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Silicon Waveguide Analysis

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Abstract. The Maxwell's equations are a basic formulation using for solve propagation equations within a waveguide. In this research, the wave propagation inside the waveguide was studied and some basic parameters of wave propagation were measured through MATLAB 2013. Silicon with a refractive index 3.46 was chosen to study this propagation and the use of core and cladding with a lower refractive index 2.46 and some parameters were extracted for the waveguide including propagation constant, cutoff wavenumber, cutoff frequency, cutoff thickness and normalized parameter. These parameters are studied at the wavelength 0.6 μm for transvers electric TE and transvers magnetic TM we changed the core thickness at wavelength constant and found that the number of modes increases as the increase of core thickness and the waveform is thinner in TM than it is in TE. We note from the measurements that the value of cutoff frequency and cutoff thickness are equal in the two types TE and TM. Moreover, when two modes (m=0, 1) and core thickness 0.24 μm the value of propagation constant in TE is equal (34.8901, 30.7737) while in TM (30.6440, 26.1967) respectively, we find that the values in TE are greater than TM. Also, the results obtained from Finite Difference Method were compared with the method used (Maxwell's equation through wave equation solutions). This work represents a short pathway for the theoretical analysis concerning the electromagnetic waves propagating through Si planar waveguide with both modes (TE and TM) considered.

Keywords: Planar Waveguide, Silicon, Transvers Electric, Transvers Magnetic.

1. Introduction

Since the development of lasers and optoelectronics the telecommunications industry has driven the rise of photonics. Waveguides allow the light from these devices to travel large distances without being obstructed and to be directed easily in small areas without the need for complicated prism, lens and mirror systems. Waveguide consist of a core of dielectric material surrounded by a medium with a lower index of refractive [1]. These fibers, (depending on the size, shape and composition), can be either multimode or single-mode. Modes are fields that maintain the same transverse distribution and polarization at all points along the fiber axis. Multimode fibers support, as the name suggests, more than one mode in the electric field. (Different modes have different propagation constants and group velocities). An optical waveguide is a structure which confines and guides the light beam by the process of Total Internal Reflection (TIR) [2]. The refractive index of the cladding material is always chosen to be lower than the refractive index of the core material in order to trap the field energy inside the core by the phenomenon of (TIR). The core thickness and refractive indices for core and cladding can be shown in the Table 1.

Table 1. The core thickness and refractive indices for core and cladding

	Refractive index	Thickness
Core	n_1= 3.46	0.24 µm, 0.36 µm and 0.48 µm
Cladding	n_2 = 2.46	- - - - -

2. Theory

2.1. Fundamentals of planar Waveguide

Planar waveguides are an optical structure in which the optical ray passes along the direction of propagation. By distributing the refractive index to the waveguide structure, waveguides can be classified into symmetric and asymmetric waveguides [3].

The waveguide is surrounded by two dielectric mediums of refractive indexes, where the refractive index of the core is greater than the refractive index of the substrate and the cladding. The refractive index of the cladding is less or equal to the refractive index of the substrate and is known as (Asymmetric step-index planar waveguide). Usually the refractive index of the cladding and the substrate are equal. In this case, the waveguide is called (Symmetric step-index planar waveguide), [4] as shown in Figure 1.

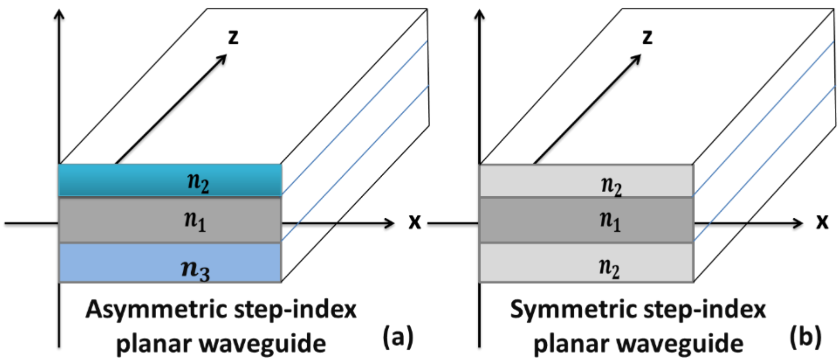

Figure 1. (a) Asymmetric step-index planar waveguide (b) Symmetric step-index planar waveguide.

