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# Some Applications on Subclasses of Analytic Functions Involving Linear Operator

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***Abstract*--The object of this paper is to give some applications of differential subordination concept on subclasses of univalent functions for some convolution operators which are defined on the space of univalent meromorphic functions in the punctured open unit disc. Moreover, we derive some sandwich theorems, we study some geometric properties like coefficient bounds, distortion theorem, and radii of starlikeness and convexity for these classes of functions. Extreme points and integral operator are also investigated.**

**Keywords— Univalent Functions, Extreme Points, Integral Operator**

## I. Introduction

Let $\mathcal{A}$ denote the class of analytic functions $h$ of the form

$$f(z) = z + \sum_{n=1}^{\infty} c_n z^n, \quad (c_n > 0, z \in Q) \tag{1}$$

and $Q$ is the unit disk such that

$$Q = \{z \in \mathbb{C} : |z| < 1\}. \tag{2}$$

A function $f$ belonging to the class $\mathcal{A}$ is said to be starlike (convex) in $U(r)$ if:[2]

b $\Re e\left\{\frac{zf'(z)}{f(z)}\right\} > 0, \left(\Re e\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0\right)$, respectively

where $z \in U(r), 0 < r \leq 1$.

The method of differential subordinations (also known as the admissible functions method) was first introduced by Miller and Mocanu in 1978 [6] and the theory started to develop in 1981 [4]. For more details see [5-7].

For given analytic functions

$f(z) = \sum_{n=0}^{\infty} c_n z^n \ and \ h(z) = \sum_{n=0}^{\infty} b_n z^n$, we

denote by $f * h$ the Hadamard product (convolution) of $f$ and $h$ defined by

$$(f * h)(z) = \sum_{n=0}^{\infty} c_n b_n z^n = (f * h)(z) \tag{3}$$

We introduce the following definition of operator generalize.

**Definition 1.1.** For $h: \mathcal{A} \to \mathcal{A}$, the generalized operator $\mathrm{N}_{\lambda,\eta}^{m,s}: \mathcal{A} \to \mathcal{A}$ is defined by

$$\mathrm{N}_{\lambda,\eta}^{m,s} f(z) = c_1 z + \sum_{n=2}^{\infty} \frac{(1+\lambda(n-1))^{m-1}}{(1+\eta(n-1))^m} \left(\frac{1+a}{n+a}\right)^s c_n z^n \tag{4}$$

where $z \in Q, c \in \mathbb{C} \setminus \{0, -1, \ldots\}, s \in \mathbb{C}, m \in N_0 = \{0, 1, \ldots\}$ and $\eta \geq \lambda \geq 0$

Special cases of this operator when $c_1 = 1$, include salagean operator $\mathrm{N}_{1,0}^{m+1,0}$ [9], the generalized salagean operator introduced by Al-oboudi $\mathrm{N}_{\lambda,0}^{m+1,0}$ [1] and the srivastava-Attiya operator $\mathrm{N}_{0,0}^{m,s}$ [10].

The authors, introduce a new subclass by using the operator $\mathrm{N}_{\lambda,\eta}^{m,s}$ as follows

**Definition 1.2.** Let $\mathrm{F}_{\lambda,\eta}^{m,s}(c, A, B, \xi)$ be the class of functions $h$ of the form

$$f(z) = c_1 z - \sum_{n=2}^{\infty} c_n z^n, (c_1 > 0, c_n \geq 0, n \in \mathbb{N} \setminus \{1\})$$

and satisfying the following

$$\frac{1}{1-\xi}\left(\frac{z\left(\mathrm{N}_{\lambda,\eta}^{m,s} f(z)\right)'}{\mathrm{N}_{\lambda,\eta}^{m,s} f(z)} - \xi\right) \prec \frac{1+Az}{1+Bz} \tag{6}$$

$(-1 \leq B < A \leq 1)$ and $(0 \leq \xi < 1)$.

On the other hand for real parameter $\varepsilon (0 < |\varepsilon| < 1)$, we define the following subclasses of the class $\mathrm{F}_{\lambda,\eta,\varepsilon}^{m,s}(c, A, B, \xi)$

$$\mathrm{F}^{m,s}_{\lambda,\eta,\varepsilon}(c,A,B,\xi)=\{f: f\in \mathrm{F}^{m,s}_{\lambda,\eta}(c,A,B,\xi) \text{ and } f(0)=f(\varepsilon)-\varepsilon=0\} \tag{7}$$

And

$$\mathrm{F}^{m,s}_{\lambda,\eta,\varepsilon_*}(c,A,B,\xi)=\{f: f\in \mathrm{F}^{m,s}_{\lambda,\eta}(c,A,B,\xi) \text{ and } f(0)=f'(\varepsilon)-1=0\} \tag{8}$$

Several authors used the above class in different form like [3-8].

