrix} \qquad (٣)$$

RESULTS AND DISCUSSION

The reduced Hamiltonian of chain I $(\hat{H}^{(1)})$ after the parameters a_o and $a_٢$ are vanished can be written as [٨]:

$$\hat{H}^{(I)} = \in \hat{n}_d + a_1 \hat{L}^2 + a_3 \hat{T}_3^2 + a_4 \hat{T}_4^2 \qquad (4)$$

The SU(٣) dynamical symmetry(chain II) occurs wherever the quadrupole-quadrupole interacting between bosons are dominating, $\in = a_o = a_3 = a_4 = 0$. The general reduced Hamiltonian of this chain is [٧,٨]:

$$H^{(II)} = a_1 \hat{L}^2 + a_2 \hat{Q}^2 \qquad (5)$$

While the reduced Hamiltonian of chain III is [٧]:

$$H^{(III)} = a_0 (\hat{P}^\dagger . \hat{P}) + a_1 (\hat{L} . \hat{L}) + a_3 (\hat{T}_3 . \hat{T}_3) \qquad (6)$$

Table (١) shows the corresponding parameters obtained with the best fitting from equations (١),(٤)and(٥)according to dynamical symmetry of chosen nuclei.

Table (١): The parameters values of Hamiltonian operator for some even-even nuclei.

Parameters / Nuclei	N_π	N_υ	N	EPS (MeV)	$\hat{P}^\dagger.\hat{P}$ (MeV)	$\hat{L}.\hat{L}$ (MeV)	$\hat{Q}.\hat{Q}$ (MeV)	$\hat{T}_3.\hat{T}_3$ (MeV)	$\hat{T}_4.\hat{T}_4$ (MeV)	CHI (MeV)
${}^{74}_{34}Se_{40}$	٣	٥	٨	٠.٢٦٦٢	٠.١١٠٠	٠.٠٣١٤	٠.٠٠٦٨	٠.١٠٢٠	٠.٠٦٦٠	٠.٠٨٠٠
${}^{106}_{46}Pd_{60}$	٢	٥	٧	٠.٥١٠٠	-٠.٠٠٦٠	٠.٠١١٠	٠.٠٠٠٠	-٠.٠٠٧٠	٠.٠٠٧٠	٠.٢٠٠٠
${}^{156}_{64}Gd_{92}$	٧	٥	١٢	٠.٠٠٠٠	٠.٠٠٤٤٤	٠.٠٠٧٧	-٠.٠١٤٨	٠.٠٠٥٥	٠.٠٠٠٠	-١.٠٣٣٠
${}^{186}_{74}W_{112}$	٤	٧	١١	٠.٠٠٠٠	٠.٠٠٤٥	٠.٠٠٩٥	-٠.٠١٧٠	٠.٠٥٠٠	٠.٠٧٠١	-٠.٨٣٠٠

The parameters of IBM-١ Hamiltonian in the multipole expansion have been fixed by fits to the energy levels of experimental values for low spin. These parameters have been shown in table(١)and used to calculate the values of energy levels and find the energy transitions of even parity states for each chosen nuclei .

The energy bands arrangement(g,β,γ) and the presence of their appearance is one of specific features ,which are used to classified the nuclear behavior of each selected nuclei .

The general known arrangement is the appearance g-band with sequence $(0_1^+, 2_1^+, 4_1^+, ...)$,β-band $(0_2^+, 2_2^+, 4_2^+, ...)$ and γ-band $(2_3^+, 3_1^+, 4_3^+, 5_1^+, ...)$ (i.e 0_2^+ is below 2_2^+). In this case the dynamical symmetry is either SU(٣) or SU(٥) as in ${}^{156}_{64}Gd_{92}$ nucleus , which is belonging to SU(٣)-SU(٥) dynamical symmetry, whereas in case of γ-band appearance before β-band (i.e 2_2^+ is below 0_2^+) this means occurrence of " ***breaking symmetry"*** to which, the nucleus belongs to it, In other words, if the state 2_2^+ comes before 0_2^+ the dynamical symmetry is O(٦). This breaking symmetry shows in $^{106}_{46}Pd_{60}$,and $^{186}_{74}W_{112}$ nuclei ,leading to appearance of γ-unstable band , within the general behavior SU(٣) for $^{186}_{74}W_{112}$,and SU(٥) for $^{106}_{46}Pd_{60}$. In this case the term of($\hat{P}^{\dagger}.\hat{P}$) in Hamiltonian equations (١) and (٦), which is belonging to the dynamical symmetry O(٦) is the dominated.