By solving Maxwell's equations, a more accurate description of the spread of the electromagnetic wave can be propagation inside the waveguide. Only detailed solutions of the wave equations are allowed when the boundary conditions are entered into the media interfaces. This means that only separate waves are able to propagate (i.e. situations characterized by separate speeds and capacities) [5].

2.2. Waveguide mode

The light guiding loss-less within a particular waveguide is considered possible via the entire internal reflection phenomenon which occurs at the interface between low to large index of the refraction dielectric materials just above the so-called critical angle. Herein, light has the ability to be occur in a confined space when a constructive interface is happening. Therefore, in contrast within a free space whereby light is able to propagate in all directions, an optical wave within a waveguide can only propagate at a separated state set; these are known as modes. Continuously, these modes are categorized based on their constant of propagations. The constant of propagation is defined as a speed measure by which the phase fronts propagate along the structure [6]. It must be mentioned that more than one mode can be obtained when an electromagnetic wave is propagated through any waveguide. Every particular mode possesses a specific cut-off frequency whereby the pre-described wave number is zero at the propagation direction. When a mode has the smallest cut-off frequency, it is considered a dominant mode. The cut-off frequency is determined by the physical dimensions of the waveguide. There should be a multimode and single mode fibers which are classified based on the number of modes that propagate through that particular fiber. A mode is a mathematical representation which describes the propagation nature of an electromagnetic wave through a waveguide [7]. In other words, the mode can be also defined as the electromagnetic configuration nature along the path of the light within a fiber. Currently, there are two different slab waveguides which are utilized, these are single-mode and multi-mode slab waveguides [8].

2.2.1. Single mode. A single-mode waveguide is recognized as having extremely small optical dispersion value, high bandwidth occurs via the allowance of the mode of zero-order to be propagated. Single-mode is capable of transporting high quantities of optical information because of lower optical dispersion over long distances. However single-mode waveguides suffer from high coupling losses from any kind of misalignment or fabrication defects [9]. A low NA (Numerical aperture) can reduce modal dispersion by limiting the number of modes and can reduce scattering loss. A single-mode possesses a relatively low core which permits only a particular mode of the propagating light at a time. Consequently, the signal mode fidelity is better reserved upon far distance, and modal dispersion is considered to be significantly decreased. The factors which mainly contribute in greater capacity of bandwidth as compared to multimode fiber can be accommodated [10].

2.2.2. Multimode. The second general type of waveguide modes, multimode possesses greater core in comparison to the single-mode fiber. It obtained this particular name as for that several modes, or light beams, could be passed instantaneously over a waveguide. Note that both mentioned modes ought to pass diverse distances to approach their targets. The disparity between the light beams arrival time is named a modal dispersion. Modal dispersion leads to humble signal quality at the receive point and eventually confines the transmission space, by which a multimode fiber is not utilized in wide-range of requests [11].

In order to compensate for the dispersion disadvantages, fiber with graded-index was developed. The greater refraction at the core center decelerates some of light beams speed which allow all the beams to approach their destinations at around a similar time and decreasing the so-called modal dispersion. [12].

2.3. Types of transverse modes

A non-directed electromagnetic wave in an open space and/or in isotropic dielectric, which is considered bulk, could be defined as an overlap of flat wave which in turn could be defined as a TEM mode as shown in the following subsections [13].

The waveguide modes are classified as follows:

- **Transverse electromagnetic (TEM) modes: -** The magnetic field as well as the electrical field are not in the propagation direction.
- **Transverse electric (TE) modes: -** The magnetic field is only in the propagation direction and is also entitled as H mode (H is the symbol that indicates the magnetic field). The electrical field is lacked in the propagation direction.
- **Transverse magnetic (TM) modes: -** The electrical field is only in the propagation direction and may be entitled as E mode (E is the symbol that indicates the electric field). The magnetic field is lacked in the propagation direction.

2.4. Symmetric step-index planar waveguide

There is refractive index discontinuity only in x-direction, z and y direction is infinitely extended.

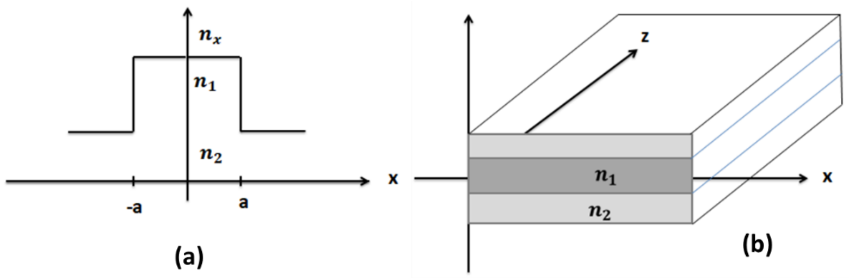

Figure 2. (a) shows the boundary of the waveguide and refractive index. (b) Confinement in x-direction and propagating in z-direction ($n_1 > n_2$).