## II. COEFFICIENT INEQUALITY

We derive a necessary and sufficient condition for function f belonging in the class $\mathrm{F}^{m,1}_{\lambda,\eta}(c,A,B,\xi)$.

**Theorem 2.1.** A function f of the form (5) is in the class $\mathrm{F}^{m,s}_{\lambda,\eta}(c,A,B,\xi)$ if and only if

$$\sum_{n=2}^{\infty}[n(1-B)+(A-1)+\xi(B-A)]\,D_n c_n < [(A-B)+\xi(B-A)]c_1 \tag{9}$$

Where

$$D_n=\frac{(1+\lambda(n-1))^{m-1}}{(1+\eta(n-1))^{m}}\left(\frac{1+n}{n+n}\right)^{s},\quad n\in\mathbb{N} \tag{10}$$

**Proof.** Let $h\in \mathrm{F}^{m,s}_{\lambda,\eta}(c,A,B,\xi)$. Then we obtain

$$\frac{1}{1-\xi}\left(\frac{z\left(\mathrm{N}^{m,s}_{\lambda,\eta}f(z)\right)'}{\mathrm{N}^{m,s}_{\lambda,\eta}f(z)}-\xi\right)=\frac{1+Aw(z)}{1+Bw(z)} \tag{11}$$

when $w(0)=0$ and $|w(z)|<1$ for every $z\neq 0$, then

$$\left|\frac{z\left(\mathrm{N}^{m,s}_{\lambda,\eta}f(z)\right)'-\mathrm{N}^{m,s}_{\lambda,\eta}f(z)}{(A-B)(1-\xi)\mathrm{N}^{m,s}_{\lambda,\eta}f(z)-B\left[z\left(\mathrm{N}^{m,s}_{\lambda,\eta}f(z)\right)'-\mathrm{N}^{m,s}_{\lambda,\eta}f(z)\right]}\right|<1.$$

Thus,

$$\left|\frac{\sum_{n=2}^{\infty}(n-1)\,D_n c_n z^{n-1}}{c_1\frac{(A-B)(1-\xi)}{w}-\sum_{n=2}^{\infty}\big((A-B)(1-\xi)-B(n-1)\big)D_n c_n z^{n-1}}\right|<1.$$

Setting $z=r,\ (0<r<1)$ we get

$$\sum_{n=2}^{\infty}(n-1)\,D_n c_n r^{n-1}< c_1\frac{(A-B)(1-\xi)}{w}-\sum_{n=2}^{\infty}\big((A-B)(1-\xi)-B(n-1)\big)D_n c_n r^{n-1},$$

Letting $r\to 1^{-}$. Then we obtain the required result.
Conversely, it suffices to show that

$$\left|z\left(\mathrm{N}^{m,s}_{\lambda,\eta}f(z)\right)'-\mathrm{N}^{m,s}_{\lambda,\eta}f(z)\right|-\Big|(A-B)(1-\xi)\mathrm{N}^{m,s}_{\lambda,\eta}f(z)-B\left[z\left(\mathrm{N}^{m,s}_{\lambda,\eta}f(z)\right)'-\mathrm{N}^{m,s}_{\lambda,\eta}f(z)\right]\Big|<0$$

Choosing $z=r,\ (0<r<1)$, we get

$$\left|\sum_{n=2}^{\infty}(n-1)\,D_n c_n z^{n}\right|-\left|c_1\frac{(A-B)(1-\xi)}{w}z-\sum_{n=2}^{\infty}\big((A-B)(1-\xi)-B(n-1)\big)D_n c_n z^{n}\right|\le \sum_{n=2}^{\infty}(n-1)\,D_n c_n r^{n}$$

$$\Big(c_1\frac{(A-B)(1-\xi)}{w}-\sum_{n=2}^{\infty}\big((A-B)(1-\xi)-B(n-1)\big)D_n c_n r^{n}\Big)$$

$$=\sum_{n=2}^{\infty}[n(1-B)+(A-1)+\xi(B-A)]\,D_n c_n r^{n}-[(A-B)+\xi(B-A)]c_1 r$$

$$<\sum_{n=2}^{\infty}[n(1-B)+(A-1)+\xi(B-A)]D_n c_n-[(A-B)+\xi(B-A)]c_1\le 0$$

then, we obtain $f\in \mathrm{F}^{m,s}_{\lambda,\eta}(c,A,B,\xi)$.

