Investigation of the Order and Disorder of the Transition Phase of the Ising Model of $NiOFe_2O_3$ in Different Square Lattices

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Abstract

In this work, the stage transition from an order-to-disorder of Nickel II Iron III oxide (ferromagnetic) of the two-dimensional Ising Model is determined using Monte Carlo method with important samples. The magnetization per site (μ), energy per site (j) of a NiOFe2O3 have been calculated as a function of inverse temperature (βJ) for different lattice sizes (L×L= 4, 8, 16, 32), in the absence of external magnetic field (B=0). The critical inverse temperature (βC=0.435 KB/J) has been determined. The analysis of the outcomes appears that (βCJ=0.435) is set by measuring the magnetic characteristics in which the ferromagnetic suffers of the stage transition from an order to disorder. For clarity, Metropolis algorithm method was used to assess the behavior of the lattice and the critical inverse temperature in which the stage transition between NiOFe2O3 of ferromagnetic and paramagnetic states occurs was noted. It was noted that above (βCJ) the substance (NiOFe2O3) becomes a ferromagnetic state (order), leading to increase in average magnetization and the average energy decreases, while below (βCJ) the substance is in a paramagnetic state (disorder).

Key Words: Ferromagnetic Material, Ising Model, Monte Carlo Steps, Metropolis Algorithm Method, Nickel.

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Introduction

Basically a stage transition and critical region are the foremost wide phenomenon in nature. The square-lattice sizes (L^2) of the Ising model are the simplest efficient system appears partial transitions (that is, the partial a stage transitions between the paramagnetic and ferromagnetic phase) and critical region at limited temperatures. The square-lattice sizes (L^2) Ising Model has played a central role in the comprehension of stage transitions and critical conditions [1,2].

The critical inverse temperature indicates the highest temperature for which there can be non-zero magnetization. At this point, the system suffers an order-to-disorder transition, named stage transition [3]. The model ISING regarded is a square-lattice of spin locations with cyclic limit conditions, varying in dimensions from ($L\times L=N^2=4\times4$ to $L\times L=N^2=32\times32$) with the Hamiltonian criterion. Together, the coupling constant J and the Boltzmann constant K_B are set to 1, such that $T^{-1}=\beta J$ is a denoted to as the inverse temperature. Spin locations are appointed a worth of (±1) to represent spin up and spin down respectively. Thus, the entire magnetization M is simply the integer sum of all lattice spin locations, and in addition, the total energy E is defined as minus the coupling steady J times all the neighboring spin pairs which can be aligned [4].

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Computational physics technique has played an increasingly significant role in addressing complicated problems of this kind in latest research. By implementing these techniques, at least in static critical behavior research, highly precise and reliable calculations of critical parameters can be performed [5].

Several works were well performed by multiple researchers on the two-dimensional Ising model of ferromagnetic. Bhanot [6] assessed the Ising model's accurate partition functions in (L^2) square lattice sizes with free boarders stipulations up to (L=10) using Cray XMP. Bhanot counted all $2^{L\times L}$ = 2^{100} (≈ 1.27 x 10^{30}) states for (L=10), and start-up get to some beneficial outcomes. Stodolsky and Wosiek [7] obtained the precise partition function for L=13 (corresponding to 2^{169} ≈ 7.48 x 10^{50} states) using IBM RISC 6000, and studied stage transitions based on the entropy as a function of the energy.

In this work, we calculated the precise partition function of the Ising model and emulate the critical conditions of $NiOFe_2O_3$ on ($L\times L$) square lattices with free boarders stipulations for ($L\times L$=16) and ($L\times L$=32), is acquired after categorize all $2^{L\times L}$=$2^{16\times 16}$ (≈1.1579×10^{77}), $2^{32\times 32}$ (≈1.79×10^{308}) shapes of spin and we observe how the system evolves across steps to achieve balance. The energy and the magnetization were determined as a function of time of the Monte Carlo Step (***MCS***) and inverse temperature respectively. The correlation between magnetization and energy was determined in various square lattices in absence an magnetic field (B=0).

The Ising Model

Hamiltonian system depends on the order of the grid spins and we conclude from this characteristics, for example, magnetization [8,9]. Let $S_{i,j}$ denote a spin in lattice coordinates i and j with either spin up or spin down, $S_{i,j}$= ±1. Assume which Hamiltonian is [10]:

$$H = -J \sum_{\langle i\, j\rangle} s_i\, s_j - B \sum_i s_i \qquad (1)$$

where <*ij*> an strategy that sums up over the closest neighboring spin pair because the spin at location *ij* interacts with spins at locations $i(j\pm1)$ and $j(i\pm1)$ respectively. J_{ij} is the exchange energy between the spins and B is an external magnetic field. In the absence of an external magnetic field, $B = 0$, and so the Hamiltonian reduces to [11]