By solving Maxwell's equations, the wave propagation inside the waveguide can be analyzed as shown below.

2.5. Propagation Constant
It is a parameter that describes the spread of light in a waveguide and depends on the situation and frequency. The propagation constant is determined in some way by the waveguide and is denoted by the symbol (β), as the amplitude and phase of that light vary with a specific frequency along the direction of propagation z [14]. The spread constant depends on the wavelength and frequency, and that its measurement unit in the basic units is $\frac{1}{m}$. The propagation constant is specified between two values ($n_2 k_\circ < \beta < n_1 k_\circ$) this condition is a basic and universal condition for any waveguide, regardless of the geometrical shape. Different types of wave solutions contain many common features, regardless of the structure and shape of the waveguide [15].

2.6. Numerical Aperture (NA)
In the optical system, the numerical aperture is a measure of its angular acceptance of incoming light. It is a theoretical parameter that is calculated by visual design and defined based on engineering considerations [16]. Although the guide is considered a special kind of optical system, there are special aspects of the term numerical aperture in the waveguide. In the waveguide, the numerical aperture can be determined based on the input beam with the maximum angle at which the total internal reflection can occur [17]. The light beam is refracted when entering the waveguide and then subjected to the total internal reflection at the core-cladding interface. This only occurs when the angle of entry is not very large, as shown in Figure 3.

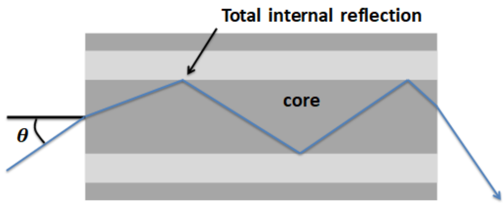

Figure 3. Total internal reflection (TIR).

The numerical aperture (NA) of the waveguide can be calculated by the difference between the refractive index of the core and the cladding, by the following relationship.

$$NA = \frac{1}{n_\circ}\sqrt{n_1^2 - n_2^2} \tag{1}$$

$n_\circ$: Refractive index of air equation (1).

n_1: Refractive index of the core.
n_2: Refractive index of the cladding.

When the input medium is different, i.e. with a higher refractive index, the maximum input angle will be smaller, but the numerical aperture remains the same [18]. There is no close relationship between the properties of waveguide modes and the numerical aperture. But only the waveguide with a high numerical aperture is due to the presence of modes with a greater deviation of the light coming out of the waveguide [19].

When the numerical aperture is high it has the following consequences:

- The high numerical aperture (NA) waveguides are the most directive, that is, they will generally support a large number of modes.
- Random differences in the refractive index increase when the numerical aperture decreases, so the waveguide that contains a low percentage of NA has increased propagation losses.
- Orientation sensitivity is reduced for refractive index fluctuations.

2.7. General solution of TEM, TE and TM wave using Maxwell's equations

By solving Maxwell's equations we find

$$\nabla \times \vec{E} = -i\omega\mu\vec{H} \quad (2)$$

$$\nabla \times \vec{H} = i\omega\varepsilon\vec{E} \quad (3)$$

[σ – not included because there are no source or material]

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -i\omega\mu(H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z) \quad (4)$$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = i\omega\varepsilon(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z) \quad (5)$$

Consider electromagnetic wave which is propagation in z-direction, so z-direction will be longitudinal and x-y plan will be the transverse direction so, if the propagation in z-direction we will have.

$$\left.\begin{matrix} E_x, & H_x \\ E_y, & H_y \end{matrix}\right\} transverse\ component$$

$$E_z, \quad H_z\} are\ longituodanal\ component$$

From solving equations (4 and 5) we get.

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -i\omega\mu H_x \quad (6) \qquad \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = i\omega\varepsilon E_x \quad (9)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -i\omega\mu H_y \quad (7) \qquad \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = i\omega\varepsilon E_y \quad (10)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu H_z \quad (8) \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon E_z \quad (11)$$

$$\vec{E} = E_\circ \exp^{-i\beta z} \quad (12)$$

$$\frac{\partial E}{\partial z} = -i\beta\vec{E} \quad (13)$$

[$E_\circ$ - could be a function of x].