We classified the selected nuclei according to above bands arrangement (ground, beta, gamma-bands). These bands are tabulated in table(٢). This table shows the energy levels E(L) for each chosen nuclei in comparison with available experimental data .The comparison shows quite well agreement in most of them at low angular momentum . This comparison can be shown more clearly in figures (١ → ٤).

These figures show the comparison between the calculated IBM-١ energy spectrum (pw) of even-parity levels with available experimental results for $^{74}_{34}Se_{40}$, $^{106}_{46}Pd_{60}$, $^{156}_{64}Gd_{92}$, $^{186}_{74}W_{112}$ belonging to the dynamical symmetries SU(٥)-O(٦)-SU(٣), O(٦)-SU(٥), SU(٣)-SU(٥),SU(٣)-O(٦) respectively .. From these figures we can see that very good reasonable agreement between the values of energy ground state (g-band) of sequence (0_1^+, 2_1^+, 4_1^+,...) and their experimental state best than other bands.

In fact table (٢) and figures (١→ ٤) show that the SU(٣), SU(٥), O(٦)-Hamiltonian give a very reasonable description of the energy spectrum for all chosen nuclei.

Table (٢): The comparison between the experimental and theoretical (pw) energy bands (g, β ,γ) in (MeV) for the chosen even-even nuclei using (IBM- ١).