**Corollary2.1.** A function f of the form (5) is in the subclass $\mathrm{F}^{m,s}_{\lambda,\eta,\varepsilon}(c,A,B,\xi)$ if and only if

$$\sum_{n=2}^{\infty}[n(1-B)+(A-1)+\xi(B-A)]\,D_n-[(A-B)+\xi(B-A)]\varepsilon^{n-1}]D_n\le[(A-B)+\xi(B-A)] \tag{12}$$

**Proof.** Form (5), we have

$$\frac{f(\varepsilon)}{\varepsilon} - 1 = c_1 - \sum_{n=2}^{\infty} c_n\, \varepsilon^{n-1},$$

the result follows by substitute $c_1 = 1 + \sum_{n=2}^{\infty} c_n\, \varepsilon^{n-1}$, in Theorem 2.1

**Corollary 2.2.** A function $f$ of the form (5) is in the subclass $\mathrm{F}_{\lambda,\eta,\varepsilon_*}^{m,s}(c, A, B, \xi)$ if and only if

$$\sum_{n=2}^{\infty} \left[\frac{[n(1-B)+(A-1)+\xi(B-A)]D_n}{(A-B)+\xi(B-A)}\right] c_n \leq 1 \qquad (13)$$

## III. Growth and Distortion Theorems

We derived the growth and distortion theorem in the class $\mathrm{F}_{\lambda,\eta}^{m,s}(c, A, B, \xi)$.

**Theorem3.1.** Let the function $h$ given by (5), be in the

class$\mathrm{F}_{\lambda,\eta}^{m,s}(c, A, B, \xi)$.

Then for ( $0 < |z| = r < 1$), we have

$$c_1 r - \frac{[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2} r^2 \leq |f(z)| \leq c_1 r + \frac{[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2} r^2 \qquad (14)$$

provided the sequence

$$\{[n(1-B)+(A-1)+\xi(B-A)]D_n\}_{n=2}^{\infty} \qquad (15)$$

is non- decreasing and positive. Also,

$$c_1 - \frac{[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2} r \leq |f'(z)| \leq c_1 + \frac{[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2} \qquad (16)$$

provided the sequence

$$\left\{\frac{[n(1-B)+(A-1)+\xi(B-A)]D_n}{n}\right\}_{n=2}^{\infty} \qquad (17)$$

is non- decreasing and positive.

**Proof.** By assumption, we have

$$\{[n(1-B)+(A-1)+\xi(B-A)]D_n\}_{n=2}^{\infty},$$

is non- decreasing and positive, then

$$\sum_{n=2}^{\infty} c_n \leq \frac{[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2}$$

For theorem,

$$\left\{\frac{[n(1-B)+(A-1)+\xi(B-A)]D_n}{n}\right\}_{n=2}^{\infty}$$

Is non -decreasing and positive, then

$$\sum_{n=2}^{\infty} n c_n \leq \frac{2[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2}$$

By (5), we get

$$|f(z)| = \left|c_1 z - \sum_{n=2}^{\infty} c_n z^n\right| \leq c_1 r + \sum_{n=2}^{\infty} c_n\, r^n$$

$$= c_1 r + r^2 \sum_{n=2}^{\infty} c_n\, r^{n-2} \leq c_1 r + r^2 \sum_{n=2}^{\infty} c_n \leq c_1 r + \frac{[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2}$$

and

$$|f(z)| \geq c_1 r - \frac{[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2} r^2$$

We obtain

$$|f'(z)| = \left|c_1 - \sum_{n=2}^{\infty} n c_n z^{n-1}\right| \leq c_1 + \sum_{n=2}^{\infty} n c_n\, r^{n-1} = c_1 + r \sum_{n=2}^{\infty} n c_n\, r^{n-2}$$

$$\leq c_1 + r \sum_{n=2}^{\infty} n c_n \leq c_1 + \frac{2[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2} r$$

and

$$|f'(z)| \geq c_1 - \frac{2[(A-B)+\xi(B-A)]c_1}{[1-2B+A+\xi(B-A)]D_2} r$$

Take the function

$$f_2(z) = c_1 z - \frac{((A-B)+\xi(B-A))c_1}{[1-2B+A+\xi(B-A)]D_2} z^2$$

**Theorem3.2.** Let the function $f$ of the form (5) in

$\mathrm{F}_{\lambda,\eta,\varepsilon}^{m,s}(c, A, B, \xi)$. Then

$$\varphi(r) \le |f(z)| \le \frac{[1-2B+A+\xi(B-A)]D_2 r + ((A-B)+\xi(B-A))r^2}{[1-2B+A+\xi(B-A)]D_2 - ((A-B)+\xi(B-A))\varepsilon} \quad (18)$$