Replacing ($\frac{\partial}{\partial x}$) with ($-i\beta$) in equations (6, 7, 9 and 11) we will get equations.

$$\frac{\partial E_z}{\partial y} + i\beta E_y = -i\omega\mu H_x \quad (14) \quad \text{contains } (E_z, E_y, H_x)$$
$$-i\beta E_x - \frac{\partial E_z}{\partial x} = -i\omega\mu H_y \quad (15) \quad \text{contains } (E_x, E_z, H_y)$$
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu H_z \quad (16) \text{ same } (8)$$
$$\frac{\partial H_z}{\partial y} + i\beta H_y = i\omega\varepsilon E_x \quad (17) \quad \text{contains } (H_z, H_y, E_x)$$
$$-i\beta H_x - \frac{\partial H_z}{\partial x} = i\omega\varepsilon E_y \quad (18) \quad \text{contains } (H_x, H_z, E_y)$$
$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon E_z \quad (19)$$

In all the equations above, one longitudinal component with two transverse components.
So from equations (14 and 18) we can write.

$$H_x = \frac{i}{ķ^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right) \quad \text{by eliminating } E_y \quad (20)$$
$$E_y = \frac{i}{ķ^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right) \quad \text{by eliminating } H_x \quad (21)$$

Similarly, from equations (15 and 17) we can write.

$$H_y = \frac{-i}{ķ^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right) \quad \text{by eliminating } E_x \quad (22)$$
$$E_x = \frac{-i}{ķ^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right) \quad \text{by eliminating } H_y \quad (23)$$

Where:
$$ķ^2 = k^2 - \beta^2 = \omega^2\mu\varepsilon - \beta \quad (24)$$
ķ: Cutoff wave number, E: Electric field, H: Magnetic field, ε: Permittivity, μ:Permeability, ω: Angular frequency, β: Propagation constant, k: Wavenumber

Through the solutions of Maxwell's equations, we notice that both the electric field and the magnetic field are independent. Through these solutions, two modes are referred to (TE and TM) modes.

2.7.1. TEM (Transverse Electron Magnetic) mode. Means that electric and magnetic fields both are transvers and there is no longitudinal components, so, $E_z = H_z = 0$ (longitudinal components). We can't use equations (20, 21, 22 and 23) in this mode because all will becomes zero. So, we use the equations (14, 15, 17 and 19).
Combine (14) and (17) we will get
$$i\beta E_y = -i\omega\mu H_x \quad (25)$$
$$-i\beta H_x = i\omega\varepsilon E_y \quad (26)$$
Combing we get
$$i\beta E_y = -i\omega\mu\left[\frac{i\omega\varepsilon E_y}{-i\beta}\right] \quad (27)$$
$$\beta^2 = \omega^2\mu\,\varepsilon \quad (28)$$
Where: - $\beta^2 = k^2$
Which means for TEM mode there is no cutoff wave number ķ=0.

In TEM mode all frequencies will have Helmholtz equation. For TEM mode we can write a Helmholtz equation in terms of $(E_x \text{ or } H_x \text{ or } E_y \text{ or } H_y)$ [18].
$$(\nabla^2 + \mathrm{k}^2)\mathrm{E_x} = 0 \quad (29)$$
$$\left(\frac{\partial^2}{\partial \mathrm{x}^2} + \frac{\partial^2}{\partial \mathrm{y}^2} + \frac{\partial^2}{\partial \mathrm{z}^2} + \mathrm{k}^2\right)\mathrm{E_x} = 0 \quad (30)$$

$$\frac{\partial^2}{\partial z^2} = (-i\beta)(-i\beta) = -\beta^2 \tag{31}$$

$$\left.\begin{aligned}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 0\right) E_x = 0\\ \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 0\right) H_x = 0\\ \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 0\right) E_y = 0\\ \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 0\right) H_y = 0\end{aligned}\right\} \tag{32}$$

*2.7.2. TE (Transverse Electric) mode.*In this mode ($E_z = 0$) because it longitudinal component. From solving equations (20, 21, 22 and 23) we will get.

$$\left.\begin{aligned}H_x &= \frac{-i\beta}{ķ^2}\left(\frac{\partial H_z}{\partial x}\right)\\ E_y &= \frac{i\omega\mu}{ķ^2}\left(\frac{\partial H_z}{\partial x}\right)\\ H_y &= \frac{-i\beta}{ķ^2}\left(\frac{\partial H_z}{\partial y}\right)\\ E_x &= \frac{-i\omega\mu}{ķ^2}\left(\frac{\partial H_z}{\partial y}\right)\end{aligned}\right\} \tag{33}$$

Amply the Helmholtz equation to solve for H_z

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + ķ^2\right) H_z = 0 \tag{34}$$

$$ķ^2 = k^2 - \beta^2 \tag{35}$$

Then substituting H_z in equation (34) above we can get (E_x *or* H_x *or* E_y *or* H_y).