$$H = -J \sum_{\langle ij\rangle} s_i\, s_j \qquad (2)$$

The following distribution of probabilities should be used to calculate the expected values such as mean energy $\langle E\rangle$ or magnetization $\langle M\rangle$ in thermodynamic physics at a specific temperature

$$P_i(\beta) = \frac{e^{-\beta E_i}}{Z} \qquad (3)$$

with $\beta=1/K_BT$ is the inverse temperature, K_B the Boltzmann constant, E_i the energy of state i, whereas Z the partition function for the canonical ensemble we may write

$$Z = \sum_{i=1}^{M} e^{-\beta E_i},$$

The specified configuration energy is given ***i*** by

$$E_i = -J \sum_{\langle i\, j\rangle}^{N} s_j\, s_i$$

Computational Observables

For getting the result, I have altered the Fortran 90 code that was developed by Lisa Larimore [12]. we can measure of the effect inverse temperature (βJ) on the energy and the magnetization at each step. The critical inverse temperature (β_C=0.435 K_B/J) has been determined. Then, by taking the amount of all the spins in the lattice, we can evaluate the magnetization and the energy as a function of time from Monte Carlo simulation (*MCS*). Two dimensional shapes are plotted using Grapher version 1.09 [13]. The computational observables of special concern are $\langle E\rangle$, $\langle M\rangle$. We determine in the next method:

$$\langle M\rangle = \frac{1}{N}\sum_{s}^{N} M(s) \qquad (4)$$

Where $\langle M\rangle$: represented the mean magnetization and $M(s)$, the magnetization per spin.

To calculate the energy provided in equation (1), we use the Hamiltonian the factor of (1/2) is presented accounting for the fact that every pair is calculated twice in the sum.

$$\langle E\rangle = \frac{1}{2}\left\langle \sum_{i}^{N} H_i \right\rangle = \frac{1}{2}\left\langle -J \sum_{i}^{N} \sum_{jnn} S_i\, S_j \right\rangle \qquad (5)$$

Boundary conditions for inverse temperature (βJ):

For a given $\beta = (K_B T)^{-1}$, the initiation lattice is set as the settled lattice of the former (β):

- At ($\beta = (K_B T)^{-1}=1$), i.e. at a fully low temperature, completely aligned spins are obtained. There is maximum magnetization. Then, as the temperature will increase the change of spins gradually.
- When ($\beta=(K_B T)^{-1}$) is such that $\beta \approx \beta_C$ there are various clusters of aligned spins, magnetization is utmost in each cluster, however, the magnetization of the group is generally cancelled due to the eventuality of being in the α_i configuration is equal to the eventuality of being in the $-\alpha_i$ configuration.
- The dipoles are routed randomly at a fairly high temperature ($\beta = (K_B T)^{-1}=0$), while the lattice is initialized at every value of ($\beta = (K_B T)^{-1}$), the findings differ: At a totally low temperature, there are several clusters of aligned spins. These domains stop evolution: We usually tend to get domain names from Weiss and walls Bloch. As a consequence, magnetization is random. The matrix of size limits the number of possible clusters[14].

Fluctuations in Monte Carlo Time "Evolution"

In the simulation system, whenever the flip is done, the interaction energy will be reduced. If the energy increases, the flip is only obliged with an eventuality of $\{exp\ (-\beta E)\}$ whereby ($\beta=1/K_B T$) and ($E>0$) is the energy variance between upturned and non-upturned case (metropolis algorithm method).The applicable temperature will be in modules of (βJ) named the decreased temperature and is the "natural" temperature module used for the While of the execution. The emulation time repeatedly calculates what is called (Monte-Carlo Step), generally indicated to as time, each of which involves the potential flipping of all spins within the vicinity [15].

Simple Sampling

The essential elementary sampling manner comprises of simply randomly selecting points within the configuration space from anywhere. A huge numeral of spins patterns is generated randomly (for the whole lattice) and data are used to calculate the average energy and magnetization. However, this mechanism needs to suffer from precisely the same problems as the quadrature manner, often sampling from unimportant areas of the stage size. The chances of producing a randomly created spin array and up / down spin patterns are remote ($\sim 2^{-L}$), and high-temperature random spin array is highly likely. The most common way to avoiding this problem is by using the Metropolis importance sampling, which works by applying weights to the microstates [16].

Results and Discussion

Location of the critical transition phenomenon

Critical region shows the maximum temperature of non-zero magnetization. In this situation, the system is subject to transition called partial transition (a stage transition) from order-to-disorder [17]. In order to locate the essential (critical) inverse temperature (β_C), the most realistic value is given in the thermodynamic limit, where ($L \times L = N^2$), the boundary of infinity is regarded. Thus, we aimed to calculate the essential (critical) inverse temperature with fully different lattice sizes. Figure (1) shows the critical inverse temperature as N approaches infinity, and has been calculated the critical region for many lattice sizes and it value ($\beta_C=0.435\ K_B/J$) without effect of the magnetic field ($B=0$).