$$\varphi(r) = \begin{cases} r & r \le \varepsilon \\ \frac{[1-2B+A+\xi(B-A)]D_2 r - ((A-B)+\xi(B-A))r^2}{[1-2B+A+\xi(B-A)]D_2 - ((A-B)+\xi(B-A))\varepsilon} & otherwise \end{cases} \quad (19)$$

provided the sequence

$$\{[n(1-B)+(A-1)+\xi(B-A)]D_n - [(A-b)+\xi(B-A)\varepsilon^{n-1}\}_{n=2}^{\infty} \quad (20)$$

is non- decreasing and positive. And

$$c_1 - \frac{2((A-B)+\xi(B-A))r}{[1-2B+A+\xi(B-A)]D_2 - ((A-B)+\xi(B-A))\varepsilon} \le |f'(z)|$$

$$\le \frac{[1-2B+A+\xi(B-A)]D_2 r + 2((A-B)+\xi(B-A))r}{[1-2B+A+\xi(B-A)]D_2 r - ((A-B)+\xi(B-A))r}, \quad (21)$$

provided the sequence

$$\left\{\frac{[n(1-B)+(A-1)+\xi(B-A)]D_n - ((A-B)+\xi(B-A))\varepsilon^{n-1}}{n}\right\}_{n=2}^{\infty} \quad (22)$$

is non- decreasing and positive, The result is sharp.

**Proof.** Let $f \in F_{\lambda,\eta,\varepsilon}^{m,s}(c, A, B, \xi)$. Then by assumption, we have

$$\sum_{n=2}^{\infty} c_n \le \frac{[(A-B)+\xi(B-A)]}{[1-2B+A+\xi(B-A)]D_2 - [(A-B)+\xi(B-A)]\varepsilon}$$

And,

$$\sum_{n=2}^{\infty} n c_n \le \frac{2[(A-B)+\xi(B-A)]}{[1-2B+A+\xi(B-A)]D_2 - 2[(A-B)+\xi(B-A)]\varepsilon}$$

we get

$$|f(z)| = |c_1 z - \sum_{n=2}^{\infty} c_n z^n| \le r\left(c_1 + \sum_{n=2}^{\infty} c_n r^{n-1}\right)$$

$$\le r\left(1 + \sum_{n=2}^{\infty} c_n \varepsilon^{n-1} + \sum_{n=2}^{\infty} c_n r^{n-1}\right) < r\left(1 + (\varepsilon + r)\sum_{n=2}^{\infty} c_n\right)$$

$$\le \frac{[1-2B+A+\xi(B-A)]D_2 r + [(A-B)+\xi(B-A)]r^2}{[1-2B+A+\xi(B-A)]D_2 - [(A-B)+\xi(B-A)]\varepsilon}$$

also,

$$|f(z)| = |c_1 z - \sum_{n=2}^{\infty} c_n z^n| \ge r\left(c_1 - \sum_{n=2}^{\infty} c_n r^{n-1}\right)$$

$$= r\left(1 + \sum_{n=2}^{\infty} c_n (\varepsilon^{n-1} - r^{n-1})\right)$$

then $|f(z)| \ge r$ if $r \le \varepsilon$, while if $r > \varepsilon$ we have

$$\{\varepsilon^{n-1} - r^{n-1}\}_{n=2}^{\infty},$$

is negative and decreasing, therefore we obtain

$$|f(z)| \ge r\left(1 + (\varepsilon - r)\sum_{n=2}^{\infty} c_n\right) \ge$$

$$\frac{[1-2B+A+\xi(B-A)]D_2 r - [(A-B)+\xi(B-A)]r^2}{[1-2B+A+\xi(B-A)]D_2 - [(A-B)+\xi(B-A)]\varepsilon}$$

and by the same technique we obtain the result (21).

Take $f(z) = z$ ,and

$$f_2(z) = \frac{[1-2B+A+\xi(B-A)]D_2 z - ((A-B)+\xi(B-A))z^2}{[1-2B+A+\xi(B-A)]D_2 - ((A-B)+\xi(B-A))\varepsilon}$$

## IV. Integral operator and Extreme points

In this section we give the integral operator property and find the extreme points for the class $F_{\lambda,\eta}^{m,s}(c, A, B, \xi)$.

**Theorem4.1.** The function $j(z)$ defined by

$$j(z) = \frac{a+1}{z^a}\int_0^z e^{a-1} h(e)\,de, \quad (23)$$

where $a$ is real number such that $a > -1$, belong of the class $F_{\lambda,\eta}^{m,s}(c, A, B, \xi)$ if the function $f \in F_{\lambda,\eta}^{m,s}(c, A, B, \xi)$ .