2.7.3. TM (Transverse Magnetic) mode. In this mode ($H_z = 0$) because it longitudinal component. Similarly, we get all the transverse components of (m) mode in terms of E_z only.

$$\left.\begin{aligned}H_x &= \frac{i\omega\varepsilon}{ķ^2}\left(\frac{\partial E_z}{\partial y}\right)\\ E_y &= \frac{-i\beta}{ķ^2}\left(\frac{\partial E_z}{\partial y}\right)\\ H_y &= \frac{-i\omega\varepsilon}{ķ^2}\left(\frac{\partial E_z}{\partial x}\right)\\ E_x &= \frac{-i\beta}{ķ^2}\left(\frac{\partial E_z}{\partial x}\right)\end{aligned}\right\} \tag{36}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + ķ^2\right) E_z = 0 \tag{37}$$

Then substituting E_z in equation (37) above we can get (E_x *or* H_x *or* E_y *or* H_y).

2.8. Cutoff frequency
The waveguide carries signals that will carry only above a certain frequency known as the cutoff frequency and below the waveguide cutoff frequency it is unable to carry the signals.
The waveguide should be able to spread the signals, and this depends on the wavelength of the signal. If the wavelength is too long, the waveguide will not operate in a position where it can carry signals [20].

The cutoff frequency depends on the dimensions of the waveguide due to mechanical limitations. In order for the waveguide to transmit the signals, the waveguide width must be the same size as the propagating wavelength. The cutoff frequency is known as the following relationship [21].

$$f_c = \frac{mc}{4aNA} \tag{38}$$

Where m is the number of mode, c is the speed of light in free space and NA is the numerical aperture.

2.9. Normalized Frequency (V number)

The normalized frequency determines the number of modes in the waveguides. The normalized frequency is a dimensionless parameter which is often used in the context of waveguides [22]. It is defined as:

$$V = \frac{2\pi}{\lambda}\frac{2a}{2}NA \tag{39}$$

Where V: normalized frequency, λ: wavelength, 2a: thickness of the waveguide core and NA: numerical aperture.

The normalized frequency is proportional to the optical frequency, but it is re-measured according to the characteristics of the waveguide, and it is considered suitable for various basic characteristics of waveguides [23].

2.10. Normalized parameter (b)

We have the normalized frequency from equation (39) define the normalized propagation constant as:

$$b = \frac{\left(\frac{\beta}{k_\circ}\right)^2 - n_2^2}{n_1^2 - n_2^2} \tag{40}$$

Which makes the value of (b) lies between (0) and (1)$(0 < b < 1)$.
b: Normalized parameter, $k_\circ$: Vacuum wavenumber.

3. Results

A specific wavelength has been studied within the silicon waveguide with its value 0.6 μm. The silicon refractive index 3.46 (core) and the refractive index of the cladding 2.46. Moreover, core thickness has values (0.24, 0.36, 0.48 μm), while thickness of the cladding was unknown.