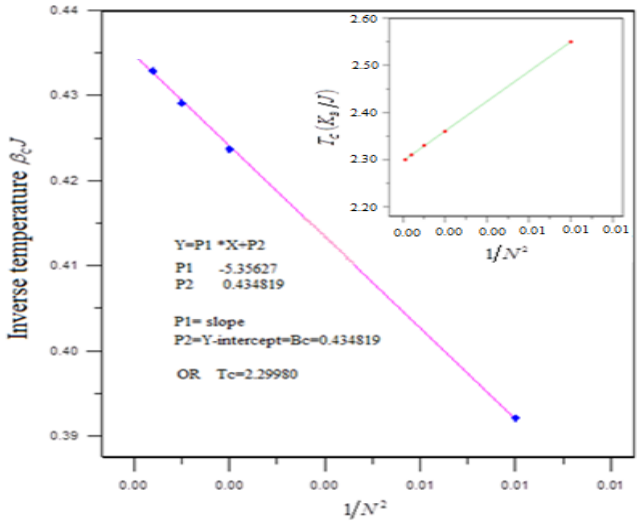

Figure 1: Critical inverse temperature (βC=0.435) depend on different lattice sizes

The impact of the lattice size on the stage transition of ($NiOFe_2O_3$)

To sight the impact of the magnitude of the lattice on the transition of the stage, the thermodynamic quantities are plotted in ($L \times L$= 4, 8, 16, 32) without effect of the magnetic field (B=0) as shown in figure (2). We have started with random spin at the lattice locations and calculated magnetization and energy

using Ising Model. We executed (Metropolis algorithm) Monte Carlo simulation of an Ising Model in Fortran 90 Code. Each the emulations were in ($L\times L$= 4, 8, 16, 32) the square lattice sizes of ferromagnetic ($NiOFe_2O_3$), and the simulation for inverse temperature (β) of 0.09 K_B/J through 1.75 K_B/J with intervals of 0.02. As the temperature raise, the system was allowed to equilibrate for 10,000 steps, and then the averages were performed over the entire lattice. It is obvious from absolute magnetization that a stage transition takes place at a critical inverse temperature ($\beta_C J$=0.435) or (T_C=2.29) from a decreased value to a maximal magnetization value. Magnetization at higher inverse temperatures (low temperatures) above ($\beta_C J$) is the most stable (called order). While at low inverse temperatures (high temperatures) below ($\beta_C J$) the magnetization becomes instable (called disorder) and the fluctuation is bigger for all lattice sizes.

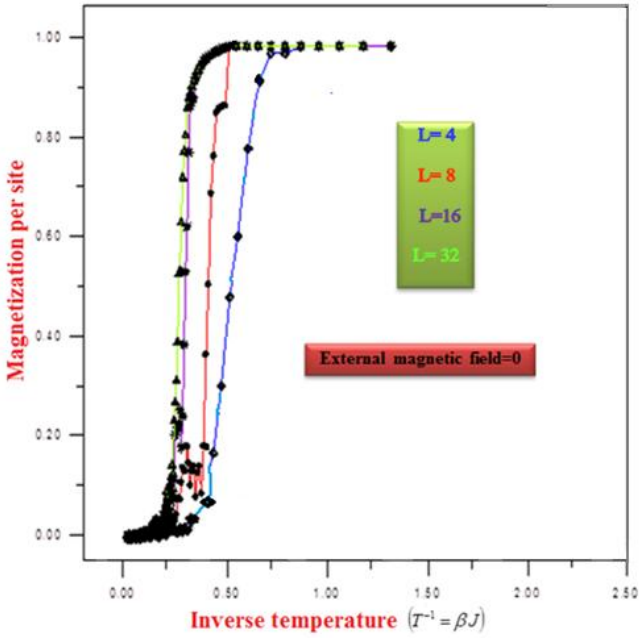

Figure 2: Magnetization per site of a $NiOFe_2O_3$ as a function of inverse temperature for $L^2=N\times N$ various square lattice sizes in an external magnetic field (B=0)

Thermodynamics technique

Thermodynamics is the process of fetching the lattice to equilibrium at a certain temperature T ($1/\beta J$). This is attained pick from two initial states of the lattice, either evenly distributed spins identical to an inverse temperature of (β = 0) or all spins aligned in the same trend identical to (β = ∞), and then applying the Monte Carlo algorithm of choice until equilibrium is reached at a given inverse temperature (β), beginning from these primary cases [18]. The technique used to predict this time τ of equilibrium was to analyze the conduct of the total energy and magnetization of a ($L\times L$= 16, 32) square lattice sizes of ferromagnetic ($NiOFe_2O_3$) using the Metropolis algorithm as these achieve a stable state of equilibrium as shown in figures (3,4). It can be seen that equilibrium has been reached for all temperatures at τ =200, τ =650 lattices sweeps (steps per site) of a ($L\times L$= 16, 32) square lattice sizes of ferromagnetic ($NiOFe_2O_3$) respectively. We can see clearly that time of thermalization will rise with (β) and that equilibrium is reached much quicker while starting from a polarized state.