**Proof.** Take

$$j(z) = c_1 z - \sum_{n=2}^{\infty} b_n z^n, \; b_n \ge 0 \quad (24)$$

where

$$b_n = \frac{a+1}{a+n} c_n \quad (25)$$

Then,

$$\sum_{n=2}^{\infty}[n(1-B)+(A-1)+\xi(B-A)]\, D_n b_n$$

$$= \sum_{n=2}^{\infty}[n(1-B)+(A-1)+\xi(B-A)]D_n \frac{a+1}{a+n} c_n$$

$$\le [n(1-B)+(A-1)+\xi(B-A)]D_n c_n \le [(A-B)+\xi(B-A)]c_1.$$

By assumption $f(z) \in F_{\lambda,\eta}^{m,s}(c, A, B, \xi)$. Therefore

$j(z) \in F_{\lambda,\eta}^{m,s}(c, A, B, \xi)$.

**Theorem4.2.** Let $f(z) = z$ and

$$f_n(z) = \frac{[n(1-B)+(A-1)+\xi(B-A)]D_n z - ((A-B)+\xi(B-A))z^n}{[n(1-B)+(A-1)+\xi(B-A)]D_n - ((A-B)+\xi(B-A))\varepsilon^{n-1}} \quad (26)$$

Then $f \in F^{m,s}_{\lambda,\eta,\varepsilon}(c, A, B, \xi)$ if it can be expressed in the form

$$f(z) = \sum_{n=1}^{\infty} \lambda_n h_n(z), \tag{27}$$

Where $\lambda_n \geq 0$, and $\sum_{n=1}^{\infty} \lambda_n = 1.$ (28)

**Proof.** Let $f(z) = \sum_{n=1}^{\infty} \lambda_n h_n(z)$, where $\lambda_n \geq 0$ and $\sum_{n=1}^{\infty} \lambda_n = 1.$

Therefore,

$$f(z) = \lambda_1 z + \sum_{n=2}^{\infty} \frac{\lambda_n [n(1-B)+(A-1)+\xi(B-A)]D_n z}{[n(1-B)+(A-1)+\xi(B-A)]D_n - ((A-B)+\xi(B-A))\varepsilon^{n-1}} \tag{29}$$

By (5), we get

$$\sum_{n=2}^{\infty} \frac{\lambda_n [(A-B)+\xi(B-A)]}{[n(1-B)+(A-1)+\xi(B-A)]D_n - ((A-B)+\xi(B-A))\varepsilon^{n-1}} \times$$

$$\left[\frac{[n(1-B)+(A-1)+\xi(B-A)]D_n}{(A-B)+\xi(B-A)} - \varepsilon^{n-1}\right] = \sum_{n=2}^{\infty} \lambda_n = 1 - \lambda_1 < 1.$$

show that $f(z) \in F^{m,s}_{\lambda,\eta,\varepsilon}(c, A, B, \xi)$.

Conversely, let $f(z) \in F^{m,s}_{\lambda,\eta,\varepsilon}(c, A, B, \xi)$. Then

$$c_n \leq \frac{(A-B)+\xi(B-A)}{[n(1-B)+(A-1)+\xi(B-A)]D_n - ((A-B)+\varepsilon(B-A))\varepsilon^{n-1}}, \quad n \geq 2$$

putting

$$\lambda_n = \frac{[n(1-B)+(A-1)+\xi(B-A)]D_n - ((A-B)+\xi(B-A))\varepsilon^{n-1}}{(A-B)+\varepsilon(B-A)}, \quad n \geq 2$$

and $\lambda_1 = 1 - \sum_{n=2}^{\infty} \lambda_n$, we obtain $f(z) = \sum_{n=1}^{\infty} \lambda_n h_n(z)$.

## V. CONCLUSIONS

1. The explore to the subclass of analytical univalent function associated with concept differential subordination.
2. We studied some differential subordination and superordination results involving of certain class, which are defined on the space of univalent meromorphic functions in the punctured open unit disc.
3. Obtain some geometric properties like coefficient boundary, coefficient inequality, distortion theorem, closure theorem, extreme points, radii of starlikeness, convexity, close to convex and values of integration.
4. By using properties of the generalized derivative operator, we explored some properties of subordinations and superordinations.
5. We derived some results of second order differential subordination involving linear operator and we derived some sandwich theorems.
6. We give some applications of differential subordination concept on subclasses of univalent functions for some convolution operators.

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