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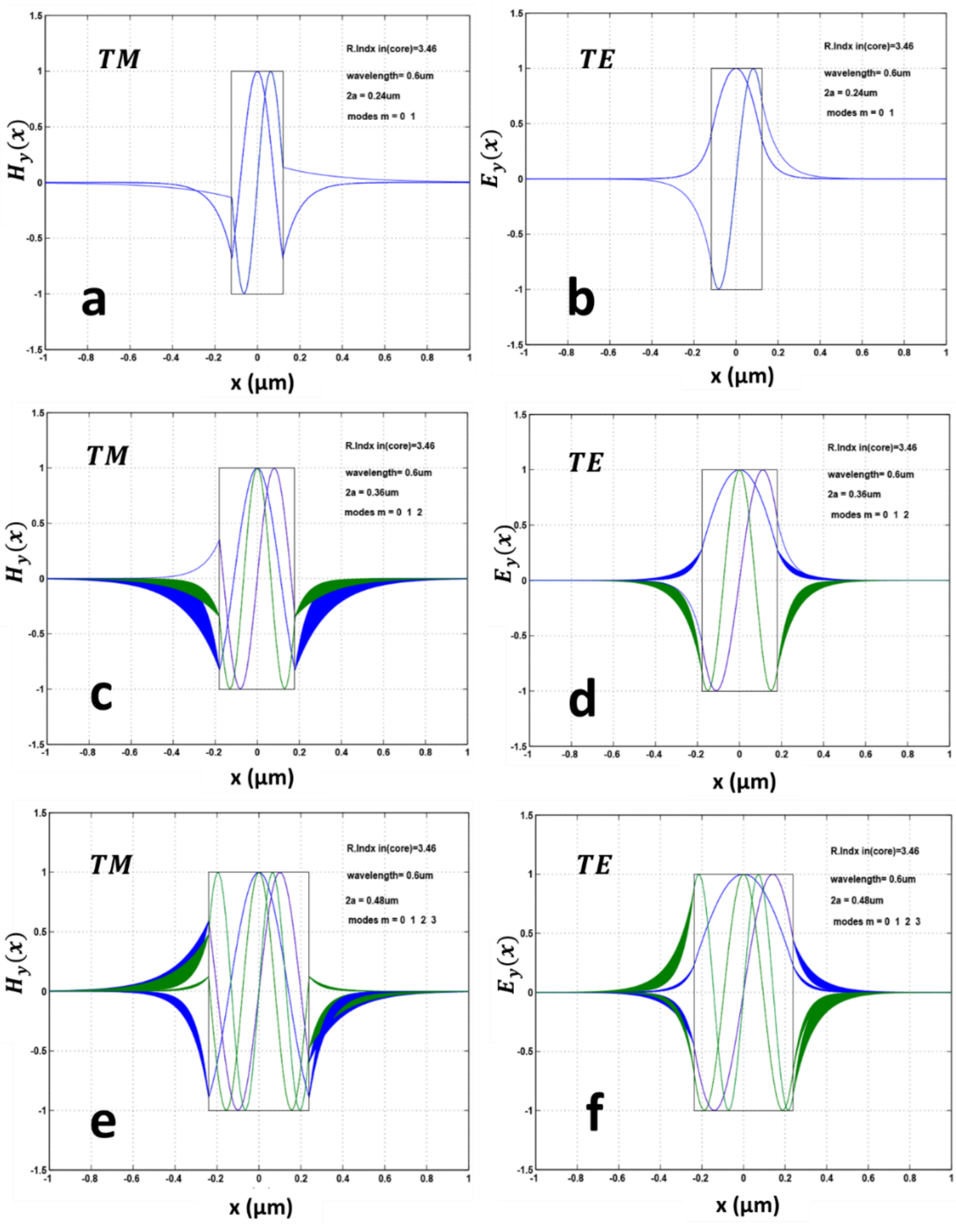

Figure 4. The intensity of the electric and magnetic field when the wavelength 0.6 μm, thickness 0.24 μm (a, b), 0.36 μm (c, d) and 0.48(e, f) μm and m=0,1, m=0,1,2 and m=0,1,2,3, respectively.

Figure 4 (a, b) shows the difference in the pattern of patterns at each thickness. We have noticed that when the thickness of the core is 0.24 μm, we get two modes, one symmetric and the other asymmetric in both types, but it was found that the waveform in TM is thinner than it is in TE. From the same Figure we noticed that when the core thickness increases, the number of modes increases, as in (c, d), we got three modes when the thickness of the core 0.36 μm, while we got four modes when the thickness of the core 0.48 μm as shown in (e, f). Through Figure 4 in all cases, TM modes appear much thinner than they are in TE due to the difference between the refractive index of the core and the refractive index of the cladding where the core is larger than in the cladding.