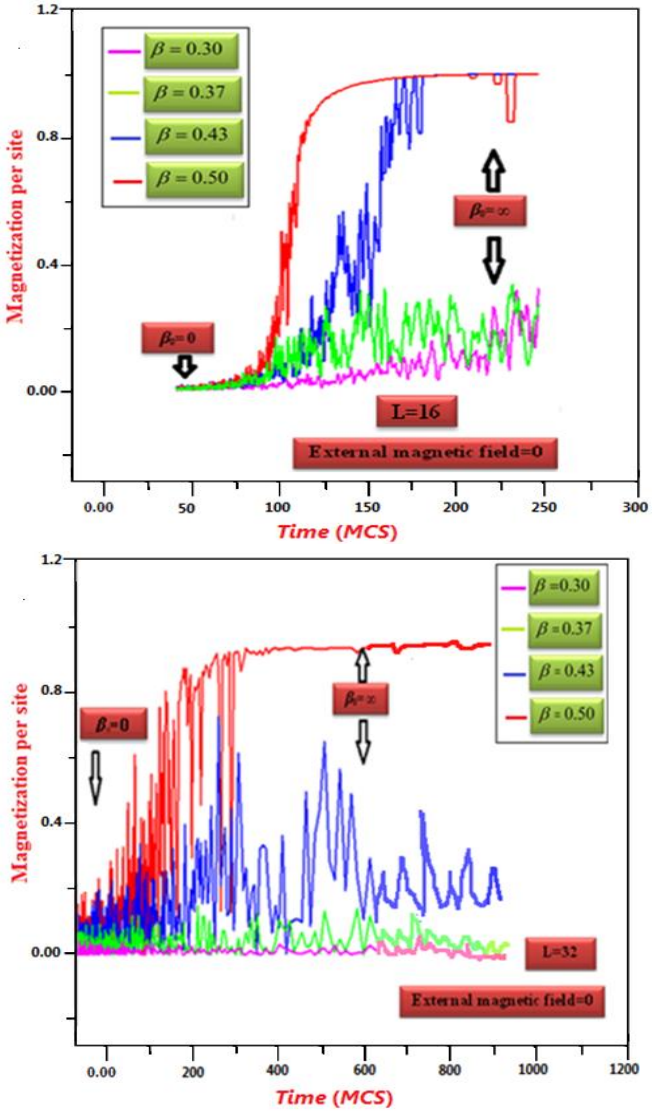

Figure 3: Thermalization processes of the total magnetization M of a NiOFe2O3 at various inverse temperature in an external magnetic field (B=0).