Nuclei	Band \ spin	٠+ / ٢+	٢+ / ٣+	٤+ / ٤+	٦+ / ٥+	٨+ / ٦+	١٠+ / ٧+	١٢+ / ٨+	١٤+ / ٩+	١٦+ / ١٠+
$^{74}_{34}Se_{40}$	g – exp	٠.٠٠٠٠	٠.٦٣٤٨	١.٣٦٣٢	٢.٢٣١٤	٣.١٩٨٤	٤.٢٥٦٣	٥.٤٤٣٠	٦.٧٣٥٣	٨.١١٨٠
	g – pw	٠.٠٠٠٠	٠٤٨٢٧	١.٢٦٠٣	٢.٣٢٩١	٣.٦٨٧٤	٥.٣٣٣٩	٧.٢٦٨١	٩.٤٨٩٦	١١.٩٩٨٠
	β ١– exp	٠.٨٥٣٨	١.٢٦٩٠	٢.١٠٨٠	٣.٢٠٠٨	———	———	———	———	———
	β ١– pw	٠.٩٦٢٠	١.٠٢٩٦	١.٩٦٦٦	٣.١٩٢٨	٤.٧٠٧٤	٦.٥٠٩٧	٨.٥٩٩٢	١٠.٩٧٥٨	———
	γ١– exp.	١.٨٣٨٧	١.٨٨٤٢	٢.٦٦٢٩	٢.٩٨٦٧	٣.٩٢٨٧	٤.١٩٨٧	———	٤.٤٤٩٩	———
	γ١– pw	١.٦٨٩٥	١.٨٣٤٦	٢.٧٠٩٧	٢.٩٩٤٩	٤.٠٢١١	٤.٤٤٣٦	٥.٦٢٢٦	٦.١٧٩٩	٧.٥١٣١
	β ٢–exp	٢.١٣٠٠	٢.٥٦٣٥	٢.٨٣١٥	٣.٩٨٠٠	———	———	———	———	———
	β ٢– pw	١.٦٣٦٩	٢.٤٧٨٩	٢.٨٣٠١	٤.٢١٢٨	٥.٨٨٣١	٧.٨٤٠٨	١٠.٠٨٥٤	———	———
	β ٣–exp	٢.٧١٨٠	٢.٨١٨٥	٣.٦٧٤٨	———	———	———	———	———	———
	β ٣– pw	٢.٣١٦٣	٢.٥٩٩٣	٣.٦٥٨٥	٥.١٢٨٠	٦.٨٨٦٦	٨.٩٣٣٥	———	———	———
	β ٤–exp	٢.٩١٨٠	٣.٣٧٩٤	٣.٧٧١٩	———	———	———	———	———	———
	β ٤– pw	٣.٣٢٨٧	٣.٣٣٧٥	٣.٨٥٠١	٥.٣٨٨٥	٧.٢١٤٢	٩.٣٢٦٩	———	———	———
$^{106}_{46}Pd_{60}$	g – exp	٠.٠٠٠٠	٠.٥١١٨	١.٢٢٩٢	٢.٠٧٦٣	٢.٩٦٢٥	٣.٥٣٣٠	٤.٠٨٨٢	٤.٨٩٣٣	٥.٨٩٤٥
	g – pw	٠.٠٠٠٠	٠.٥٩٦٨	١.٢٧٧٥	٢.٠٤٢١	٢.٨٩٠٥	٣.٨٢٢٧	٤.٨٣٨٧	٥.٩٣٨٣	———
	γ١– exp.	١.١٢٨٠	١.٥٥٧٦	١.٩٣٢٣	٢.٧٥٧٠	———	———	———	———	———
	γ١– pw	١.١٢٠٧	١.٧٠٦١	١.٧٩٥٧	٢.٤٢٠١	٢.٥٥٤٥	٣.٢١٧٩	٣.٣٩٧١	٤.٠٩٩٥	٤.٣٢٣٥
	β ١–exp	١.١٣٣٨	١.٥٦٢٢	٢.٠٧٦٦	———	———	———	———	———	———
	β ١– pw	١.٠٨٩٠	١.٦٨٨٤	٢.٣٠٨١	٣.٠٦١١	٣.٨٩٧٩	٤.٨١٨٣	———	———	———
	β ٢–exp	١.٧٠٦٤	٢.٢٤٢٥	٢.٢٨٢٩	———	———	———	———	———	———
	B ٢– pw	١.٥٧١٧	٢.١٥١٣	٢.٣٧١٧	٣.١٣٨٦	٣.٩٨٩٢	٤.٩٢٣٤	———	———	———

Table (٢) / To be continued : (٢/٤)