Table 2. Parameters of the silicon (Si) waveguide of the four lower order modes in TE and TM mode.

mode	λ (μm)	2a (μm)	m	β	k_c	t_{co} (μm)	f_c (THz)	b
TE	≤ 0.6 μm	0.24	0	34.8901	9.7729	0	0	0.8529
			1	30.7737	19.1263	0.1233	256.87	0.4365
		0.36	0	35.5196	7.1550	0	0	0.9211
			1	33.3524	14.1580	0.1233	171.25	0.6912
			2	26.6209	20.8671	0.2466	342.50	0.3293
		0.48	0	35.7938	5.6247	0	0	0.9513
			1	34.4615	11.1910	0.1233	128.44	0.8071
			2	32.1755	16.6604	0.2466	256.87	0.5724
			3	28.8760	21.8863	0.3699	385.31	0.2622
TM	≤ 0.6 μm	0.24	0	30.6440	19.3334	0	0	0.4242
			1	26.1967	25.0313	0.1233	256.87	0.0349
		0.36	0	33.3539	14.1544	0	0	0.6914
			1	30.5565	19.4713	0.1233	171.25	0.4160
			2	26.9605	24.2067	0.2466	342.50	0.0974
		0.48	0	34.4822	11.1270	0	0	0.8093
			1	32.6376	15.7360	0.1233	128.44	0.6186
			2	30.1111	20.1533	0.2466	256.87	0.3744
			3	27.0661	24.0886	0.3699	385.31	0.1062

Through the Table 2 which contains several variables, propagation constant, cutoff wavenumber, cutoff thickness, cutoff frequency and normalized parameter. These variables have different values according to the mode. We note that the values of the propagation constant are in TE higher than they are in TM at every core thickness, while cutoff wavenumber is in TM greater than in TE. Moreover, the cutoff thickness and cutoff frequency are equal in both modes. It was also observed that normalized parameter it has higher values in TE than TM. Relationships were also drawn between most of the parameters, and the difference between TE and TM was noted.

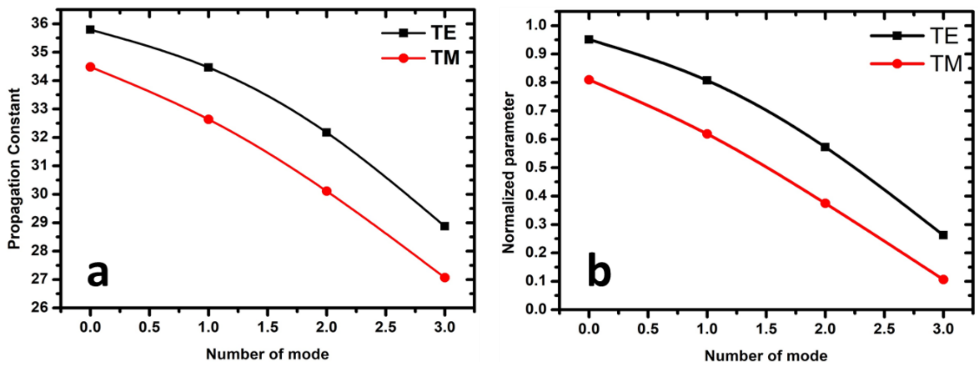

Figure 5. (a) Shows the relationship between the number of modes and the propagation constant. (b)Shows the relationship between the number of modes and the normalized parameter.

Figure (5 a, b) shows the relationship between the propagation constant and the number of modes at the wavelength of 0.6 μm and the core thickness of 0.48 μm, observed there are four modes in TE and

TM modes. Through the Figures shown, it was found that the inverse relationship between the number of modes and each of the propagation constant and normalized parameter. The numerical values of both for each variable are greater in TE than TM at each mode.

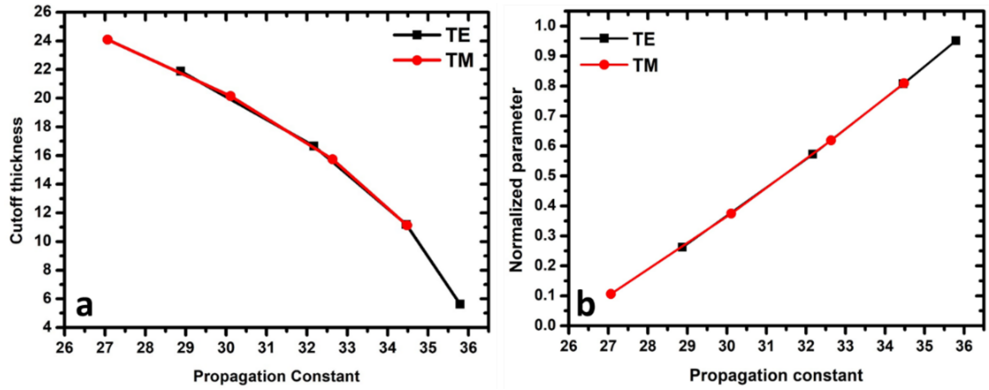

Figure 6. (a) Shows the relationship between the propagation constant and the cutoff thickness. (b)Shows the relationship between the propagation constant and the normalized parameter.