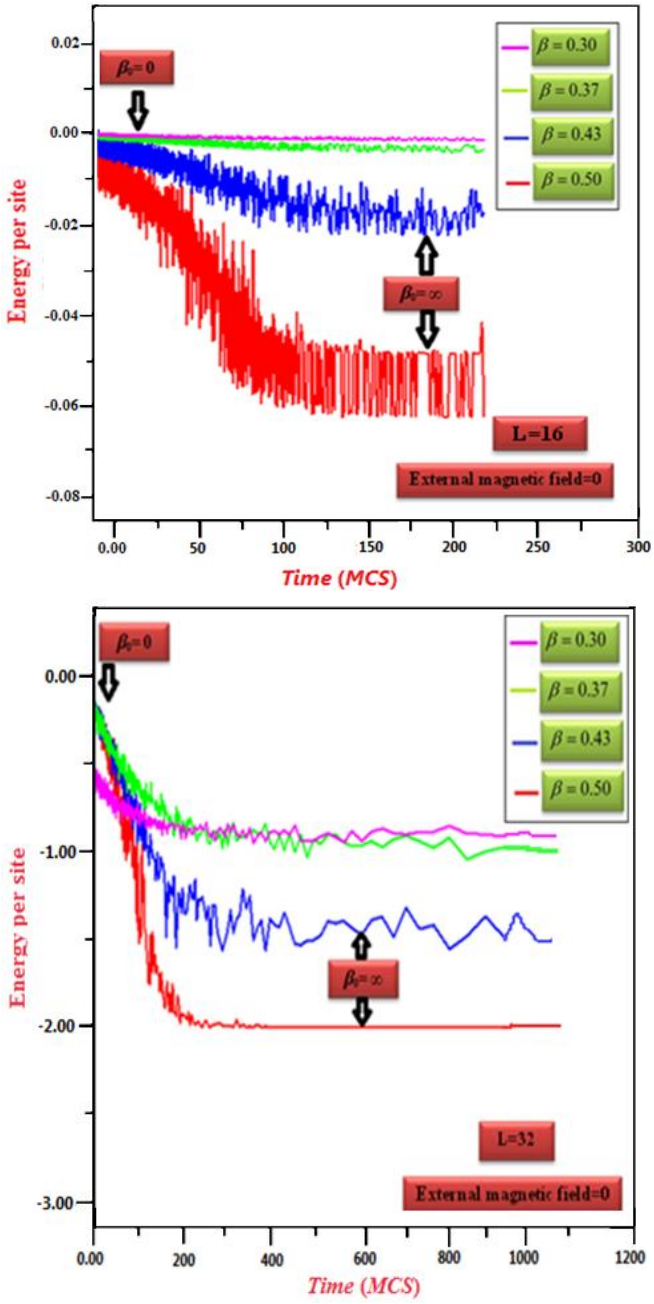

Figure 4: Thermalization processes of the total energy E of a $NiOFe_2O_3$ at various inverse temperature in an external magnetic field (B=0)

The impact inverse temperature on magnetization and energy of ($NiOFe_2O_3$).

Figures (5,6) show the impact of inverse temperature at β_C=0.435 on energy and magnetization with a great range of spin-lattice interactions ($L\times L$=16) of ferromagnetic ($NiOFe_2O_3$). It should be observed that stage transition is already important if the scale is sufficiently big. Figure (5) obviously indicates a transition stage under $\beta_C J$=0.435 for ($L\times L$=16) lattice size without the magnetic field ($B=0$). At higher inverse temperatures βJ (lower temperatures), the regime highly prefer the ground cases. There are instances where spins line up either to up and the magnetization becomes +1 or down and the magnetization becomes -1. At lower inverse temperatures βJ (higher temperatures) than a stage transition, the spins tend to line up randomly, which outcomes in (Magnetization=0). This corresponds to extreme temperature, since ($J= -E/K_BT$) which implies that (J) is inversely proportional to T. Nevertheless, the spins tend to align as the inverse temperature increases. The transition stage is much more evident in figure (6). We start with a ($L\times L=N^2$) square spin lattice and observe how the system evolves across ($2^{L\times L}=2^{16\times 16}$ ($\approx 1.1579\times 10^{77}$)) step to achieve balance. With growing inverse temperature, the division of the line is likely to become more evident owing to the different energy of the lattice with domains, which means the magnetization be zero ($M=0$), as well as a full magnetization lattice of (-1 or 1). Figure (7) shows the relationship between magnetization and energy for ($L\times L$=16) square lattice size of ferromagnetic ($NiOFe_2O_3$). We will see that two low-energy zones are the identical for spin up and spin-down orientation. Low-energy cases are also not exhibiting high net magnetization. It occurs when there are comparatively large clusters of spins both up and down. This is often similar to ferromagnetes domains. It is sensible to note that reduced energy outcomes from net magnetization or domains formation.

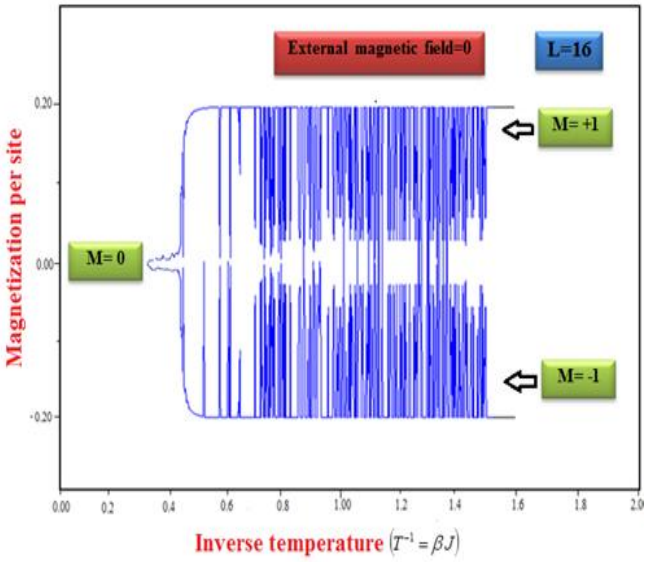

Figure 5: Magnetization per site of a $NiOFe_2O_3$ as function of inverse temperature for $L^2=N\times N$ square lattice size in an external magnetic field (B=0).