Nuclei	Band \ spin	٠+ ٢+	٢+ ٣+	٤+ ٤+	٦+ ٥+	٨+ ٦+	١٠+ ٧+	١٢+ ٨+	١٤+ ٩+	١٦+ ١٠+
$^{10}_{4}$	β ٣–exp	٢.٠٠١٥	٢.٣٠٨٨	٢.٣٦٦٠	———	———	———	———	———	———
	β ٣–pw	٢.١٨٣٠	٢.٢١٤٩	٢.٨١٤٧	٣.٥٦١٩	٤.٣٩٢٧	———	———	———	———
	β ٤– exp	٢.٢٧٨١	٢.٤٣٩١	٢.٦٤٨٩	———	———	———	———	———	———
	β ٤– pw	٢.٦٦٨٢	٢.٦٨٨٠	٢.٨٩٢٢	٣.٥٦١٩	٤.٣٩٢٧	———	———	———	———
$^{156}_{64}Gd_{92}$	g – exp	٠.٠٠٠٠	٠.٠٨٩٠	٠.٢٨٨٢	٠.٥٨٤٧	٠.٩٦٥١	١.٤١٦٠	١.٩٢٤٤	٢.٤٧٥٧	٣.٠٥٩٥
	g – pw	٠.٠٠٠٠	٠.٠٨١٦	٠.٢٧١٩	٠.٥٧٠٣	٠.٩٧٦٠	١.٤٨٨٠	٢.١٠٥٢	٢.٨٢٦٦	٣.٦٥٠٩
	β ١–exp	١.٠٤٩٥	١.١٢٩٤	١.٢٩٧٨	١.٥٤٠٢	١.٨٤٨٩	٢.٢١٩٩	٢.٧٠٧٧	———	———
	β ١–pw	٠.٩٣٤٧	٠.٩٣٩٥	١.١٣٥٢	١.٤٤٠٤	١.٨٥٣٠	٢.٣٧٠٤	٢.٩٩٠١	———	———
	γ١– exp.	١.١٥٤١	١.٢٤٨٠	١.٣٥٥٤	١.٥٠٦٨	١.٦١٣٨	١.٨٤٩٨	٢.٠١٠٧	———	———
	γ١– pw	١.٠١٧٣	١.٠٢٣٧	١.٢٤٩٨	١.٢٧٥٥	١.٥٥١٦	١.٦٣٧٣	١.٩٦١٩	———	———
	β ٢– exp	١.١٦٨٢	١.٢٥٨٠	١.٥١٠٧	١.٧٦٥٦	٢.١٣٤١	٢.٥٢٣٠	———	———	———
	β ٢– exp	١.٦٣٢٦	١.٧٢١٣	١.٨٣١٩	٢.١٥٢٢	٢.٥٧٨٣	٣.١٠٤٢	———	———	———
	β ٣– exp	١.٧١٥٢	١.٧٧١١	١.٨٩٣٤	٢.١٩٥٣	———	———	———	———	———
	β ٣– pw	١.٧٣٥٠	١.٧٥٣٤	١.٩٢١٥	٢.٢٢٩١	٢.٦٤٥٦	٣.١٧٣٦	———	———	———
	γ ٢– exp	١.٨٢٧٨	١.٩١٦٣	٢.٠٢٠٦	٢.٠٦٦٦	٢.٤٨٤٠	———	———	———	———
	γ٢– pw	١.٨١٩١	١.٨٤٠٩	١.٩٦٢٢	١.٩٧٨٨	٢.٢٨٨٣	٢.٣٥٣٨	٢.٧٢٥٣	———	———
$^{186}_{74}W_{112}$	g – exp	٠.٠٠٠٠	٠.١٢٢٦	٠.٣٩٦٥	٠.٨٠٨٦	١.٣٤٨٦	٢.٠٠٢٠	———	———	———
	g – pw	٠.٠٠٠٠	٠.١٢٤٦	٠.٣٨٩٧	٠.٧٨٨٧	١.٣٢٠٥	١.٩٨٧٤	٢.٧٩٢١	٣.٧٣٧١	٤.٨٢٤٦
	γ١ – exp.	٠.٧٣٧٩	١.٠١٤٩	١.١٧٠٠	———	———	———	———	———	———
	γ١ – pw	٠.٨١٤٢	١.١٠٢٤	١.١٨٠٩	١.٥٩٧٧	١.٦٦٠٦	٢.٢٣٥٨	٢.٢٨١٠	٣.٠١٨٣	٣.٠٤٨٩
	β ١– exp	٠.٨٨٢٠	١.٢٨٤٠	١.٤٥٧١	———	———	———	———	———	———
	β ١– pw	٠.٨٦٩٤	١.١٢١٦	١.٥٣٠٥	٢.٠٩٨٢	٢.٨١٧٠	٣.٦٨٤٩	٤.٧٠١٣	٥.٨٦٦٢	٧.١٧٩٧
	β ٢– exp	١.١٥٣٠	١.٥٢٠٢	١.٦٠٨٠	———	———	———	———	———	———
	β ٢– pw	١.٥٠٠٤	١.٦٧٦٨	١.٧٥٢٧	٢.٤٠٢٣	٣.١٧٨٣	٤.٠٩٦٦	٥.١٦٢٢	٦.٣٧٦٧	٧.٧٤٠٤

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