Through Figure (6, a), we find that the inverse proportion between each of the two variables, and both TE and TM. Figure (6, b) we find that the proportional between each of the two variables shown in the Figure, but the values of TM are greater than the values of TE.

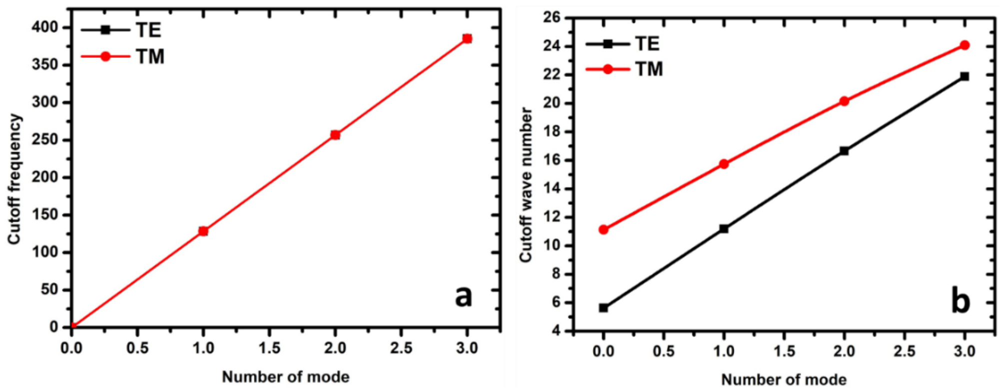

Figure 7. (a) The relationship between the number of modes and the cutoff frequency (b) The relationship between the number of modes and cutoff wavenumber.

We can observe from the Figure (7, a) the relationship between number of modes and cutoff frequency, as we note that the proportional for both TE and TM. While the Figure (7, b) shows the relationship between the number of modes and cutoff wavenumber we notice through this form that the proportional between them, Also, through the Figure (7, b), we notice that the values of the cutoff wavenumber in TM are greater than in TE. Moreover, it shows the approximation of the values cutoff wavenumber in TE and TM when increasing the number of modes.

3.1. Finite Difference Method

When solving Maxwell's equations using a finite difference method that depends on the method of solving them on using the algebra of matrices, the shape of the waves is obtained at certain wavelengths and core thicknesses, and also results are obtained for the parameters of the waveguide in the TE mode field with core thickness (0.24, 0.36, 0.48 μm) at 0.6 μm wavelength. The results were obtained as shown in the Table 3.

Table 3. Parameters of silicon (Si) waveguide of the four lower order modes in Finite Difference Method in TE mode.

λ (μm)	**2a(μm)**	**m**	**β**	**kς**	**t_{co} (μm)**	**f_c (THz)**	**b**
0.6 μm	0.24	0	34.9322	9.6216	0	0	0.8574
		1	31.0133	18.7352	0.1233	256.87	0.4593
	0.36	0	35.5404	7.0507	0	0	0.9235
		1	33.4408	13.9480	0.1233	171.25	0.7004
		2	29.8995	20.4659	0.2466	342.50	0.3548
	0.48	0	35.8040	5.5594	0	0	0.9524
		1	34.5061	11.0527	0.1233	128.44	0.8118
		2	32.3081	16.4018	0.2466	256.87	0.5856
		3	29.1845	21.4732	0.3699	385.31	0.2897

4. Conclusion

These parameters are studied at the wavelength 0.6 μm for transvers electric TE and transvers magnetic TM we changed the core thickness at wavelength constant and found that the number of modes increases as the increase in core thickness and the waveform is thinner in TM than it is in TE. We note from the measurements that have been found that value of cutoff frequency and cutoff thickness are equal in the two types TE and TM. Moreover, when two modes (m=0, 1) and core thickness 0.24 μm the value of propagation constant in TE is equal (34.8901, 30.7737) while in TM (30.6440, 26.1967) respectively, we find that the values in TE are greater than TM. In this study, the waveguide parameters were acquired using Maxwell's equation through wave equation solutions and Finite Different Method by using MATLAB program. Hereinafter, it was concluded that the difference between the two utilized approaches exhibited small values.

This means that both aforementioned techniques can be considered successful in determining the waveguide parameters in such arrangement and this work represents a short pathway for the theoretical analysis concerning the electromagnetic waves propagating through Si planar waveguide with both modes (TE and TM) considered.

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