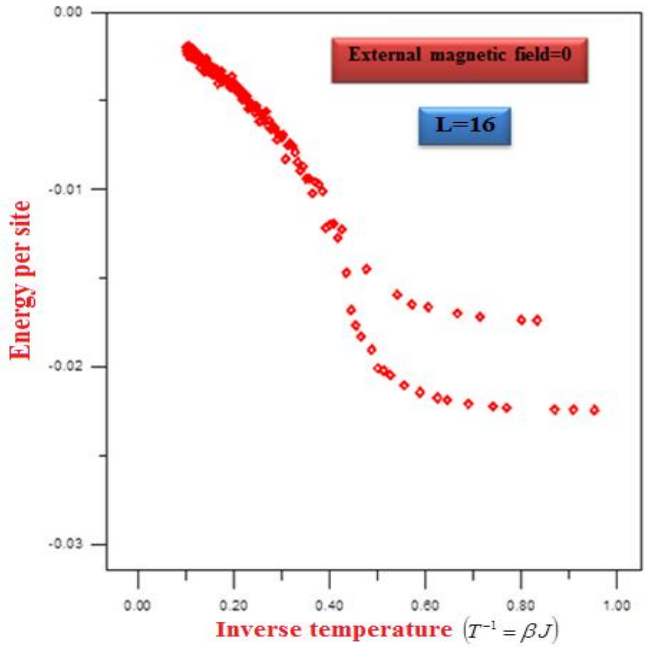

Figure 6: Energy per site of *a* $NiOFe_2O_3$ as a function of inverse temperature for $L^2=N\times N$ square lattice size in an external magnetic field (B=0)

In additionally, it is interesting to draw magnetization versus energy, which is completed in Figure (7). The zones of higher density refer the four states. Concerning (M=±1.0, E=-2.0) we find the two high inverse temperature stage ground states; centered at (*M=0, E=-1.2*) we find the high inverse temperature semi settled states; and concerning (*M= 0, E=-0.4*) we find the low inverse temperature stage.

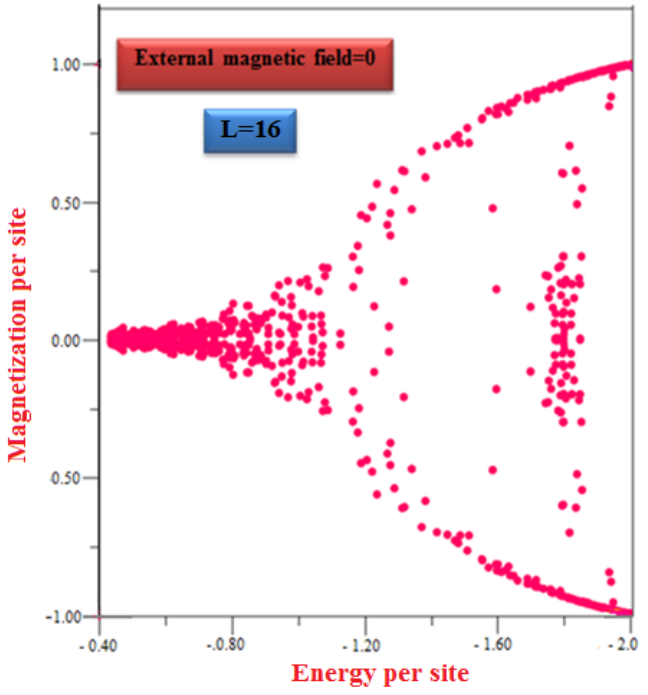

Figure 7: Magnetization per site of a $NiOFe_2O_3$ as a function of energy for $L^2=N\times N$ square lattice size in an external magnetic field (B=0)

Conclusion

The Monte Carlo method implemented in the Ising Model that prescribes the magnetic characteristics of a ferromagnetic a ($NiOFe_2O_3$), permits to attain the thermodynamic quantities variation with a ($L\times L$= 4, 8, 16, 32) various lattice sizes at Curie region ($\beta_C J$=0.435). The square-lattice size (L^2) Ising Model is the simplest system showing partial transitions (the transition between the ferromagnetic phase and the paramagnetic section) and critical conditions at limited temperatures. Under a certain temperature (critical inverse temperature βc=0.435 K_B/J), the material will be in the paramagnetic state, which means that the average magnetization decreases and the average energy increases, while above a certain temperature (critical inverse temperature βc=0.435 K_B/J) the material will be in the case of ferromagnetic, thus, the average magnetization increases and the average energy decreases. Moreover, under a certain temperature (critical inverse temperature βc=0.435 K_B/J), spontaneous magnetization is zero.

- There are two explanations in a wider lattice for an increased computational time:

1. Since the flipping of spins is greater, it is important to settle on more particles.
2. This takes longer for a larger system to achieve a balance which that has to permit the system to develop over a greater number of steps. Because the computations are more demanding in this situation.

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Appendix (A)

Table 1: Calculation magnetization as a function of inverse temperature Beta at ($L \times L = N^2$) lattice sizes in external an magnetic fielf ($B=0$).

*	A	B	C	D	E
1	beta	ave_magnetization8*8	ave_magnetization16*16	ave_magnetization32*3	
2	0.25	2.230562E-2	1.005891E-2	1.369705E-2	
3	0.2631579	3.100313E-3	5.949742E-2	5.014297E-3	
4	0.2777778	2.565062E-2	2.075461E-2	3.993836E-2	
5	0.2941176	1.679344E-2	0.0154082	4.101594E-2	
6	0.3125	1.963688E-2	4.482773E-2	1.235457E-2	
7	0.3333333	2.569781E-2	1.236836E-2	8.661707E-2	
8	0.3571429	8.215313E-2	6.494766E-3	2.110117E-2	
9	0.3846154	0.1351894	0.2851027	0.129877	
10	0.4166667	0.2997716	0.2305326	0.2485087	
11	0.4545454	0.1854881	0.8167112	0.8054909	
12	0.5	0.916545	0.9223817	0.9994117	
13	0.5555556	0.9589047	0.9592512	0.9451892	
14	0.6250001	0.980031	0.9808226	0.9997045	
15	0.7142857	0.9914312	0.9994117	0.9909622	
16	0.8333334	0.9967628	0.9973462	0.977351	
17	1	0.9995225	0.9974266	0.9502508	
18	1.25	0.9999434	1	0.9994117	
19	1.666667	1	1	0.9997045	
20	2.5	1	1	0.9909622	
21					
22					
23					
24					
25					
26					
27					
28					
29					
30					
31					
32					

Table 2: Calculation energy as a function of Time of the Monte Carlo Step (MSC) at ($L \times L = N^2$) lattice sizse in external an magnetic fielf ($B=0$)

	Time	ave_energy 4*4	ave_energy 8*8	ave_energy16*1	ave_energy32*32
2	0.25	-3.776656E-2	-8.336484E-3	-1.894883E-3	-8.143603E-4
3	0.2506266	-0.039295	-0.076172E-3	-2.585156E-3	-6.621631E-4
4	0.2512563	-4.276156E-2	-9.868594E-3	-2.409531E-3	-6.598535E-4
5	0.2518892	-3.820906E-2	-8.051407E-3	-2.56918E-3	-5.534619E-4
6	0.2525252	-4.151125E-2	-9.859063E-3	-9.043555E-4	-5.370508E-4
7	0.2531646	-3.929375E-2	-8.938047E-3	-2.096504E-3	-7.519678E-4
8	0.2538071	-4.151937E-2	-9.807422E-3	-2.356719E-3	-8.207178E-4
9	0.2544529	-0.039025	-8.132813E-3	-2.711152E-3	-2.820557E-4
10	0.255102	-4.040219E-2	-0.592344E-3	-2.954844E-3	-7.05205E-5
11	0.2557545	-3.994656E-2	-9.127109E-3	-2.576016E-3	-2.438184E-4
12	0.2564102	-4.075344E-2	-7.63125E-3	-2.173164E-3	-7.130176E-4
13	0.2570694	-4.407125E-2	-7.913594E-3	-1.884434E-3	-4.67334E-5
14	0.2577319	-4.184062E-2	-1.066937E-2	-2.407988E-3	-3.555908E-4
15	0.2583979	-4.290594E-2	-8.93375E-3	-2.373164E-3	-6.573779E-4
16	0.2590674	-4.353969E-2	-9.560469E-3	-2.112656E-3	-3.830615E-4
17	0.2597403	-0.03992	-9.753281E-3	-2.493066E-3	-2.605127E-4
18	0.2604167	-4.483937E-2	-9.396954E-3	-2.199707E-3	-4.639697E-4
19	0.2610966	-3.933937E-2	-0.010285	-1.337422E-3	-4.008789E-4
20	0.2617801	-0.0425875	-9.862266E-3	-2.021836E-3	-5.076465E-4
21	0.2624672	-4.404656E-2	-8.807578E-3	-2.965723E-3	-4.730127E-4
22	0.2631579	-4.573781E-2	-1.081859E-2	-2.339121E-3	-8.819336E-4
23	0.2638522	-0.0413075	-9.501406E-3	-2.638535E-3	-8.581592E-4
24	0.2645503	-4.369075E-2	-9.324141E-3	-2.038418E-3	-5.095557E-4
25	0.265252	-3.911563E-2	-9.602344E-3	-1.946738E-3	-8.609472E-4
26	0.2659574	-4.245562E-2	-8.604922E-3	-2.682031E-3	-1.606397E-4
33	0.2710027	-4.298875E-2	-9.25961E-3	-2.636777E-3	-7.021045E-4
34	0.2717391	-4.571813E-2	-9.725234E-3	-0.0019525	-6.48003E-4
35	0.2724796	-4.404188E-2	-1.062945E-2	-3.461074E-3	-2.917041E-4
36	0.273224	-4.501281E-2	-0.0115068	-1.851777E-3	-1.802197E-4
37	0.2739726	-4.212406E-2	-1.094992E-2	-2.770605E-3	-6.696093E-4
38	0.2747253	-4.601562E-2	-9.624922E-3	-2.178106E-3	-8.96001E-4
39	0.2754821	-4.428687E-2	-9.186407E-3	-2.792363E-3	-6.289063E-5
40	0.2762431	-4.620437E-2	-1.057789E-2	-2.513496E-3	-3.523096E-4
41	0.2770083	-4.643875E-2	-9.142031E-3	-3.670781E-3	-4.866162E-4
42	0.2777778	-4.496531E-2	-1.020594E-2	-2.187598E-3	-6.346289E-4
43	0.2785515	-4.801781E-2	-9.40711E-3	-2.197793E-3	-9.440185E-4
44	0.2793296	-4.814656E-2	-1.009461E-2	-2.757187E-3	-7.883447E-4
45	0.2801121	-4.437063E-2	-9.955547E-3	-3.200371E-3	-4.381592E-4
46	0.2808989	-4.692281E-2	-9.941953E-3	-2.479141E-3	-3.551269E-4
47	0.2816902	-4.743813E-2	-1.128305E-2	-2.532793E-3	-5.644287E-4
48	0.2824859	-4.452937E-2	-1.048906E-2	-2.10252E-3	-2.931006E-4
49	0.2832861	-5.005594E-2	-1.025867E-2	-2.223828E-3	-9.957813E-4
50	0.2840909	-4.579969E-2	-8.899531E-3	-2.463242E-3	-1.914844E-4
51	0.2849003	-5.097562E-2	-1.145633E-2	-2.734043E-3	-8.173438E-4
52	0.2857143	-4.621375E-2	-9.58289E-3	-2.644434E-3	-1.240532E-3
53	0.2865329	-4.794406E-2	-1.006266E-2	-2.461934E-3	-6.733057E-4
54	0.2873563	-5.202969E-2	-8.513828E-3	-2.868203E-3	-8.354444E-4
55	0.2881844	-4.499875E-2	-1.005781E-2	-2.808223E-3	-5.66294E-4
56	0.2890173	-5.177625E-2	-1.048094E-2	-2.537969E-3	-7.253418E-4
57	0.2898551	-4.957531E-2	-0.0100057	-2.096488E-3	-9.567373E-4
58	0.2906977	-5.404313E-2	-1.033805E-2	-2.309609E-3	-8.287158E-4
59	0.2915452	-4.950313E-2	-1.080766E-2	-3.33916E-3	-8.428271E-4
60	0.2923976	-5.127063E-2	-9.592422E-3	-3.769492E-3	-6.111963E-4
61	0.2932551	-5.496344E-2	-1.127133E-2	-2.898379E-3	-3.336133E-4
62	0.2941176	-5.274906E-2	-1.078602E-2	-2.583887E-3	-1.131103E-3
63	0.2949852	-4.808313E-2	-1.153359E-2	-1.913694E-3	-8.422461E-4
64	0.295858	-5.112125E-2	-1.266344E-2	-1.535449E-3	-6.050977E-4
65	0.2967359	-5.101312E-2	-1.017734E-2	-2.133789E-3	-2.658057E-4
66	0.297619	-5.324937E-2	-1.139328E-2	-3.362891E-3	-6.020166E-4
67	0.2985075	-5.226344E-2	-1.185266E-2	-2.880449E-3	-7.472754E-4
68	0.2994012	-5.219406E-2	-1.085797E-2	-3.300371E-3	-7.095703E-4
69	0.3003003	-5.398031E-2	-1.110406E-2	-2.225078E-3	-6.777051E-4
70	0.3012048	-5.497719E-2	-0.0107218	-2.951465E-3	-1.07895E-3
71	0.3021148	-5.321813E-2	-1.120695E-2	-1.968106E-3	-1.071148E-3
72	0.3030303	-5.340875E-2	-1.051602E-2	-3.259063E-3	-5.430566E-4
73	0.3039514	-5.434531E-2	-1.191656E-2	-2.342715E-3	-9.457861E-4
74	0.3048781	-0.052965	-1.360266E-2	-2.536484E-3	-3.174414E-4
75	0.3058104	-5.481625E-2	-1.186234E-2	-3.157949E-3	-1.675781E-5
76	0.3067485	-5.097719E-2	-1.115094E-2	-3.550703E-3	-7.603076E-4
77	0.3076923	-5.122906E-2	-1.361992E-2	-3.206758E-3	-7.902734E-4
78	0.308642	-5.559281E-2	-1.100406E-2	-2.466816E-3	-9.954785E-4
79	0.3095975	-5.281875E-2	-1.152016E-2	-2.918457E-3	-9.109522E-4
80	0.310559	-5.902906E-2	-0.0109167	-3.380977E-3	-6.699951E-4